Global stability and control analysis of axial compressor stall and surge phenomena using bifurcation methods

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Abstract: Active controllers that allow the compressor to operate safely at its peak pressure rise point by preventing surge and controlling entry into rotating stall need to be globally stabilizing. Bifurcation methods have emerged as a useful tool for the analysis of global stability and control of nonlinear dynamical systems such as the axial compressor. In this paper we show how bifurcation analysis can be used effectively to obtain answers to questions of global stability of the compressor dynamics. Since the bifurcation method does not directly provide information on transient behaviour, we demonstrate how bifurcation results need to be carefully interpreted to be of use when dealing with practical systems. We then present a novel nonlinear bifurcation-based stall/surge controller that is globally stabilizing. With such a controller, it becomes possible to avoid surge entirely and to prevent abrupt entry into rotating stall. The controller also eliminates the hysteresis between entry into and recovery from a rotating stall, and maintains system stability under all perturbations, small and large.

Keywords: compressor stall and surge, global stability, control, bifurcation methods

NOTATION

K_1, K_2	controller gain parameters
R	square of the amplitude of first mode rotating stall
β	parameter related to the Greitzer 'B' parameter
γ	parameter representing throttle area
σ	constant, fixed at 7
φ	nondimensional mass flow coefficient
ψ	nondimensional compressor pressure rise
ψ_{c0}	constant, fixed at 1.67

Superscript

$'$	derivative with respect to nondimensional time

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Subscripts

0	nominal value under closed-loop conditions
c	compressor
T	throttle

1 INTRODUCTION

Axial compressors are designed to operate under conditions of steady axisymmetric flow, where the pressure rise across the compressor is a function of the compressor mass flow rate. Throttling down the mass flow results in higher blade loading and a larger compressor pressure rise, but only up to a point. Beyond this point, lower mass flow rates result in stalled flow at one or more circumferential blade stations, resulting in the phenomenon of rotating stall. These regions of stalled flow, called stall cells, rotate at a fraction of the compressor rotor speed, thus loading and unloading the blades as they pass in and out of the stall cells. The presence of stall cells results in a loss of pressure rise and creates circumferential asymmetry in the flow through the compressor. In addition, the annulus averaged mass flow, and hence the pressure rise, may show a time periodic oscillation along the axis of the compressor. This phenomenon is called classic surge. In extreme cases, the oscillation of the averaged flow quantities along the compressor axis may be severe (even reverse flow over part of the cycle) with mild or insignificant rotating stall behaviour. This is called deep surge. Besides the obvious loss in compressor pressure rise and the possibility of engine flame out, stall and surge can damage compressor blades and can cause substantial over-temperatures in the burner and turbine. Thus, stall and surge act as primary design constraints in axial flow compressors, effectively reducing gas turbine engine performance.

The traditional approach to avoid stall and surge has been to impose a surge margin, which discourages operation near the unstable region. However, since the desired point of axisymmetric operation with maximum pressure rise occurs at the verge of this unstable region, use of the surge margin, unfortunately, precludes operation near the peak pressure rise point, thus limiting the compressor performance. Instead, recent research has focused on the use of active control techniques to suppress rotating stall and surge in order to extend the stable operating range, and to improve engine performance by allowing the compressor to operate near the peak pressure rise point without the danger of entering rotating stall or surge. Most of the developments in the field of compressor flow stability and active control over the last two decades have been reviewed in a recent paper [**1**]. An introduction to the problem of compressor stall/surge modeling and control is available in reference [**2**]. An overview of the various control techniques applied to compressor stall/surge dynamics has been provided in [**3, 4**].

