PMU-based model-free approach for short term voltage stability monitoring

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Abstract—In this paper, we propose a model-free approach for short term voltage stability monitoring of power system. Our data-driven approach make use of Phasor Measurement Units (PMU) data to generate the stability certificate for verifying the voltage stability. We employ Lyapunov exponents, a stability tool adapted from ergodic theory of dynamical system, to generate the stability certificate. The time-series voltage data from PMU is used for the online computation of Lyapunov exponent. The proposed method can not only be used to determine the voltage stability of the entire system but can also be used to determine stability/instability contribution of individual buses to the overall system stability. Simulation results are presented on WSCC nine bus system using different load models.

Index Terms—Phasor Measurement Units, Online stability monitoring, Lyapunov exponents, Short term voltage stability.

I. INTRODUCTION

The introduction of Phasor measurement technology in the late 1980s has provided with an opportunity to modernize electric power grid by enabling wide area monitoring, detection, protection, and control of power grid [1], [2]. Phasor Measurement Unit (PMU) provides time-stamped synchronized phasor measurement of system voltage, current, and power throughout power grid.

There is an increased research efforts to use PMU for improved operation and design of electric power grid [3], [4]. Potential applications ranges from monitoring in real-time, system state estimation, and wide-area control. This paper focus on the real-time monitoring application of power grid. In particular, we propose a novel approach for real-time short-term voltage stability prediction of electric power grid.

Voltage stability refers to the ability of the power system to maintain steady voltages at all the buses following a fault or disturbance. Voltage stability problem for power system can be classified in two categories namely small disturbance and large disturbance voltage stability. Based on the time frame of interest voltage stability is further classified into short term and long term phenomenon [5]. Short term large disturbance voltage stability is an increasing concern for industry because of the increasing penetration of induction motor and electronically controlled loads. The short term voltage instability is mainly caused by stalling of induction motor loads, which draws six times reactive power than that of the nominal one, following a contingency within a time frame of few seconds. Most of the research work in short term voltage stability is focused on the dynamic modeling and aggregation of fast acting load components like induction motor loads. The description of various past incidents and analysis of short term voltage stability problem is given in [6]. The use of synchronized phasor measurements to detect long term voltage instability based on wide area monitoring is presented in [7], and methods based on local measurements is given in [8]. The methods based on local measurements rely on Thevenin impedance matching criteria. However these methods are not directly applicable for online short term large disturbance voltage stability monitoring. Transient stability analysis using Lyapunov exponent for a finite time span was proposed in [9]. First, model parameters (e.g. real power injection, damping etc) are estimated using least square optimization from the available PMU data over a fixed time span. The estimated model parameters are used for calculation of the Lyapunov exponents. This method is computationally expensive and stability could be ascertained only after a certain time window. Moreover, system parameters are assumed to be constant during the interval, for which the transient stability of the system is predicted.

Short term voltage stability analysis of power system is a challenging problem because of the non-equilibrium nature of dynamics that are involved during the transient phase. The system state is perturbed away from equilibrium for short time period following a fault or disturbance. Due to non-equilibrium nature of dynamics that are involved during the transient phase, we require non-equilibrium notion of stability. The non-equilibrium notion of stability that we use in this paper is adopted from ergodic theory of dynamical system and is captured by Lyapunov exponents. In particular, if the maximum Lyapunov exponents of the system is negative (positive) then the nearby trajectories of the system converges (respectively diverges) and hence stable (respectively unstable) system dynamics. The use of Lyapunov exponents for transient rotor angle stability of power system is proposed in [9], [10]. In particular, the system Lyapunov exponent was computed using the data from the PMU and the system model. In this paper, we propose the application of PMU data for short term voltage stability monitoring and provide data-driven, model-free, approach for online computation of Lyapunov exponents.

Our objective is to make use of Phasor Measurement Units (PMU) data for the online computation of the Lyapunov exponents. One of the important contributing factor to the voltage instability is the system load [5], [6]. Typically load models that are used for voltage stability analysis are not accurate due to uncertainty associated with the load and unmodeled dynamics. The lack of appropriate load model is one of the main challenge in the development of model-based method for short term voltage stability analysis. Our
proposed model-free approach for short term voltage stability monitoring circumvent this problem.

