$$\boldsymbol{Res(f,c)} = [(z-c)f(z)]|_{z=c} = \left.\frac{[(z-c)^n f(z)]^{(n-1)}}{(n-1)!}\right|_{z=c} \quad \text{(when } f \text{ has nth order pole at } c\text{)}$$

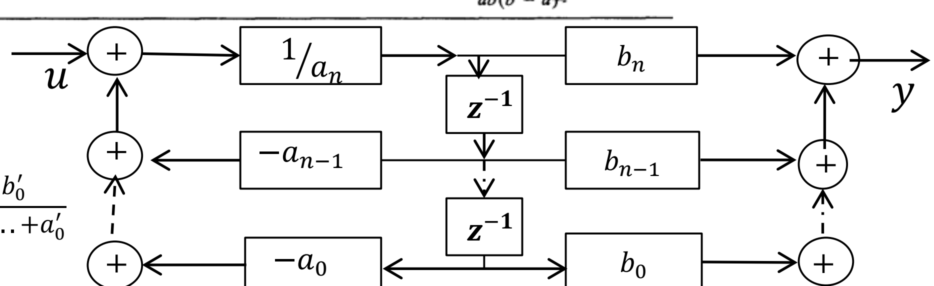
$$\boldsymbol{X(z)} = \sum_{p\in poles(X)} Res\left[\frac{X(\lambda)}{1-z^{-1}e^{T\lambda}}, p\right]; \quad \boldsymbol{X^*(s)} = X(z)|_{z=e^{Ts}}; \quad \boldsymbol{PTF}: G(z) = (1-z^{-1})\,\boldsymbol{Z}\left(\frac{G_p(s)}{s}\right)$$