Most of the theoretically developed stall/surge control strategies have been based on the one-mode approximation of the Moore–Greitzer (MG86) model [**5**]. (A list of existing compressor flow dynamics models and their properties is given in reference [**2**].) The one-mode MG86 model consists of three first-order ordinary differential equations in three variables (representing compressor pressure rise, mass flow, and the amplitude of the first mode of rotating stall) and two parameters (representing the compressor rpm and throttle area). Many studies using the one-mode MG86 model have chosen the pressure rise and mass flow as sensed variables, and the throttle parameter as the control variable for implementation of their stall/surge control law. A recent study suggests that a close-coupled valve actuator and a mass flow sensor may be the most promising choice for surge control [**6**]. However, other methods of actuation such as air injection have been successfully demonstrated in experimental programmes [**7**]. A few of the theoretically derived control laws have also been experimentally validated [**8–10**], although there are persisting concerns on the limitations imposed by sensor availability and present-day actuator bandwidths. Prompted by these concerns, linear fixed-order dynamic compensators have been proposed in reference [**11**] which, the authors claim, considerably reduce the sensing requirements for control. However, most of the control strategies developed in recent years have relied on nonlinear feedback laws for stall/surge suppression, and nonlinear bifurcation-based control appears to be capable of handling actuator rate and magnitude limits [**12, 13**].

Multi-mode MG86 models have been analysed for their dynamics and control, and there is some evidence to suggest that controllers designed for the one-mode MG86 model may not always work in the multi-mode case [**14, 15**]. There have been some attempts to examine alternate models that capture compressor dynamic effects ignored by the MG86 model; for example, control of compressors that exhibit the phenomenon of deep hysteresis [**16**], control of high-speed multi-stage compressors [**17**], and active stabilization in the presence of inlet distortion effects [**18**]. A robust disturbance rejecting control law has been devised [**19**] to handle uncertain pressure-flow maps (compressor characteristics), while a high-gain adaptive control for surge stabilization has been proposed [**20**].

It has been recognized in recent years that successful active stall/surge controllers would need to be globally stabilizing, and that, in turn, depends on the global stability properties of the axial compressor dynamic system. Jet engines often operate under noisy environments where large fluctuations are possible, and the operating point needs to be globally stable in order to recover satisfactorily from perturbations encountered under these conditions. Globally stabilizing controllers for jet engine surge and stall using a back-stepping approach have been designed [**21, 22**].

An efficient technique to compute numerically global stability information is the continuation and bifurcation method. In this method, a continuation algorithm is first used to compute all equilibrium states and periodic solutions, and their stability. Bifurcation theory is then used to predict the global stability behaviour of the system, based on the continuation results. Numerical simulation is required only for a few parameter values indicated by the bifurcation method, which provides significant savings in time and money. The bifurcation method has been applied to the analysis of compressor flow dynamics [**23–26**]. The idea that the bifurcation method can be naturally used to design nonlinear controllers to alter the global stability behaviour of dynamical systems first emerged in the early 1990s [**27, 28**]. There are obvious advantages if the globally stabilizing control could itself be derived using bifurcation methods, because the requirements for global stability are naturally posed in terms of the presence or absence of certain bifurcations. Although there have been a few papers [**29, 30**] where bifurcation-based nonlinear controllers for axial compressor flow dynamics have been devised, these controllers have not been globally stabilizing. It is therefore of interest to consider globally stabilizing controllers for the compressor stall/surge problem obtained using bifurcation methods.

In the present paper, we first show how bifurcation analysis can be effectively used to obtain answers to questions of global stability of the compressor dynamics. It must be understood that the bifurcation method does not directly provide information on transient behaviour, and bifurcation results need to be carefully interpreted to be of use when dealing with practical systems. Following this, we present a novel nonlinear bifurcation-based stall/surge controller that is globally stabilizing. This controller will permit the compressor to operate near its peak pressure rise point without the danger of surging or encountering abrupt rotating stall.

2 BIFURCATION ANALYSIS

The most widely used model for studies of axial compressor flow dynamics and control has been the MG86 model [**5**]. This model features the following set of three nonlinear ordinary differential equations, following the notation in reference [**23**]:

$$
\begin{aligned}
\varphi' &= -\psi + \psi_c(\varphi) - 3\varphi R \\
\psi' &= \frac{1}{\beta^2}[\varphi - \varphi_T(\psi)], \\
R' &= \sigma R(1 - \varphi^2 - R),
\end{aligned} \tag{1}
$$

where φ is the nondimensional mass flow coefficient, ψ is the nondimensional pressure rise of the compressor, R is the square of the amplitude of the first mode of the rotating stall disturbance, and the '$'$' refers to differentiation with respect to a suitable nondimensional time parameter. Clearly, R is non-negative, with $R = 0$ representing axisymmetric flow, while $R > 0$ indicates the presence of regions of stalled flow. The compressor characteristic, $\psi_c(\varphi)$, which represents the response of the compressor in steady axisymmetric flow, is taken to be a cubic function:

$$\psi_c(\varphi) = \psi_{c0} + 1 + \tfrac{3}{2}\varphi - \tfrac{1}{2}\varphi^3 \tag{2}$$

while the throttle characteristic $\varphi_T(\psi)$ is modeled as:

$$\varphi_T(\psi) = \sqrt{\gamma\psi} - 1 \tag{3}$$

There are two parameters in equation (1), the throttle parameter γ, which represents the area of the throttle, and the parameter β, which is related to the Greitzer 'B' parameter [**31**] and represents the compressor rotational speed. The constant ψ_{c0} is chosen to be 1.67, and σ is fixed at 7.

A complete bifurcation analysis of the MG86 model in equation (1) has been carried out [**25**]. The AUTO97 continuation and bifurcation software [**32**] has been used to carry out the computations. The computed bifurcation diagram, for pressure rise ψ with varying throttle parameter γ, for the case $\beta = 0.75$, is presented in Fig. 1.* (Similar bifurcation diagrams for the other variables, φ and R, can also be plotted.) Figure 1 contains information about all

*Note that in all bifurcation diagrams, full lines are stable equilibria, dashed lines are unstable equilibria, filled circles represent peak amplitude of stable limit cycles, empty circles denote peak amplitude of unstable limit cycles, filled squares are Hopf bifurcations, and empty squares are sub/supercritical bifurcations.

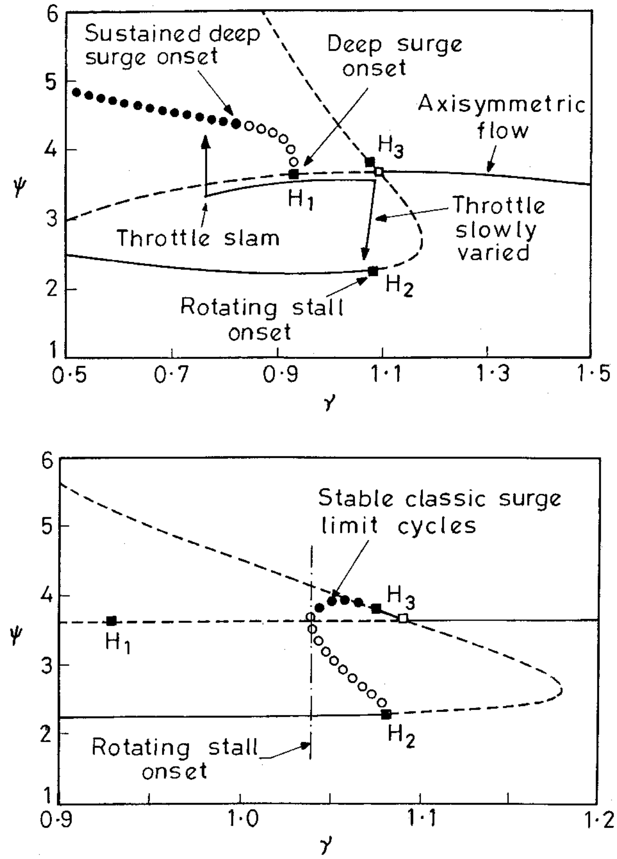

Fig. 1 Bifurcation diagram of pressure rise ψ with varying throttle parameter γ for MG86 model with $\beta = 0.75$. Arrows labeled 'Throttle slowly varied' and 'Throttle slam' indicate typical modes of entry into rotating stall and surge. Part (b) shows on enlargement with different parameter axis scale showing classic surge cycles in detail

steady states (equilibrium states and limit cycles) and their stability, and identifies bifurcation points where steady states exchange stability, new steady states are created, or existing steady states disappear. The subcritical bifurcation point represents the peak value of pressure rise where the axisymmetric flow loses stability. Limit cycles originating from Hopf bifurcation point H_1 represent deep surge with $R = 0$, while those from Hopf bifurcation point H_2 are classic surge cycles with $R > 0$. Classic surge cycles occur only for a narrow range of values of the throttle parameter γ, and are therefore plotted separately in Fig. 1b on a different parameter axis scale. Limit cycles from Hopf point H_3 are not plotted because they are found to have negative R and are therefore nonphysical (this is one of the shortcomings of the one-mode MG86 model). Bifurcation diagrams, such as that in Fig. 1, can be used to identify parameter regions of different global stability behaviour.