There are three main contributions of this paper. 1) We adapt a stability notion from ergodic theory of dynamical system to develop data-driven approach, employing real-time data from PMU devices, for short term voltage stability monitoring. 2) Extensive simulations are performed on a WSCC nine bus system with different load models to verify the applicability of the developed method. 3) Our proposed method is also used to provide information on the instability (stability) contributions of the individual buses to overall system instability (stability). The information regarding the contributions of individual buses to overall system stability can be used to determine appropriate local control action to prevent the fault propagation to the entire system.

The organization of the paper is as follows. The mathematical preliminaries the Lyapunov exponent and our proposed approach for its online computation are presented in section II. Simulation results are presented in section III followed by conclusions in section IV.

II. MATHEMATICAL PRELIMINARIES: LYAPUNOV EXPONENT

Our proposed stability notion is adapted from ergodic theory of dynamical system and is captured by Lyapunov exponents [11]. If the maximum Lyapunov exponent of the system is negative (positive) then the nearby system trajectories will converge (respectively diverge) to each other. For a continuous time dynamical systems if the maximum Lyapunov exponents of the systems are negative, then the steady state dynamics of the system consists of stable equilibrium point. Lack of stability as implied by positive value of Lyapunov exponent is shown to be responsible for the chaotic behavior and hence unstable dynamics. However, the focus of this paper is not to use Lyapunov exponents for the analysis of chaotic dynamics but for short term stability prediction in the form of divergence of nearby system trajectories. We now provide mathematical definition of maximum or principal Lyapunov exponent [11].

Definition 1. Consider a continuous time dynamical system \( \dot{x} = f(x) \), with \( x \in X \subset \mathbb{R}^N \), let \( \phi(t,x) \) be the solution of differential equation. Define the following limiting matrix

\[
\Lambda(x) = \lim_{t\to\infty} \left[ \frac{\partial \phi(t,x)^T}{\partial x} \frac{\partial \phi(t,x)}{\partial x} \right] \tag{1}
\]

Let \( \Lambda_i(x) \) be the eigenvalues of the limiting matrix \( \Lambda(x) \), then the Lyapunov exponents \( \lambda_i(x) \) are defined as

\[
\lambda_i(x) = \log \Lambda_i(x) \tag{2}
\]

Let \( \lambda_1(x) \geq \lambda_2(x) \cdots \geq \lambda_N(x) \), then \( \lambda_1(x) \) is called the maximum Lyapunov exponent.

Using results from Multiplicative ergodic theorem it is known that the limit in [12] and [13] is well defined. Furthermore the limit in [12] is independent of initial condition \( x \) under the assumption of unique ergodicity of the system. Roughly speaking Lyapunov exponents can be thought of as the generalization of eigenvalues from linear systems to nonlinear systems.

The negative value of maximum Lyapunov exponent implies exponential convergence of nearby system trajectories. Hence the Lyapunov exponent is a stability certificate for trajectories as opposed to an equilibrium point. This stability property of trajectories captured using Lyapunov exponent makes them ideal candidate for short term stability analysis where the system state is away from equilibrium point. Although Lyapunov exponent is a stability certificate of trajectories it has important consequence on the steady state system dynamics. In particular, if the maximum Lyapunov exponent of the system is negative then the steady state dynamics of the system will consist of stable equilibrium point. However in this paper, we are not interested in the asymptotic computation of Lyapunov exponent. Instead, we will compute Lyapunov exponent over finite time interval. Furthermore, instead of using system model for the computation of Lyapunov exponent we will employ time series data for the computation.

A. Computation of Lyapunov exponent from time-series data

Computation of Lyapunov exponent using time series data is proposed in [14]. Data-driven approach for the computation of Lyapunov exponent relies on the reconstruction of phase space dynamics. This higher dimensional phase space is constructed using time-delayed embedding technique [14]. Approximate system Jacobian matrix is constructed in the embedded phase space for the computation of Lyapunov exponents. One of the main challenge in the computation of Lyapunov exponents using time-series data is the determination of phase space appropriate dimension for embedding the time series data [15]. Typically, the embedding dimension is determined by the complexity of the phase space dynamics, more complex the dynamics larger the dimension of the embedding phase space. One of the computationally expensive operation in the computation of Lyapunov exponent is the construction of system Jacobian. Jacobian free approaches are also proposed for the computation of exponents [16], [17].