Laplace transform $E(s)$	Time function $e(t)$	z-Transform $E(z)$	Modified z-transform $E(z,m)$ $= \boldsymbol{Z}(e(t-(1-m)T)) = z^{-1}\boldsymbol{Z}(E(s)e^{mTs})$
$\frac{1}{s}$	$u(t)$	$\frac{z}{z-1}$	$\frac{1}{z-1}$
$\frac{1}{s^2}$	t	$\frac{Tz}{(z-1)^2}$	$\frac{mT}{z-1}+\frac{T}{(z-1)^2}$
$\frac{1}{s^3}$	$\frac{t^2}{2}$	$\frac{T^2z(z+1)}{2(z-1)^3}$	$\frac{T^2}{2}\left[\frac{m^2}{z-1}+\frac{2m+1}{(z-1)^2}+\frac{2}{(z-1)^3}\right]$
$\frac{(k-1)!}{s^k}$	t^{k-1}	$\lim_{a\to 0}(-1)^{k-1}\frac{\partial^{k-1}}{\partial a^{k-1}}\left[\frac{z}{z-\epsilon^{-aT}}\right]$	$\lim_{a\to 0}(-1)^{k-1}\frac{\partial^{k-1}}{\partial a^{k-1}}\left[\frac{\epsilon^{-amT}}{z-\epsilon^{-aT}}\right]$
$\frac{1}{s+a}$	ϵ^{-at}	$\frac{z}{z-\epsilon^{-aT}}$	$\frac{\epsilon^{-amT}}{z-\epsilon^{-aT}}$
$\frac{1}{(s+a)^2}$	$t\epsilon^{-at}$	$\frac{Tz\epsilon^{-aT}}{(z-\epsilon^{-aT})^2}$	$\frac{T\epsilon^{-amT}[\epsilon^{-aT}+m(z-\epsilon^{-aT})]}{(z-\epsilon^{-aT})^2}$
$\frac{(k-1)!}{(s+a)^k}$	$t^k\epsilon^{-at}$	$(-1)^k\frac{\partial^k}{\partial a^k}\left[\frac{z}{z-\epsilon^{-aT}}\right]$	$(-1)^k\frac{\partial^k}{\partial a^k}\left[\frac{\epsilon^{-amT}}{z-\epsilon^{-aT}}\right]$
$\frac{a}{s(s+a)}$	$1-\epsilon^{-at}$	$\frac{z(1-\epsilon^{-aT})}{(z-1)(z-\epsilon^{-aT})}$	$\frac{1}{z-1}-\frac{\epsilon^{-amT}}{z-\epsilon^{-aT}}$
$\frac{a}{s^2(s+a)}$	$t-\frac{1-\epsilon^{-at}}{a}$	$\frac{z[(aT-1+\epsilon^{-aT})z+(1-\epsilon^{-aT}-aT\epsilon^{-aT})]}{a(z-1)^2(z-\epsilon^{-aT})}$	$\frac{T}{(z-1)^2}+\frac{amT-1}{a(z-1)}+\frac{\epsilon^{-amT}}{a(z-\epsilon^{-aT})}$
$\frac{a^2}{s(s+a)^2}$	$1-(1+at)\epsilon^{-at}$	$\frac{z}{z-1}-\frac{z}{z-\epsilon^{-aT}}-\frac{aT\epsilon^{-aT}z}{(z-\epsilon^{-aT})^2}$	$\frac{1}{z-1}-\left[\frac{1+amT}{z-\epsilon^{-aT}}+\frac{aT\epsilon^{-aT}}{(z-\epsilon^{-aT})^2}\right]\epsilon^{-amT}$
$\frac{b-a}{(s+a)(s+b)}$	$\epsilon^{-at}-\epsilon^{-bt}$	$\frac{(e^{-aT}-\epsilon^{-bT})z}{(z-\epsilon^{-aT})(z-\epsilon^{-bT})}$	$\frac{\epsilon^{-amT}}{z-\epsilon^{-aT}}-\frac{\epsilon^{-bmT}}{z-\epsilon^{-bT}}$
$\frac{a}{s^2+a^2}$	$\sin(at)$	$\frac{z\sin(aT)}{z^2-2z\cos(aT)+1}$	$\frac{z\sin(amT)+\sin(1-m)aT}{z^2-2z\cos(aT)+1}$
$\frac{s}{s^2+a^2}$	$\cos(at)$	$\frac{z(z-\cos(aT))}{z^2-2z\cos aT+1}$	$\frac{z\cos(amT)-\cos(1-m)aT}{z^2-2z\cos(aT)+1}$
$\frac{1}{(s+a)^2+b^2}$	$\frac{1}{b}\epsilon^{-at}\sin bt$	$\frac{1}{b}\left[\frac{z\epsilon^{-aT}\sin bT}{z^2-2z\epsilon^{-aT}\cos(bT)+\epsilon^{-2aT}}\right]$	$\frac{1}{b}\left[\frac{\epsilon^{-amT}[z\sin bmT+\epsilon^{-aT}\sin(1-m)bT]}{z^2-2z\epsilon^{-aT}\cos bT+\epsilon^{-2aT}}\right]$
$\frac{s+a}{(s+a)^2+b^2}$	$\epsilon^{-at}\cos bt$	$\frac{z^2-z\epsilon^{-aT}\cos bT}{z^2-2z\epsilon^{-aT}\cos bT+\epsilon^{-2aT}}$	$\frac{\epsilon^{-amT}[z\cos bmT+\epsilon^{-aT}\sin(1-m)bT]}{z^2-2z\epsilon^{-aT}\cos bT+\epsilon^{-2aT}}$
$\frac{a^2+b^2}{s[(s+a)^2+b^2]}$	$1-\epsilon^{-at}\left(\cos bt+\frac{a}{b}\sin bt\right)$	$\frac{z(Az+B)}{(z-1)(z^2-2z\epsilon^{-aT}\cos bT+\epsilon^{-2aT})}$ $A=1-\epsilon^{-aT}\left(\cos bT+\frac{a}{b}\sin bT\right)$ $B=\epsilon^{-2aT}+\epsilon^{-aT}\left(\frac{a}{b}\sin bT-\cos bT\right)$	$\frac{1}{z-1}$ $-\frac{\epsilon^{-amT}[z\cos bmT+\epsilon^{-aT}\sin(1-m)bT]}{z^2-2z\epsilon^{-aT}\cos bT+\epsilon^{-2aT}}$ $+\frac{\frac{a}{b}\{\epsilon^{-amT}[z\sin bmT-\epsilon^{-aT}\sin(1-m)bT]\}}{z^2-2z\epsilon^{-aT}\cos bT+\epsilon^{-2aT}}$
$\frac{1}{s(s+a)(s+b)}$	$\frac{1}{ab}+\frac{\epsilon^{-at}}{a(a-b)}+\frac{\epsilon^{-bt}}{b(b-a)}$	$\frac{(Az+B)z}{(z-\epsilon^{-aT})(z-\epsilon^{-bT})(z-1)}$	$A=\frac{b(1-\epsilon^{-aT})-a(1-\epsilon^{-bT})}{ab(b-a)}$ $B=\frac{a\epsilon^{-aT}(1-\epsilon^{-bT})-b\epsilon^{-bT}(1-\epsilon^{-aT})}{ab(b-a)}$

