Practice Problems

Problem 1. What single amount on April 1, 2002, is equivalent to a series of equal, quarterly cash flows of $1000, which started with a cash flow on July 1, 2002, and will end with a cash flow on October 1, 2008? Use an interest rate of 18% and quarterly compounding.

Solution:

\[ A_{\text{qr}} = \$1000. \quad \text{First A @ 7/2002.} \quad \text{Last A @ 10/2008.} \]

\[ T = 4/2002 \text{ To } 10/2008 = 6.5 \text{ Years.} \]

\[ t_A = \frac{1}{4} \text{ Year.} \quad n_{\text{qr}} = 6.5 \times 4 = 26. \quad r = 18\%. \quad t_c = \frac{1}{4} \text{ Year.} \quad i_{\text{qr}} = \frac{18\%}{4} = 4.5\%. \]

Single amount in April 2002,

\[ P_{4/02} = 1000 \times (P/A, 4.5\%, 26) = 1000 \times (15.147) = \$15,147 \]

Problem 2. Mary Smith took a car loan of $12,000 to pay back in 60 monthly installments at a nominal interest rate of 12% on the understanding that the interest rate may be changed sometime in the future. Compute

i. the monthly payment for Mary.

ii. the loan balance immediately after the 24th payment.

iii. the monthly payment for the remainder of the loan if the interest rate is reduced to 9%.

Solution:

i. Nominal interest rate = 12%

Monthly interest rate = 12%/12 = 1%

Monthly payment at 1% int. per month, \( A_1 = \$12,000 \times (A/P, 1\%, 60) = \$266.4 \)

ii. After the 24th payment, there will be 36 more payments before the loan is retired.

Therefore, loan balance after the 24th payment = $266.40(P/A, 1\%, 36) = \$8,020.50

iii. Since the nominal interest rate is 9%, monthly interest rate = 9%/12 = 3/4%

Monthly payment at 3/4% int. per month, \( A_2 = \$8,020.50 \times (A/P, 3/4\%, 36) = \$255.05 \)

Problem 3. A series of equal semiannual cash flows started with the first cash flow occurring on July 1, 1991, and ends with the last cash flow occurring on January 1, 2008. Each cash flow is equal to $128,000. The nominal interest rate is 12% and compounding is semiannual. What single amount on July 1, 2001, is equivalent to this cash flow system?

Solution:

\[ n = 17 \times 2 = 34 \text{ periods} \]

\[ i/6\text{-month period} = \frac{12\%}{2} = 6\% \]

\[ P \text{ on } 7/1, 2001 = \$128,000 \times (F/A, 6\% 34) / (F/P, 6\% 13) = \$6,252,016.88 \]
Problem 4. How much would you need to invest at 6% interest on December 31, 2004, in order to accumulate $1850 on December 31, 2011? Present the economic functions required, showing first the functional notation and then its numerical value.

Solution:

\( F = 1,850 \). \( n = \frac{12}{2004} \text{ To } 12/2011 = 7 \text{ Years} \). \( i = 6\% \).

\[
P = F \times (P/F, i, n) = 1,850 \times (0.7050) = 1,230.44
\]

Problem 5. Money is rather tight this month, and so you decide to borrow $1000 from your local loan shark, “Mr. E.Z. Loan”. He is willing to lend you the $1000 if you will repay him $1050 one month later.

(a). What nominal annual interest rate are you being charged?

(b). What effective annual interest rate are you being charged? Assume monthly compounding.

Solution:

(a) Nominal interest rate per year = \( 12 \{(1,050-1,000)/1,000\} = 0.60 \text{ or } 60\% \)

(b) Effective interest rate = \( (1+0.05)^{12} - 1 = 0.7959 \text{ or } 79.59\% \)

Problem 6. Your family is expanding in number, and so you decide to sell your current home and to upgrade to a larger home. You estimate that you can sell your current home for $100,000 and can buy a larger home for $175,000. You plan to use the entire $100,000 home sale proceeds as a down payment on the new home and will finance the remainder for 10 years at 6% nominal annual interest compounded monthly. What is your estimated monthly mortgage payment?

Solution:

Loan amount = 175,000 – 100,000 = $75,000

Monthly interest rate = 0.06/12 =0.005 \( \text{ Number of payments =10 (12) = 120.} \)

Monthly mortgage payment = 75,000(A/P, ½ %, 120) = 75,000(0.0111) = $832.50

Problem 7. What is the equivalent worth on December 31, 2003, of $1295 deposited on December 31, 1996? Use an interest rate of 6%. Present the economic equivalence function required, showing first the functional notation and then its numerical value.

Solution:

\( P = 1,295 \). \( n = \frac{12/1996 \text{ To } 12/2003 = 7 \text{ Years}}{i = 6\%} \).

\[
F = P \times (F/P, i, n) = 1,295 \times (F/P, 6\%, 7) = 1,295 \times 1.504 = 1,947.68
\]

**Problem 8.** Joe wants to be able to purchase a dream car for about $19,000 on January 1, 2004, just after he graduates from college. Joe has had a part time job and started making deposits of $275 each month into an account that pays 9% compounded monthly beginning with the first deposit on February 1, 1999. The last deposit is to be made on January 1, 2004. Determine how much money he would have saved to buy the car. Will he be able to buy his dream car?

**Solution:**

Monthly deposits, \( A = $275 \). \( n = 2/1999 \) To 1/2004 = 60 periods.

\[
F = A \times (F/A, 0.75\%, 60) = 275 \times (69.770) = $19,168.75
\]

By making a deposit of $275 a month, Joe will have enough money saved to buy his dream car.