CprE 310 Theoretical Foundations of Computer Engineering

Homework 1

Due: 5 Sept 2002, 9.30am

Policy

This is a *collaborative* homework. You are allowed to discuss the problems with other students. But, the write up should be written by yourself without help from others.

Please state your assumptions and try to be **concise** and clear.

Problem 1.1

Venn Diagrams.

a. $A = \{1, 3\}$, $B = \{3, 4, 5\}$ and $C = \{3, 5\}$. Draw a Venn diagram showing A,B and C.

b. R is the set of real numbers, Z the set of integers, N the set of natural numbers, and Q the set of rational numbers. Draw a Venn diagram showing R, Z, N and Q.

Problem 1.2

Find two sets A and B, such that $A \in B$ and $A \subset B$.

Problem 1.3

Cross Products. In class, we saw the definition of the cross product of two sets. This problem deals with the cross product of three sets. Lookup page 44 of the textbook for the definition of the cross product of 3 or more sets.

a. If $A = \{a, b, c\}$ and $B = \{0, 1\}$ and $C = \{x, y\}$, what is $A \times B \times C$?

b. If A, B and C have m, n, l elements respectively, how many elements does $A \times B \times C$ have?

Problem 1.4

Define the minimum element of a set to be the smallest element in the set. Not every set of numbers has a minimum element. For example, the set of all integers does not have a minimum element.

Question: Give an example of a set which is a subset of the non-negative real numbers, and *which does not have a minimum element.*

Problem 1.5

For the following problems, *do not use truth tables*, but you can use the propositional equivalences listed in Table 5 on page 17 of the textbook.

a. Show that the following are equivalent: $p \rightarrow q$ and $\neg q \rightarrow \neg p$

b. Show that this formula is a tautology. $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$

Hint: The following logical equivalence might prove to be useful. $(p \rightarrow q) \leftrightarrow (\neg p \vee q)$.

Problem 1.6

A propositional formula F is said to be in Conjunctive Normal Form (CNF for short) if:

F is an AND of *conjuncts* $C_1, C_2 \ldots$, where
each conjunct C_i is an OR of *literals* and
each literal is a symbol (representing a proposition) or its negation.

For example, the following formulas are in CNF:

- $(b \vee \neg c) \wedge (a \vee d)$
- $a \vee \neg b$

The following are not:

- $(b \wedge c) \vee d$
- $a \rightarrow b$

Question: Convert the following propositional formula into an equivalent one which is in CNF.

$$(p \to q) \to r$$