$A = e^{A_cT}; B = \int_0^T e^{A_c\tau}B_c d\tau; C = C_c; D = D_C.$

$$\sum_{j=o}^{n} a_k y[k+j] = \sum_{k=0}^{n} b_k u[k+j]$$

$$G(z) = \frac{b_nz^n+\ldots+b_0}{a_nz^n+\ldots+a_0} = \frac{b_n}{a_n} + \frac{b'_{n-1}z^{n-1}+\ldots+b'_0}{z^n+a'_{n-1}z^{n-1}+\ldots+a'_0}$$



$$A = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ 0 & 0 & \ddots & 0 \\ \cdots & \cdots & \cdots & 1 \\ -a'_0 & -a'_1 & \cdots & -a'_{n-1} \end{bmatrix},\ B = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ \mathbf{1} \end{bmatrix},\ C = [\,b'_0\ \ b'_1\ \ \ldots\ \ b'_{n-1}],\ D = [b_n/a_n].$$

$G(z) = C(zI-A)^{-1}B + D$, with characteristic polynomial, $Q(z) = \det(zI-A) \Rightarrow G(z)$ poles are eigen-values of A.

Bilinear transform: $s = \frac{2(z-1)}{T(z+1)}$; $z = \frac{1+\frac{T}{2}s}{1-\frac{T}{2}s}$; pre-warped frequency: $\omega = \frac{2}{T}\tan\left(\frac{T}{2}\omega_z\right)$.

Control requirements: Stability and its margins; Transients: peak-overshoot, response-time, settling time; Steady-state: dc gain, dc error, BW; Sensitivities: noise/disturbance/model-error; Safety; Security.

z-plane pole at: $r\angle \pm \theta \Rightarrow$ s-plane pole at: $\frac{\ln r}{T} \pm j\,\frac{\theta}{T} \Rightarrow$ (for char. poly. $s^2 + 2\zeta\omega_n s + \omega_n^2$): $-\omega_n\zeta = \frac{\ln r}{T}$; $\omega_n\sqrt{1-\zeta^2} = \frac{\theta}{T}$

$\Rightarrow$ nat-freq $\omega_n = \frac{\sqrt{\ln^2 r + \theta^2}}{T}$; damping-coeff $\zeta = \frac{-\ln r}{\sqrt{\ln^2 r + \theta^2}}$; time-const $\tau = \frac{T}{|\ln r|} = \frac{1}{\omega_n\zeta}$; osc. freq. $\omega = \frac{\theta}{T} = \omega_n\sqrt{1-\zeta^2}$;

peak-overshoot $= e^{-\pi\zeta/\sqrt{1-\zeta^2}}$, at half time-period $\frac{\pi}{\omega_n\sqrt{1-\zeta^2}}$; rise-time = $\frac{\pi - \cos^{-1}\zeta}{\omega_n\sqrt{1-\zeta^2}}$; phase margin = $\tan^{-1}\left(\frac{2\zeta}{\sqrt{\left(\sqrt{4\zeta^4+1}-2\zeta^2\right)}}\right)$;

gain-crossover freq: $\omega_g = \omega_n\sqrt{\left(\sqrt{4\zeta^4+1}-2\zeta^2\right)}$.

Type: # integrators in loop-gain; $e_{ss} = \lim_{s\to 0} s\left(\frac{R(s)}{1+L(s)}\right)\Big|_{R(s)\sim O(t^n)} = \lim_{s\to 0}\frac{1}{s^n L(s)}$ $\left(\text{or } \lim_{z\to 1}\frac{1}{(z-1)^n L(z)}\right)$ (Type(n): no dc error to $O(t^{<n})$ input; finite dc error to $O(t^n)$ input; infinite dc error to $O(t^{>n})$ input; step=$O(t^0)$; ramp=$O(t^1)$).

Routh-Hurwitz: # RHP roots = # sign changes in 1st column; Auxiliary polynomial (= row above zero row) is a factor of original polynomial; it can yield imaginary-axis poles especially when aux poly is of 2nd degree.

Root-locus: #branches = max(#poles, #zeros); originations at poles & terminations at zeros; #asymptotes = |#poles - #zeros|; asymptote meeting point = (sum poles - sum zeros)/(#asymptotes); asymptote angles = (odd * π)/(#asympotoes); (sum of pole-vector angles)-(sum of zero vector angles) = odd * π; (prod of pole-vector lengths)/(prod of zero-vector lengths) = k; breakaway = roots of derivative of inverse loop-gain (necessary condition); imaginary-axis roots found by RH-test applied to char. poly. = [num(loop-gain) + den(loop-gain)] when s^1-row is zero, so s^2-row is auxiliary poly. (For z-domain, unit-circle roots found by Jury-test applied to char. poly. so $a_2 = |a_0|$.

