**Completely Mixed Activated Sludge (CMAS) Bioreactor Design Equations**

Mass Balance:

**Biomass:**

\[
Q X_O + V \left( \frac{\mu_{\text{max}} SX}{K_S + S} - k_d X \right) = (Q - Q_w) X_e + Q_w X_w
\]

**Substrate:**

\[
QS_O - V \left( \frac{\mu_{\text{max}} SX}{Y(K_S + S)} \right) = (Q - Q_w) S_e + Q_w S_w
\]

where
- \( Q, Q_w \) = influent flow and waste flow, respectively, m³/d
- \( V \) = volume of aeration basin, m³
- \( \mu_{\text{max}} \) = maximum specific growth rate coefficient, h⁻¹
- \( K_S \) = half saturation coefficient, mg/L
- \( k_d \) = decay coefficient, h⁻¹
- \( X_O, X, X_e, X_w \) = biomass in influent, bioreactor, effluent, and waste, mg/L as MLVSS
- \( S \) = soluble substrate concentration in bioreactor, mg/L as BOD or COD
- \( S_O \) = influent substrate concentration, mg/L as BOD or COD
- \( Y \) = biomass yield, mg biomass formed/ mg substrate utilized (mg VSS/ mg BOD)

**Assumptions:**

1. Influent and effluent ___________________ concentration is negligible
2. Aeration basin is a ___________________ CSTR, \( S = S_w = S_e \)
3. All reactions occur in ___________________ basin

Then:

\[
\frac{\mu_{\text{max}} S}{K_S + S} = \frac{Q_w X_w}{VX} + k_d \quad ; \quad \frac{\mu_{\text{max}} S}{K_S + S} = \frac{QY}{VX}(S_O - S)
\]
Observe:

\[ \frac{Q}{V} = \frac{1}{\theta} \quad ; \quad \frac{Q_w X_w}{V X} = \frac{1}{\theta_c} \]

Where \( \theta = \) the hydraulic retention time, HRT, and \( \theta_c = \) the solids residence time, SRT. This results in the following design equations:

\[
S = \frac{K_s (1 + k_d \theta_c)}{\theta_c (\mu_{max} - k_d) - 1} \quad ; \quad \theta_c = \frac{K_s + S}{S (\mu_{max} - k_d) - K_s k_d} \quad ; \quad X = \frac{\theta_c Y (S_o - S)}{\theta (1 + k_d \theta_c)}
\]

The minimum soluble BOD concentration that can be achieved as \( \theta_c \to \infty \):

\[
S_{\text{min}} = \frac{K_s k_d}{\mu_{max} - k_d}
\]

The minimum \( \theta_c \) achievable as \( \mu \to \mu_{max} \):

\[
\theta_{c_{\text{min}}} = \frac{K_s + S_o}{S_o (\mu_{max} - k_d) - K_s k_d}
\]

**Steps for Activated Sludge Design**

1. Establish effluent soluble BOD\(_5\) allowable to meet BOD\(_5\) and SS effluent limits.

2. Determine what \( \theta_c \) is required to meet the effluent soluble BOD\(_5\) allowable.

3. Solve for the mixed liquor volatile suspended solids, MLVSS, concentration given a particular hydraulic residence time, \( \theta \). Or solve for \( \theta \) given a particular MLVSS.

4. Calculate the return activated sludge (RAS) flow, \( Q_r \), and concentration, \( X_r' \).

\[ X_i' Q_r = X'(Q_r + Q) \quad ; \quad X_i' = 10^6/SVI \]

where \( X' = \) MLSS, mg/L (\( X' \) typically is approximately 1.2·\( X \))

\( X_i' = \) RAS concentration, mg/L

\( Q_r = \) RAS flow rate, m\(^3\)/s
Find $X_r$ using the sludge volume index, SVI, from the following figure:

5. Sludge production can be estimated as follows:

$$P_X = Y_{obs} \frac{Q (S_o - S)}{1000} \frac{kg}{g}$$

where:
- $P_X =$ sludge production, kg/d
- $Y_{obs} =$ observed growth yield, mg biomass formed, VSS/ mg
- $S_o =$ influent $BOD_5$
- $Q =$ influent flow, m$^3$/d
- $S =$ effluent $BOD_5$

$Y_{obs}$ can be estimated as:

$$Y_{obs} = \frac{Y}{1 + k_d \theta_c}$$

6. Oxygen requirement for carbonaceous BOD removal can be calculated as:

$$O_2 \text{ req} = \left( \frac{Q(S_o - S)}{f} \cdot \frac{kg}{1000 \ g} \right) - 1.42 \cdot P_X$$

where $f =$ the conversion from $BOD_5$ to $BOD_{L}$, (0.45 - 0.68)
When nitrification is occurring the oxygen requirement can be calculated as:

\[ O_2_{\text{req}} = \left( \frac{Q(S_0 - S)}{f} \cdot \frac{\text{kg}}{1000 \ \text{g}} \right) - 1.42P_X + 4.57Q(N_0 - N) \cdot \frac{\text{kg}}{1000 \ \text{g}} \]

where \( N_0 \) and \( N \) are the influent and effluent NH\textsubscript{4}-N concentrations, respectively.

