

# Homework 3

*Due: 19 Sept 2002, 9.30am*

## Policy

This is a *collaborative* homework. You are allowed to discuss the problems with other students. But, the write up should be written by yourself without help from others.

Please state your assumptions and try to be **concise** and clear.

### Problem 3.1

Following are two pairs of statements. State whether the statements in each pair are equivalent or not.

- a. If there is a snowstorm, then the school is closed.  
If the school is closed, then there is a snowstorm.
- b. If I eat spicy food, then I have strange dreams.  
If I did not have strange dreams, then I did not eat spicy food.

### Problem 3.2

Prove that if  $n$  is a positive integer (i.e.  $n \in \{1, 2, 3, \dots\}$ ), then  $n$  is odd if and only if  $5n + 6$  is odd.

Note: To prove a statement of the form “A if and only if B”, you should prove two parts (1) “if A then B” and (2) “if B then A”.

### Problem 3.3

We are given  $n$  real numbers,  $a_1, a_2, \dots, a_n$ . Prove that at least one of these numbers is greater than or equal to their average, which is  $\frac{a_1 + a_2 + \dots + a_n}{n}$ .

**Problem 3.4**

Prove the following statement using mathematical induction. If  $n$  is a positive integer (i.e.  $n \in \{1, 2, 3, \dots\}$ ), then

$$1 + 3 + 5 + \dots + (2n - 1) = n^2$$

**Problem 3.5**

Using mathematical induction, show that if  $n$  is a positive integer, then

$$1^2 + 2^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$$