Most of the existing approach for calculating Lyapunov exponents using time series data are suited for off-line computation. In particular, the entire time-series data is obtained first and then processed to compute the exponents. In this paper, we propose a modification of algorithm proposed in [15], developed for off-line computation of Lyapunov exponents using small data sets. These modifications to existing algorithm are specifically made to make it suitable for on-line computation and for improved computational efficiency. In particular, we outline the following algorithm for the online computation of maximum Lyapunov exponent.

Algorithm

1) Let \( V_t \in \mathbb{R}^n \) be vector valued time series data for \( t = 0, \Delta t, 2\Delta t, \ldots \), where \( \Delta t \) is the sampling period.

2) For a fixed small number \( \epsilon \), choose integer \( N \) such that

\[
0 < \| V_{m\Delta t} - V_{(m-1)\Delta t} \| < \epsilon \quad \text{for} \quad m = 1, 2, \ldots, N.
\]
3) Define the maximum Lyapunov exponent at time $k\Delta t$ using following formula

$$\Lambda(k\Delta t) := \frac{1}{Nk\Delta t} \sum_{m=1}^{N} \log \frac{\| V_m(k+m)\Delta t - V_m(k+m-1)\Delta t \|}{\| V_m\Delta t - V_m(1)\Delta t \|} \quad (3)$$

The basic idea behind Eq. (3) for Lyapunov exponent computation using time-series data is to take $N$ initial conditions and study the evolution of these initial conditions over time. The Lyapunov exponent at time instant $t$ is then defined as distance between the initial conditions at time $t$ normalized with the distance at time instant zero and averaged over number of initial conditions. Furthermore the dimension of embedding space in the computation of Lyapunov exponent is taken to be equal to one.

We propose the use Eq. (3) for the computation of Lyapunov exponent to determine the short term voltage stability of the system. In particular, $V_t = (v_1^t, \ldots, v_N^t)$ in (3) will represent the time-series voltage data from $n$ buses. The formula in Eq. (3) can also be used for computing the Lyapunov exponent of individual buses to determine stability/instability contribution of individual buses to overall system stability/instability. The Lyapunov exponent for $i^{th}$ bus will be computed using following equation

$$\lambda_i(k\Delta t) := \frac{1}{Nk\Delta t} \sum_{m=1}^{N} \log \frac{|v_i^{(k+m)\Delta t} - v_i^{(k+m-1)\Delta t}|}{|v_i^{m\Delta t} - v_i^{(m-1)\Delta t}|} \quad (4)$$

where $v_i^{k\Delta t}$ is the voltage measurement at $i^{th}$ bus.

B. PMU measurement for computation of Lyapunov exponent

The use of PMU measurements for the computation of Lyapunov exponent using formulas (3) and (4) is relatively straightforward. The vector valued time-series data $V_t$ in our proposed algorithm could either represent the actual voltage measurement made using PMU devices located at the $n$ system buses or it could represent the combination of PMU measurement and estimated voltages at the buses with no PMU devices. Given the fact that the total numbers of PMU still amounts to less than one percent of the number of buses in electric power grid. It is important that the method developed for online stability monitoring does not crucially depend on the availability of system state information from all the buses of power grid. Our proposed method does not suffer from this limitation. In particular, short term voltage stability prediction can be made based on the available system state information from PMU devices. Typically the system state information at the buses where the PMU devices is not placed is estimated using some estimation algorithm [18]. This state estimation process is fundamentally limited by the observability property of the electric power grid, i.e., system states at the particular bus with no PMU devices can only be estimated if the power grid is observable for the given placement configuration of the PMU device. Our proposed method for short term voltage stability monitoring employing PMU measurements is also restricted with the same fundamental limitation. In particular, Lyapunov exponent computed using voltage measurements from PMU devices at $n$ buses will provide stability information of all the buses whose states can be estimated using $n$ PMU measurements.