7. **Calculate the alkalinity consumed.**
   The conversion of NH\textsubscript{3}-N to nitrate not only requires oxygen but it also consumes considerable amount of alkalinity (7.1 mg/L as CaCO\textsubscript{3} for every mg/L NH\textsubscript{3}-N):

\[ \text{alk consumed (kg/d)} = Q(N_0 - N) \cdot 7.1 \ \text{mg/L as CaCO}_3/\text{mg NH}_4\text{-N} \cdot \text{(kg/1000 g)} \]

8. **Settling Tank Design**
   The design of primary and secondary settling tanks can be done on the basis of settling tests and/or established design criteria. In general, the design of tanks must meet established overflow rate and weir loading criteria.

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Primary Settling Tanks</th>
<th>Secondary Settling Tanks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Overflow Rate, m\textsuperscript{3}/m\textsuperscript{2}\cdot d^{-1}</td>
<td>Avg</td>
<td>Peak</td>
</tr>
<tr>
<td>OR = Q/A</td>
<td>41</td>
<td>60-120*</td>
</tr>
<tr>
<td>Weir Loading, m\textsuperscript{3}/m\cdot d^{-1}</td>
<td>&lt; 1 mgd</td>
<td>&gt; 1 mgd</td>
</tr>
<tr>
<td>WL = Q/L</td>
<td>250</td>
<td>375</td>
</tr>
</tbody>
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* for tanks not receiving waste activated sludge, use 49 m\textsuperscript{3}/m\textsuperscript{2}\cdot d for primary clarifiers receiving WAS

**Activated Sludge Operational Considerations**
An operator of an activated sludge plant is concerned with three things:
1. E______________ quality (BOD\textsubscript{5} and SS)
2. S______________ characteristics of the biomass (SVI)
3. Sludge w______________ or solids inventory (\( \theta_c \), F/M)

These three objectives/operational parameters are interrelated. A good settling sludge will produce good effluent quality. Maintaining the proper solids inventory will produce a good settling sludge. Controlling \( \theta_c \) will maintain the proper solids inventory.

**SVI** - Sludge v______________ index.
- Measure of s______________ characteristics of biomass.
- Measured in a g______________ cylinder after 30 minutes of settling.
- Units of mL/g.
- A d______________ SVI is in the range of 75 - 150.
Sludge Bulking
- Sludge bulking is the condition where the SVI is high and the suspended solids are not settling in the secondary settling tank.
- It is usually an indication of filamentous organisms - long string-like organisms which outcompete the flocculent organisms because of their large surface area.
- Filamentous organisms can be caused by
  
a) l_________________ F/M ratio  
b) l_________________ DO  
c) nutrient d_________________  
d) l_________________ pH  
e) i_________________ or toxicity  

F/M Ratio
- The f_________________ to m__________________________ (F/M) ratio is an alternative control/design parameter to $\theta_c$ for the operation of an activated sludge plant.

\[
\frac{F}{M} = \frac{Q}{MX} = \frac{mg \text{ BOD}_5/dD}{mg \text{ MLVSS}}
\]

Note: the F/M ratio is inversely proportional to $\theta_c$.

- Low F/M ratios are typical in c_________________ mixed activated sludge (CMAS) systems.
- CMAS systems, consequently, often have filamentous b_________________ problems.

CMAS with Selector

By using a s_________________, the F/M in the first compartment of an activated sludge system can be increased, giving the f_________________ microorganisms a competitive advantage.

SUMMARY OF ACRONYMS

SRT solids retention time (or solids residence time), also MCRT, mean cell residence time
MLSS mixed liquor suspended solids
MLVSS mixed liquor volatile suspended solids (used as a surrogate measurement of the biomass in an activated sludge system)
SVI sludge volume index - a measurement of the settling properties of activated sludge
F:M food:microorganisms ratio, an alternative design parameter for A.S. system
RAS, WAS return and waste activated sludge