Nyquist-plot: Polar plot of loop-gain freq-response, $L(j\bar{\omega}) = L(z)|_{z=(1+\frac{T}{2}j\bar{\omega})/(1-\frac{T}{2}j\bar{\omega})}$, where $\bar{\omega}$ is bilinear freq;

unstable poles of closed-loop = # unstable poles of open-loop + # directional encirclements of (-1,0);
Gain-margin = (1/$|L(j\bar{\omega})|$) at π-phase freq $\bar{\omega}_p$; phase-margin $\phi_m = |\pi - |\angle L(j\bar{\omega})||$ at unit-gain freq $\bar{\omega}_g$;
max-sensitivity to noise, $\max_{\omega}\left|\frac{1}{1+L(j\omega)}\right|$ = inverse of min distance between (-1,0) and Nyquist-plot.

1st -order control: $D(j\bar{\omega}) = a_0(1 + j\bar{\omega}/\bar{\omega}_0)/\,(1 + j\bar{\omega}/\bar{\omega}_p)$; phase-lag: $\bar{\omega}_p < \bar{\omega}_0$; phase-lead: $\bar{\omega}_p > \bar{\omega}_0$.

Apply bilinear $j\bar{\omega} \to \frac{2}{T}\left[\frac{z-1}{z+1}\right]$ to convert $D(j\bar{\omega}) \to D(z) = k\frac{z-z_0}{z-z_p}$. Then $k = a_0\left[\frac{\bar{\omega}_p(\bar{\omega}_0 + {}^2/_T)}{\bar{\omega}_0(\omega_p + {}^2/_T)}\right]$, $z_0 = \frac{{}^2/_T - \bar{\omega}_0}{{}^2/_T + \bar{\omega}_0}$, $z_p = \frac{{}^2/_T - \bar{\omega}_p}{{}^2/_T + \bar{\omega}_p}$

Phase-lag: Choose $\bar{\omega}_g$ s.t. $\theta = \angle D(j\bar{\omega}_g) = -\pi + \phi_m - \angle G(j\bar{\omega}_g) = -5°$; $\bar{\omega}_0 = 0.1\bar{\omega}_g$; $|G(j\bar{\omega}_g)|\frac{a_0\bar{\omega}_p}{\bar{\omega}_0} \approx 1$.

Phase-lead: Choose $\bar{\omega}_g$ s.t. $\theta = \angle D(j\bar{\omega}_g) = -\pi + \phi_m - \angle G(j\bar{\omega}_g) > 0$, and $|G(j\bar{\omega}_g)| < \frac{\cos\theta}{a_0}$.

$\Rightarrow\ \bar{\omega}_p = \frac{\bar{\omega}_g \sin\theta}{\cos\theta - a_0|G(j\bar{\omega}_g)|}$; $\bar{\omega}_0 = \frac{a_0\bar{\omega}_g|G(j\bar{\omega}_g)|\sin\theta}{1 - a_0|G(j\bar{\omega}_g)|\cos\theta}$ $\left(\bar{\omega}_g = \sqrt{\bar{\omega}_p\bar{\omega}_0} \Rightarrow \theta = \tan^{-1}\left(.5\left[\sqrt{\frac{\bar{\omega}_p}{\bar{\omega}_0}} - \sqrt{\frac{\bar{\omega}_0}{\bar{\omega}_p}}\right]\right); |G(j\bar{\omega}_g)|\left(a_0\sqrt{\frac{\bar{\omega}_p}{\bar{\omega}_0}}\right) \approx 1\right)$.