2.1 Steady-state behaviour

The bifurcation diagram of Fig. 1 indicates four main regions of throttle parameter γ with different global stability behaviour. To the right of the subcritical bifurcation, the axisymmetric flow is the only stable solution. To the immediate left of the subcritical point, there is a narrow region, amplified in Fig. 1b, where both rotating stall and classic surge are stable, and either steady state is possible depending on the initial conditions. Further to the left is a region where rotating stall is the sole stable solution. Finally, there is a leftmost region where both rotating stall and deep surge are stable, once again the choice of steady state depending on the initial conditions. Several predictions about the global dynamical behaviour of the MG86 model may be made based on the bifurcation diagram of Fig. 1:

1. As the mass flow is slowly throttled down past the subcritical bifurcation value, Fig. 1 predicts that the system will first show classic surge before entering rotating stall at the point marked "rotating stall onset" in the figure. If the throttle parameter is slowly increased from this point, one may conclude that, once past the Hopf point H_2, rotating stall should give way to classic surge briefly, followed by recovery to axisymmetric flow shortly after the subcritical bifurcation.
2. Both entry into rotating stall and recovery to axisymmetric flow appear to be accompanied by abrupt changes in the pressure rise (and mass flow), but hysteretic behaviour appears to be limited to a narrow range of the throttle parameter between the values $\gamma \approx 1.05$–1.10.
3. Deep surge seems to occur only when the throttle is slammed to values of γ less than that at the Hopf point H_1, otherwise the compressor always appears to enter rotating stall, no matter how the throttle is varied.

A more detailed bifurcation analysis, as in reference [**25**], can reveal further information. For example, it can be established that presence of classic surge is not a prerequisite for onset of deep surge, and that there is a cutoff value of β below which surge does not occur at all.

2.2 Transient behaviour

While bifurcation diagrams such as that in Fig. 1 can be easily computed, some care must be exercised in their interpretation. This is because bifurcation diagrams only provide steady state information whereas, in practical operation, there are always significant transients, which can be discovered only by numerical simulation. Two numerical simulations are presented in this section as illustrations. The first simulation in Fig. 2a shows the system response to throttle varied slowly from $\gamma = 1.2$ to 0.9 and back. The predicted entry into classic surge briefly, followed by rotating stall, and the foreseen abrupt loss in pressure rise, are all verified in Fig. 2a. However, recovery to axisymmetric flow is delayed until $\gamma \approx 1.2$, leading to a larger region of hysteresis than expected, and there is also no evidence of classic surge during the recovery transient. While this observation can be explained in terms of the topology of the classic surge cycles (their proximity to a saddle point as explained in reference [**25**]), this is not obvious from the diagram in Fig. 1.

