**Completely Mixed Activated Sludge (CMAS) Bioreactor Design Equations**

![Diagram of CMAS bioreactor](image)

**Mass Balance:**

**Biomass:**

\[
Q X_O + V \left( \frac{\mu_{\text{max}} SX}{K_S + S} - k_d X \right) = (Q - Q_w) X_e + Q_w X_w
\]

**Substrate:**

\[
QS_O - V \left( \frac{\mu_{\text{max}} SX}{Y(K_S + S)} \right) = (Q - Q_w) S_e + Q_w S_w
\]

where  
- \( Q, Q_w \) = influent flow and waste flow, respectively, m³/d  
- \( V \) = volume of aeration basin, m³  
- \( \mu_{\text{max}} \) = maximum specific growth rate coefficient, h⁻¹  
- \( K_S \) = half saturation coefficient, mg/L  
- \( k_d \) = decay coefficient, h⁻¹  
- \( X_O, X, X_e, X_w \) = biomass in influent, bioreactor, effluent, and waste, mg/L as MLVSS  
- \( S \) = soluble substrate concentration in bioreactor, mg/L as BOD or COD  
- \( S_O \) = influent substrate concentration, mg/L as BOD or COD  
- \( Y \) = biomass yield, mg biomass formed/ mg substrate utilized (mg VSS/ mg BOD)

**Assumptions:**

1. Influent and effluent ________________ concentration is negligible
2. Aeration basin is a ________________ CSTR, \( S = S_w = S_e \)
3. All reactions occur in ________________ basin

Then:

\[
\frac{\mu_{\text{max}} S}{K_S + S} = \frac{Q_w X_w}{VX} + k_d ; \quad \frac{\mu_{\text{max}} S}{K_S + S} = \frac{Q Y}{VX(S_O - S)}
\]
Observe:

\[ \frac{Q}{V} = \frac{1}{\theta} \quad ; \quad \frac{QwXw}{VX} = \frac{1}{\theta_c} \]

Where \( \theta \) = the hydraulic retention time, HRT, and \( \theta_c \) = the solids residence time, SRT. This results in the following design equations:

\[ S = \frac{K_S \left( 1 + k_d \theta_c \right)}{\theta_c \left( \mu_{max} - k_d \right) - 1} \quad ; \quad \theta_c = \frac{K_S + S}{\theta \left( \mu_{max} - k_d \right) - K_S k_d} \quad ; \quad X = \frac{\theta_c Y \left( S_o - S \right)}{\theta \left( 1 + k_d \theta_c \right)} \]

The minimum soluble BOD concentration that can be achieved as \( \theta_c \to \infty \):

\[ S_{min} = \frac{K_S k_d}{\mu_{max} - k_d} \]

The minimum \( \theta_c \) achievable as \( \mu \to \mu_{max} \):

\[ \theta_{c_{min}} = \frac{K_S + S_o}{S_o \left( \mu_{max} - k_d \right) - K_S k_d} \]

**Steps for Activated Sludge Design**

1. Establish effluent soluble BOD\(_5\) allowable to meet BOD\(_5\) and SS effluent limits.

2. Determine what \( \theta_c \) is required to meet the effluent soluble BOD\(_5\) allowable.

3. Solve for the mixed liquor volatile suspended solids, MLVSS, concentration given a particular hydraulic residence time, \( \theta \). Or solve for \( \theta \) given a particular MLVSS.

4. Calculate the return activated sludge (RAS) flow, \( Q_r \), and concentration, \( X'_r \).

\[ X'_r Q_r = X' \left( Q_r + Q \right) \quad ; \quad X'_r = 10^6 / \text{SVI} \]

where \( X' = \text{MLSS, mg/L} \) (\( X' \) typically is approximately 1.2·\( X \))

\( X'_r \) = RAS concentration, mg/L

\( Q_r \) = RAS flow rate, m\(^3\)/s
Find \( X_i \) using the sludge volume index, SVI, from the following figure:

5. Sludge production can be estimated as follows:

\[
P_X = Y_{\text{OBS}} \cdot \frac{Q \cdot (S_O - S)}{1000} \quad \text{kg/} \quad \text{g}
\]

where:
- \( P_X \) = sludge production, kg/d
- \( Y_{\text{OBS}} \) = observed growth yield, mg biomass formed, VSS/ mg
- \( S_O \) = influent BOD
- \( S \) = effluent BOD
- \( Q \) = influent flow, m\(^3\)/d