III. SIMULATION RESULTS

In this section, we present simulation results to demonstrate the application of our proposed approach on WSCC nine bus system.

A. Simulation results with different load models

The WSCC nine bus system has three generators, three loads and six transmission lines and is shown in Fig. 1. The power flow and dynamic data is available in [19]. Each of the three generators are represented by GENROU models with IEEET1 exciters. Load modeling is very important for short term voltage stability analysis. Therefore, four sets of scenarios are considered for load representation; viz., constant impedance, complex load models (CLOD), detailed induction motor model (CIM5) and composite load model (CMDL) . As we see in the simulation results, the short term voltage stability critically depends upon the load model. In particular, for a fixed fault clearing time we see that the system voltage with composite load model is short term unstable but is short term stable with constant impedance load. This fact highlights one of the main advantages of our proposed model-free approach for short term voltage stability analysis. We next present simulation results using different load models.

1) Constant Impedance Load: In Figs. 2 and 5, we show the time domain simulation results with constant impedance load model at bus 5, 6 and 8 of the nine bus system. A three phase fault is created near bus 7 at the end of line 7—5. The critical clearing time for this three phase fault is known to be equal to $t^*_c = 1.079$ sec. . The time-domain simulation result for various bus voltages with fault clearing time $t_c = 1.06 < t^*_c$ in shown in Fig. 2. As expected, we see that the system is stable and various bus voltages settles to a new steady state value after small initial transience. The stable behavior of the system voltage dynamics is verified by the negative value of the system Lyapunov exponent plot as shown in Fig. 3. In
Fig. 2. Time domain voltage simulation at various buses with fault clearing time $t_{cl} = 1.06 \ sec < t_{cr} = 1.079 \ sec$. with constant impedance load.

Fig. 3. System Lyapunov exponent (LE) evolution with fault clearing time $t_{cl} = 1.06 \ sec < t_{cr} = 1.079 \ sec$. with constant impedance load.

Fig. 4. LE evolution at individual buses with fault clearing time $t_{cl} = 1.06 \ sec < t_{cr} = 1.079 \ sec$. with constant impedance load.

Fig. 5. Time domain voltage simulation at various buses with fault clearing time $t_{cl} = 1.083 \ sec > t_{cr} = 1.079 \ sec$. (Using constant impedance load).

An unstable scenario was created by clearing the three phase fault at bus 7 at time $t_{cl} = 1.083 \ sec$, which is greater than $t_{cr}$. The time domain response of the various bus voltages in Fig. 5 clearly show the oscillatory instability of the system voltages. The positive value of system Lyapunov exponent plot in Fig. 6 clearly reflect the voltage instability. Furthermore the Lyapunov exponent plots of the individual buses in Fig. 7 provide information about the relative instability contribution of individual buses to overall system instability.

2) Composite Load Model (CLOD): The simulation results from the section III-A1 are repeated in Figs. 8-10 with composite load models. The composite load model includes six categories of load and the average load split values are used for this simulation [6]. The percentage of different categories of load are small induction motors (45%), large induction motors (15%), discharge lighting (20%), transformer saturation current (1%), constant MVA (5%), and a combination of constant current and constant impedance loads (14%).

In Fig. 8, we notice that the three phase fault at bus 7 that was stable with constant impedance model and fault clearing time of 1.06 sec. is now unstable with composite load model. Any model-based approach for short term voltage stability monitoring will provide wrong conclusion about system stability if correct load models are not used. The loads are known to be one of the most common sources of uncertainty in power system. Hence it is difficult to obtain accurate load models thereby highlighting the limitations of the model-
based approach for voltage stability analysis. Our model-free approach based on the computation of Lyapunov exponent from time series data does not suffer from this limitation.

3) Composite Load Model (CMDL): Here we simulate the dynamic behavior of an aggregate of three-phase motors, a single-phase air conditioner motor, electronic loads and static loads connected to a low-voltage load bus and the response is reflected at the high voltage bus. The percentage of three-phase motors driving constant torque loads is 7.2%, three-phase motors driving torque speed-squared loads with high inertia is 7.2% and single phase air conditioner load is 45%. This model captures the dynamics of loads more accurately under severe fault conditions. Fault occurring time and clearing time is 1 sec and 1.066 respectively. Here also we observe unstable behavior and which is also reflected in exponents. Fig. 11 and Fig. 12 show evolution of bus voltages and $\Lambda(t)$ with time. Here it stays above 0 line showing unstable behavior.