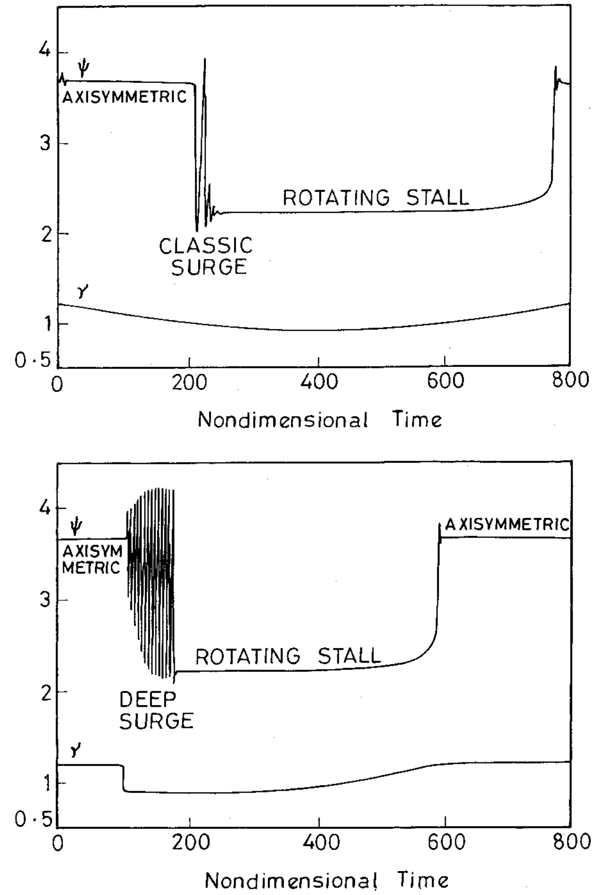

Fig. 2 Transient response of the compressor system in Fig. 1 to (a) slowly varying throttle showing jump and hysteresis, and (b) throttle slam showing deep surge

Another simulation in Fig. 2b shows the throttle being slammed from $\gamma = 1.2$ to 0.9 and being held there for a while before being slowly brought back to its original value of 1.2. Figure 1 predicts the deep surge solution for $\gamma = 0.9$ to be unstable and, hence, one may believe that deep surge cycles may not be expected to occur in this case. However, several deep surge cycles may be noticed in Fig. 2b before onset of rotating stall, while recovery to axisymmetric flow in Fig. 2b is identical to that seen in Fig. 2a. Thus, while the prediction of deep surge being caused by throttle slam turns out to be correct, it is necessary to distinguish between the point of deep surge onset and the point of sustained deep surge onset as marked in Fig. 1. It turns out that the deep surge cycles originating at the Hopf point H_1, although unstable, are in fact stable in the $R = 0$ plane. It is only at the point marked "Sustained deep surge onset" in Fig. 1 that the deep surge cycles gain stability in the direction out of the $R = 0$ plane at what is seen to be a transcritical bifurcation of cycles [**25**].

Thus, the bifurcation diagram in Fig. 1 identifies the following points. The first is the point of onset of instability in the axisymmetric flow (labeled H_3). This is also the point where classic surge oscillations originate. The second is the point of rotating stall onset (labeled H_2). The third is the point where deep surge cycles are created (labeled H_1), but these deep surge cycles are not sustained, eventually giving way to rotating stall. Finally the fourth is the point of onset of sustained deep surge oscillations. This usually requires the throttle to be slammed, as indicated by the arrow in Fig. 1. The power of the bifurcation method lies in the fact that, when accompanied by a few select numerical simulations, it provides an exhaustive picture of the global stability of the system. In terms of time and cost, it reduces the number of numerical simulations to a fraction of that required in the absence of bifurcation analysis. It also allows one to quickly confirm whether a closed-loop system with a specific controller has desired global stability properties.

3 BIFURCATION-BASED CONTROL

Linear controllers depend on local stability information and are usually unable to alter the global stability characteristics of nonlinear systems in a desired manner. For example, a hypothetical linear controller was considered in reference [**33**] that attempted to maintain the axial compressor system of equation (1) in axisymmetric flow, that is, at $R = 0$, no matter what the value of the throttle parameter γ. It was found that for certain values of γ, this controller could inadvertently force the compressor into deep surge even though the uncontrolled system did not show deep surge under those conditions. Control of the global stability behaviour of nonlinear systems therefore requires nonlinear control laws.

The bifurcation method is an effective tool for the global control analysis of nonlinear systems. Bifurcation-based control laws to prevent jump and hysteresis at onset of rotating stall have been devised in the past [**29, 30**]. These laws were based on the idea that the jump phenomenon and the accompanying hysteretic response could be avoided if the subcritical bifurcation at onset of rotating stall could be converted into a supercritical one. Essentially, this requires a feedback that alters the throttle characteristic in equation (3) such that multiple equilibrium states are avoided for a given value of throttle parameter. While the control law in reference [**29**] required feedback of the rotating stall amplitude variable R, which is not easily measurable, another bifurcation-based control law requiring only feedback of compressor pressure rise ψ has been put forward by Gu *et al.* [**30**], and is given as:

$$\gamma = \left(\sqrt{\gamma_0} + \frac{K_1}{\sqrt{\psi}}\right)^2 \tag{4}$$

where γ_0 is the new nominal value of the throttle parameter. With this control law, the second part of equation (1) now becomes:

$$\psi' = \frac{1}{\beta^2}(\varphi - \sqrt{\gamma_0 \psi} - K_1 + 1) \tag{5}$$

It can be shown that, for values of K_1 between 0.77 and 2.0, the bifurcation point at onset of rotating stall is indeed supercritical, and hence jump and hysteresis are eliminated. However, this control law does not eliminate surge. Far worse, this controller actually induces the compressor to surge whenever the throttle is reduced past the peak pressure rise point. This can be seen from Fig. 3, which shows the bifurcation diagrams for R with varying throttle parameter γ in the open-loop case ($\beta = 0.75$, as in Fig. 1), and for R with varying parameter γ_0 in the closed-loop case with the control law in equation (4). The gain parameter in equation (4) is chosen as $K_1 = 0.9$. Solutions with $R = 0$ represent the desired axisymmetric flow through the compressor. With decreasing γ, the stable axisymmetric solutions lose stability in both Figs 3a and b. However, the closed-loop bifurcation diagram in Fig. 3b shows a supercritical bifurcation as against the subcritical bifurcation in the open-loop diagram of Fig. 3a. While this does eliminate jump and hysteresis at the point of onset of instability, it must be noted that the Hopf bifurcation H_2, indicating onset of surge, is located very near the supercritical bifurcation point in Fig. 3b. In the

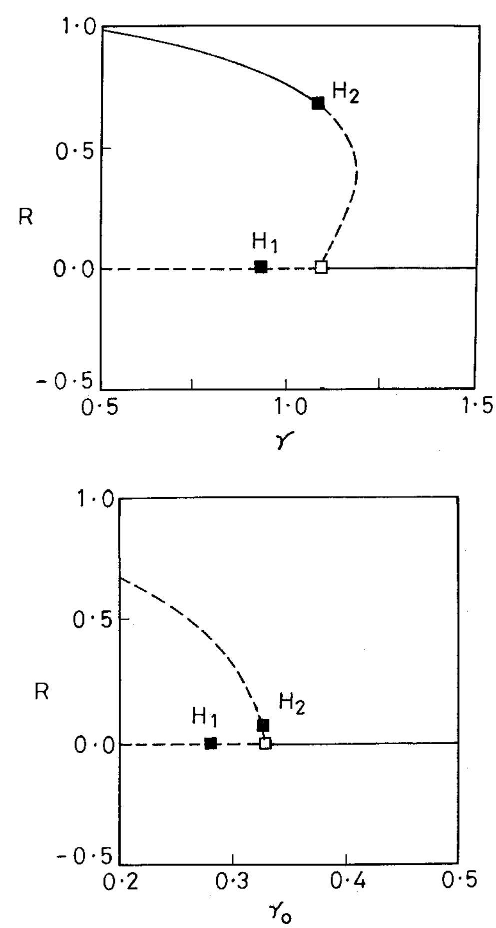

Fig. 3 Bifurcation diagram of first-mode rotating stall amplitude R with varying throttle parameter γ (or γ_0) for MG86 model with $\beta = 0.75$ for (a) open-loop system, and (b) closed-loop system with control law equation (4)

absence of any stable equilibrium solutions for γ_0 less than the H_2 Hopf value, it is almost certain that the compressor will start surging as the throttle is reduced past the peak pressure rise point marked by the supercritical bifurcation. Further evidence of this surging behaviour, in the form of a numerical simulation, is presented later in this paper.

We now aim to modify the controller in equation (4) such that it also eliminates the Hopf bifurcation point H_2, without disturbing the supercritical nature of the equilibrium states at the point of onset of instability of the axisymmetric flow solutions. The modified controller may be expected to be globally stabilizing, that is, for γ_0 greater than the supercritical bifurcation value, the only stable solutions will be the axisymmetric equilibria, while for γ_0 less than the supercritical value, the rotating stall equilibria would be the only stable solutions. This implies that there would be no stable periodic solutions for any value of γ_0, and hence no surge cycles. The first step in this procedure is to start with the control law in equation (4), with the parameter K_1 fixed at 0.9, and carry out a two-parameter continuation to track the locus of Hopf bifurcation points H_2 with varying parameters β and γ_0. This locus is plotted in Fig. 4, from which it is clearly seen that there is a minimum value of β marked by a dashed line, below which the Hopf point H_2 does not occur. Hence, operation at any value of β below this minimum guarantees that the compressor will not surge. However, β, which represents the compressor rpm, is not a suitable control parameter, and the challenge, therefore, is to devise a control strategy using the throttle γ as the control parameter that can emulate the effects of operating at lowered values of β.