\( Y_{\text{OBS}} \) can be estimated as:

\[
Y_{\text{OBS}} = \frac{Y}{1 + k_d \theta_c}
\]

6. Oxygen requirement for carbonaceous BOD removal can be calculated as:

\[
O_2 \text{ req} = \left( \frac{Q(S_O - S)}{f} \cdot \frac{kg}{1000 \text{ g}} \right) - 1.42 \cdot P_X
\]

where \( f \) = the conversion from BOD\(_3\) to BOD\(_L\), (0.45 - 0.68)
When nitrification is occurring the oxygen requirement can be calculated as:

\[ O_2 \text{ req} = \left( \frac{Q(S_o - S)}{f} \cdot \frac{\text{kg}}{1000 \text{ g}} \right) - 1.42 \cdot P_X + 4.57 \cdot Q \cdot (N_o - N) \cdot \frac{\text{kg}}{1000 \text{ g}} \]

where \( N_o \) and \( N \) are the influent and effluent \( NH_4-N \) concentrations, respectively.

7. **Calculate the alkalinity consumed.**
The conversion of \( NH_3-N \) to nitrate not only requires oxygen but it also consumes considerable amount of alkalinity (7.1 mg/L as CaCO\(_3\) for every mg/L \( NH_3-N \)):

\[ \text{alk consumed (kg/d)} = Q \cdot (N_o - N) \cdot 7.1 \text{ mg/L as CaCO}_3/\text{mg NH}_4-N \cdot \text{(kg/1000 g)} \]

8. **Settling Tank Design**
The design of primary and secondary settling tanks can be done on the basis of settling tests and/or established design criteria. In general, the design of tanks must meet established overflow rate and weir loading criteria.

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Primary Settling Tanks</th>
<th>Secondary Settling Tanks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Overflow Rate, m(^3)/m(^2)-d</td>
<td>Avg</td>
<td>Peak</td>
</tr>
<tr>
<td>OR = Q/A</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>41</td>
<td>60-120*</td>
</tr>
<tr>
<td>Weir Loading, m(^3)/m(^2)-d</td>
<td>&lt; 1 mgd</td>
<td>&gt; 1 mgd</td>
</tr>
<tr>
<td>WL = Q/L</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>250</td>
<td>375</td>
</tr>
</tbody>
</table>

*For tanks not receiving waste activated sludge, use 49 m\(^3\)/m\(^2\)-d for primary clarifiers receiving WAS.

**Activated Sludge Operational Considerations**
An operator of an activated sludge plant is concerned with three things:

1. E______________ quality (BOD \(_5\) and SS)
2. S_______________ characteristics of the biomass (SVI)
3. Sludge w_______________ or solids inventory (\( \theta_c \), F/M)

These three objectives/operational parameters are interrelated. A good settling sludge will produce good effluent quality. Maintaining the proper solids inventory will produce a good settling sludge. Controlling \( \theta_c \) will maintain the proper solids inventory.

**SVI** - Sludge v__________________ index.
- Measure of s__________________ characteristics of biomass.
- Measured in a g__________________ cylinder after 30 minutes of settling.
- Units of mL/g.
- A d__________________ SVI is in the range of 75 - 150.
Sludge Bulking
- Sludge bulking is the condition where the SVI is h_________ and the suspended solids are not settling in the secondary settling tank.
- It is usually an indication of f________________ organisms - long string-like organisms which outcompete the flocculent organisms because of their large surface area.
- Filamentous organisms can be caused by
  a) l________ F/M ratio
  b) l________ DO
  c) nutrient d________________
  d) l________ pH
  e) i_______________ or toxicity

F/M Ratio
- The f____________ to m________________________ (F/M) ratio is an alternative control/design parameter to θc for the operation of an activated sludge plant.

\[
\frac{F}{M} = \frac{Q}{V} \frac{S_O}{X} = \frac{mg \ BOD_5/DO}{mg \ MLVSS}
\]

Note: the F/M ratio is inversely proportional to θc.
- Low F/M ratios are typical in e________________ mixed activated sludge (CMAS) systems.
- CMAS systems, consequently, often have filamentous b____________ problems.

**SUMMARY OF ACRONYMS**
- SRT solids retention time (or solids residence time), also MCRT, mean cell residence time
- MLSS mixed liquor suspended solids
- MLVSS mixed liquor volatile suspended solids (used as a surrogate measurement of the biomass in an activated sludge system)
- SVI sludge volume index - a measurement of the settling properties of activated sludge
- F:M food:microorganisms ratio, an alternative design parameter for A.S. system
- RAS, WAS return and waste activated sludge