4) Induction Motor Model (CIM5): The percentage of induction motor in the system is one of the important factors which determine voltage recovery or collapse. In this load model, the rotating load dynamics and electromagnetic dynamics of the motor are represented in detail. Without the use of this detailed induction motor model, this problem of voltage recovery or collapse cannot be captured. Also, it is very difficult to identify the parameters of this aggregated induction motor model. The amount of system load represented by small motors and large motors are 45% and 15% respectively. The parameters used for this simulation can be found in [20]. For the bus 7 fault described in section III-A1, and a fault clearing time of 1.066 sec the voltage response is shown in Fig. 13. It can be observed from Fig. 13 that the degree of instability in the system substantially increases after 3.5 s. It is captured by exponent, which shoots up to 1 as depicted in Fig. 14.
B. Contribution of individual bus voltage towards overall system stability

Our proposed model-free approach for short term voltage stability monitoring can also be used to determine stability contribution of individual bus to overall system stability. In particular Eq. (4) is used to compute the Lyapunov exponent for individual bus. Larger value of Lyapunov exponent at a particular bus will imply that the bus voltage is more unstable compared to other bus. For example, in Figs. 7 and 10, we plot the Lyapunov exponents for individual bus voltages for constant impedance and CLOD load respectively. From Fig. 7, we see that the voltage at bus 6 is most unstable among all the buses. We believe that the information on the relative stability/instability contribution of the individual bus to overall system can be used to make decision on appropriate local control action to prevent further spread of cascade failures.

C. Computational consideration for online implementation

The simulation results obtained in the previous section are performed on desktop computer using MATLAB based codes. We use four samples per cycles for the computation of Lyapunov exponent. The sampling rate of four samples per cycle is much smaller than the typical sampling rate for the commercially available PMU devices. We expect the stability prediction to be more accurate with faster sampling rate. Only computation operation that is required for the computation of Lyapunov exponent is vector multiplication, which can be performed relatively quickly. Hence our proposed approach for stability monitoring is amicable to online implementation. Furthermore the approach can be easily extended for stability monitoring of large size system. With regard to Eq. (3), the number of initial conditions is taken to be equal to 30 control [21]. Our proposed approach, involving Lyapunov exponents is naturally suited to exploit the recent advance in PMU technology for wide area monitoring and control of power system.
(i.e., $N = 30$). With four samples per cycle, we have to wait for seven cycles (i.e., 50 msec) before we start computing the Lyapunov exponent. Clearly this waiting period will be considerably smaller with the increase in the sampling rate. 

The total time interval over which the Lyapunov exponents were computed is equal to five seconds. Ideally, for the purpose of online implementation the Lyapunov exponent needs to be computed over a finite time window. The appropriate size of this window will be crucial in making reliable, accurate, and timely stability prediction. In particular, smaller size window will lead to inaccurate stability prediction while larger size window will lead to accurate but untimely prediction. We expect the optimal window size to be function of fault characteristics. Deciding appropriate window size and other computational issues are being currently investigated.

### IV. CONCLUSION

In this paper we presented a novel model-free approach employing PMU data for voltage stability monitoring of power system. The novel approach employs Lyapunov exponent, a stability tool adapted from ergodic theory of dynamical system, for short-term voltage stability prediction. Our proposed approach is amicable to various extension. We have successfully used proposed method for short term stability analysis of a larger power system (IEEE 162 bus system). It has also been used to experimentally determine critical clearing times, corresponding to various fault locations, for both WSCC 9 bus system and IEEE 162 bus system. For space constraints we leave corresponding simulation studies for future presentation. Issues like appropriate choice of time window would be addressed in detail there. Our future research efforts will focus on employing the new stability metric for the purpose of local control design based on relative degree of instability of individual buses in the power grid. The proposed framework can also be used to develop model-free approach for short term rotor angle monitoring in real-time using PMU data.

### REFERENCES