Consider the following control law, which is based on a nonlinear feedback of pressure rise ψ and mass flow φ to the throttle γ:

$$\gamma = \left(\frac{1}{K_2}\right)^2 \left\{\sqrt{\gamma_0} + \frac{1}{\sqrt{\psi}}[(K_2 - 1)(\varphi + 1) + K_1]\right\}^2 \tag{6}$$

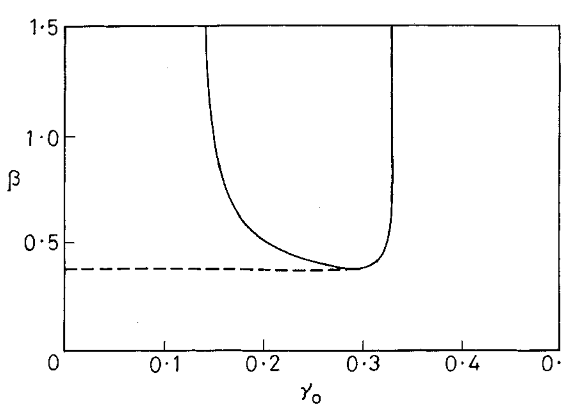

Fig. 4 Two-parameter continuation showing the locus of Hopf points H_2 with varying parameters γ_0 and β for the closed-loop compressor system with the control law equation (4)

Using this control law in the second part of equation (1) gives

$$\psi' = \frac{1}{K_2\beta^2}(\varphi - \sqrt{\gamma_0\psi} - K_1 + 1) \tag{7}$$

Surprisingly, this is identical to equation (5) except for the additional parameter K_2 that now appears in the denominator of equation (7). For a choice of $K_2 = 1$, it is easily verified that the control law in equation (6) is identical to that in equation (4), and that the closed-loop compressor dynamics in Fig. 3b is left unchanged. The effect of any other choice of K_2 is to make the compressor operate at an effective value of β given by,

$$\beta_{\mathrm{eff}}{}^2 = K_2\beta^2 \tag{8}$$

Thus, in order to make a compressor operating at $\beta = 0.75$ behave like one working at, say, $\beta = 0.2$, one only needs to choose $K_2 = (\beta_{\mathrm{eff}}/\beta)^2 = 0.0711$ in the control law of equation (6). The bifurcation diagrams for the resulting closed-loop compressor system with $K_1 = 0.9$ and $K_2 = 0.0711$ are plotted in Fig. 5. The bifurcation diagram of R against γ_0 in Fig. 5a shows the onset of instability of the axisymmetric

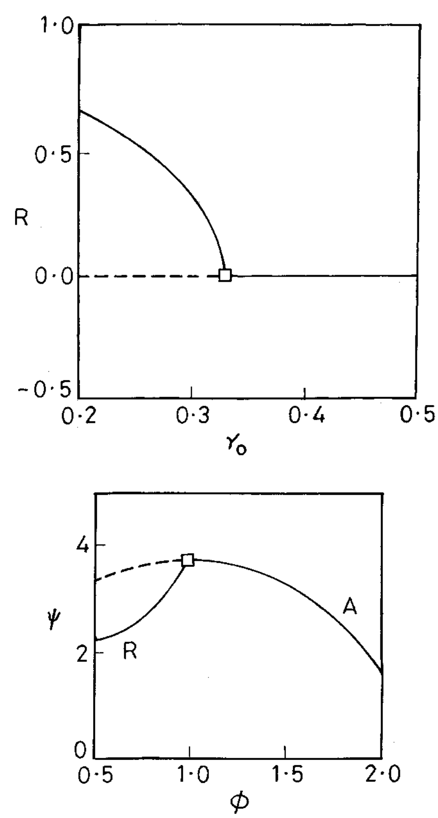

Fig. 5 Bifurcation diagram of (a) first-mode rotating stall amplitude R with varying throttle parameter γ_0, and (b) pressure rise ψ with varying mass flow φ, for the closed-loop compressor system with the control law equation (6)

