

# Homework 7

*Due: 5 Nov 2002, 9.30am*

## Policy

This is a *collaborative* homework. You are allowed to discuss the problems with other students. But, the write up should be written by yourself without help from others.

Please state your assumptions, justify your answers, and try to be **concise** and clear.

## Problem 7.1

**Practice Problems:** Note that these problems will not be graded.

- Page 285, problem 29
- Page 285, problem 33
- Page 285, problem 26 (see page 272 for the definition of a Bernoulli trial, a concept that we are already familiar with).
- On average, how many times should we throw a die before we get a six?

## Problem 7.2

The uniform random variable  $U$  is defined as follows.  $U$  takes values in the range  $1, 2, \dots, n$  and its probability distribution is: for every  $i$  in  $1 \dots n$ ,

$$\Pr[U = i] = 1/n$$

- a. What is the expected value of  $U$ , i.e.  $E[U]$ ?
- b. What is  $\Pr[U \geq E[U]]$ ?

**Hint:** Recall the alternate formula for the expectation for a random variable:

$$E[X] = \sum_{i=1}^{i=k} \Pr[X = i]i$$

### Problem 7.3

Shuffle a deck of 52 cards, and then turn up the cards one by one. Let  $X$  denote the random variable equal to the number of cards required till the first ace appears. What is the expected value of  $X$ ? Explain your reasoning.

**Note:** You don't have to simplify your answer (since that might require a lot of computation). You can leave it as a summation, and will receive credit as long as your expression is correct.

### Problem 7.4

What are the probabilities of the following events? Which of them is the most likely?

- Getting at least 1 six when 6 dice are rolled
- Getting at least 2 sixes when 12 dice are rolled
- Getting at least 3 sixes when 18 dice are rolled

### Problem 7.5

A typist types letters and envelopes to  $n$  different people. He curses his job, and then randomly puts the letters into envelopes. Let  $X$  be the random variable equal to the number of letters that went into their own envelopes.

- a. What is the expected value of  $X$ ?
- b. If  $0 \leq r \leq n$ , what is  $\Pr[X = r]$ ?

**Note:** You are not required to solve part (b) of this problem. If you do turn it in, we will grade it (will not give you any credit, though).

**Problem 7.6**

Suppose that in a sequence of  $n$  coin tosses, we got  $H$  heads and  $T$  tails. Let  $X$  denote the random variable  $|H - T|$  (note that  $X$  is always positive). If  $0 \leq r \leq n$ , what is  $\Pr[X = r]$ ?