Phase-lag-lead: $\bar{\omega}_{p1} < \bar{\omega}_{01} \ll \bar{\omega}_{02} < \bar{\omega}_{p2}$ with $\frac{\bar{\omega}_{01}}{\bar{\omega}_{p1}} = \frac{\bar{\omega}_{p2}}{\bar{\omega}_{02}} = \frac{\bar{\omega}_g\alpha}{\bar{\omega}_g/\alpha} = \alpha^2$; $\bar{\omega}_{01} = .1\bar{\omega}_g$. Choose $\bar{\omega}_g$ so $|G(j\bar{\omega}_g)|\left(\frac{a_0}{\alpha^2}\sqrt{\frac{\bar{\omega}_p}{\bar{\omega}_0}}\right) \approx 1$,

$\theta = -\pi + 5 + \phi_m - \angle G(j\bar{\omega}_g) > 0$, and $|G(j\bar{\omega}_g)| < \frac{\alpha^2\cos\theta}{a_0}$. Replace a_0 by $\frac{a_0}{\alpha^2}$ in $(\bar{\omega}_p, \bar{\omega}_0)$ of phase-lead to get $(\bar{\omega}_{p2}, \bar{\omega}_{02})$.

PID: $D(j\bar{\omega}) = k_P + \frac{k_I}{j\bar{\omega}} + \frac{k_D j\bar{\omega}}{1 + {}^T/_2 j\bar{\omega}} = \left(k_P + \frac{k_D\bar{\omega}^2\left(\frac{2}{T}\right)}{\left(\frac{2}{T}\right)^2 + \bar{\omega}^2}\right) + j\left(\frac{k_D\bar{\omega}\left(\frac{2}{T}\right)^2}{\left(\frac{2}{T}\right)^2 + \bar{\omega}^2} - \frac{k_I}{\bar{\omega}}\right)\Bigg|_{\bar{\omega}=\bar{\omega}_g} = \frac{\cos\theta}{|G(j\bar{\omega}_g)|} + j\frac{\sin\theta}{|G(j\bar{\omega}_g)|}$;

PI ($k_D = 0$) = phase-lag; PD ($k_I = 0$) = phase-lead. (use if –ve/+ve controller phase θ needed at given $\bar{\omega}_g$)

State-space: Controller: $u(k) = -K\hat{x}(k) + Nr(k)$; $\hat{x}$(k) is state-estimate with error: $e(k) = x(k) - \hat{x}(k)$.
Controlled-plant state: $x(k+1) = Ax(k) + B(-K\hat{x}(k) + Nr(k)) = (A - BK)x(k) + BKe(k) + BNr(k)$.
Controlled-plant output: $y(k) = Cx(k) + Du(k) = (C - DK)x(k) + DKe(k) + DNr(k)$.
Controlled-plant pole-placement: $K = [0\ 0\ \ldots\ 1][B\ AB\ \ldots\ A^{n-1}B]^{-1}\,Q(A)$.
Observer: $\hat{x}(k+1) = A\hat{x}(k) + Bu(k) + G(y(k) - \hat{y}(k)) = (A - BK)\hat{x}(k) + BNr(k) + GCe(k)$; $e(k+1) = (A - GC)e(k)$.
Observer pole-placement: $G^T = [0\ 0\ \ldots 1][C^T\ A^TC^T\ \ldots (A^T)^{n-1}C^T]^{-1}Q(A^T)$.
Controlled-system Eigen-values = Eigen-values($A - BK$) U Eigen-values($A - GC$).
Current-observer: prediction: $\bar{x}(k+1) = A\hat{x}(k) + Bu(k) = (A - BK)\hat{x}(k) + BNr(k)$;
update: $\hat{x}(k+1) = \bar{x}(k+1) + G(y(k+1) - \bar{y}(k+1)) = (A - BK)\hat{x}(k) + BNr(k) + GCAe(k)$;
$[x(k+1) - \bar{x}(k+1)] = \bar{e}(k+1) = Ae(k)$; $[x(k+1) - \hat{x}(k+1)] = e(k+1) = (A - GCA)e(k)$.
Current observer pole-placement: $G^T = [0\ 0\ \ldots 1][A^TC^T\ (A^T)^2C^T\ \ldots\ (A^T)^nC^T]^{-1}Q(A^T)$.
Controlled-system Eigen-values = Eigen-values($A - BK$) U Eigen-values($A - GCA$).
Since error-dynamics is uncontrollable, System TF = Reduced system TF (that ignores $e(k)$).
N can affect amplitude gain $[(C - DK)[zI - A + BK]^{-1}BN + DN]$ (and so dc gain, $(C - DK)[I - A + BK]^{-1}BN + DN$) but not the system poles at $\det\left(\left(zI - (A - BK)\right)^{-1}\right) = 0$.

For unit dc-gain, choose $N = KN_x + N_u$ such that $\begin{bmatrix} A - I & B \\ C & D \end{bmatrix}\begin{bmatrix} N_x \\ N_u \end{bmatrix} = \begin{bmatrix} 0 \\ I \end{bmatrix}$.