($R = 0$) solutions at a supercritical bifurcation. The bifurcated rotating stall solutions ($R > 0$) are all stable, and jump and hysteresis do not occur at onset of rotating stall. There are no Hopf bifurcations and, hence, surge limit cycles have been eliminated. For each value of γ_0, there is only one stable equilibrium state, and that is globally asymptotically stable. The equilibrium solutions can also be viewed on a plot of pressure rise ψ versus mass flow φ in Fig. 5b. States marked 'A' are the axisymmetric solutions, those marked 'R' are the rotating stall solutions, and the supercritical bifurcation can be seen to occur at the peak pressure rise point. Incidentally, the control law in equation (6) also succeeds in displacing the deep surge Hopf bifurcation point H_1 to smaller values of γ_0, out of the range of values in Fig. 5, virtually eliminating the possibility of deep surge for any reasonable throttle slam.

To close this section, a couple of numerical simulations are presented illustrating the conclusions drawn from the bifurcation diagrams of Figs 3 and 5. Figures 6a and b show the response in pressure rise of the closed-loop compressor system with the control laws in equations (4) and (6), respectively, to the identical throttle input shown in the figures. The control law in equation (4) without the global stabilization property manages to avoid hysteresis, but pushes the compressor into large-amplitude surge cycles instead. The new globally stabilizing controller in equation (6), on the other hand, takes the compressor into and out of rotating stall without hysteresis, and avoids surge altogether.

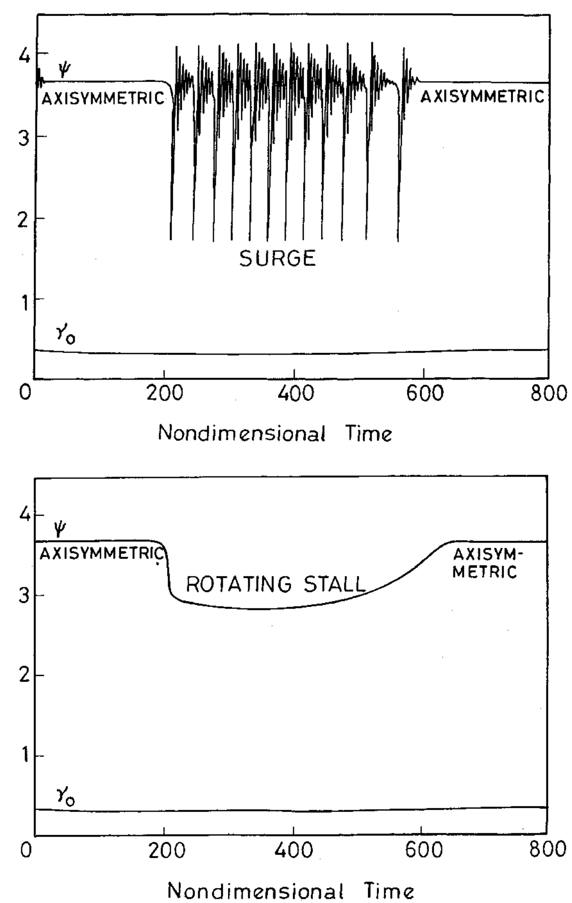

Fig. 6 Closed-loop response of compressor system with (a) control law equation (4), and (b) control law equation (6)

4 CONCLUSIONS

Axial compressors in jet engines operate in a severe environment of high noise and large fluctuations due to disturbances, both external and from within the engine. Active control techniques appear to promise safe, stable, and efficient compressor operation under these harsh operating conditions, provided the control laws are globally stabilizing. This ensures that the compressor returns to the desired operating condition after experiencing any perturbation, large or small. The best way to check for global stability of a system is to carry out a numerical bifurcation analysis, and this has been demonstrated in section 2 for the compressor dynamic system given by the MG86 model. Secondly, there are obvious advantages if the globally stabilizing control could itself be devised using bifurcation methods since the requirements for global stability are naturally posed in terms of the presence or absence of certain bifurcations. In section 3, we have derived a bifurcation-based globally stabilizing controller, which appears to provide a successful active control strategy for jet engine axial compressors. It would, however, be of interest to test the performance of the newly devised control law on a 2–3 stage, low-speed compressor rig.

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