Region automaton R for timed automaton G

- State space $X \times \mathbb{R}_+^{|C|}$ is infinite

However, only finitely many Nerode equivalence classes, called regions, exist.

- $n_x \in \mathbb{N}$ be the largest integer against which clock x compared

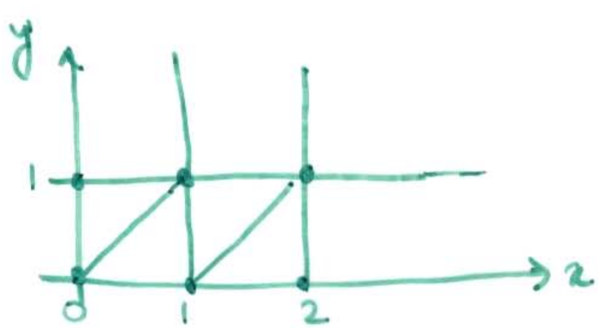
- $\vec{v}_1 \cong \vec{v}_2$ iff

(1) $\forall x \in C : \lfloor \vec{v}_1(x) \rfloor = \lfloor \vec{v}_2(x) \rfloor$ or $[\vec{v}_1(x) > n_x \wedge \vec{v}_2(x) > n_x]$

(2) $\forall x_1, x_2 \in C$ such that $\vec{v}_i(x_j) \leq n_{x_j}$:

$$[\text{frac}(\vec{v}_1(x_1)) \leq \text{frac}(\vec{v}_1(x_2))] \Leftrightarrow [\text{frac}(\vec{v}_2(x_1)) \leq \text{frac}(\vec{v}_2(x_2))]$$

Example: $C = \{x, y\}$; $n_x = 2$, $n_y = 1$

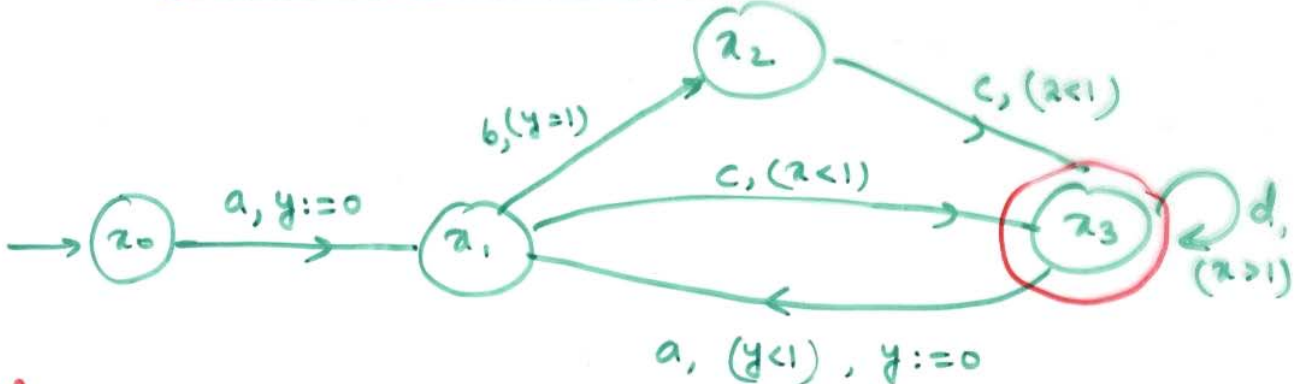


clock space = $\mathbb{R}_+^2$

6 corner points : $(1,1)$
14 open line segments : $0 < x = y < 1$
8 open regions : $0 < x < y < 1$

$\Rightarrow$ 28 clock regions

Construction of region automaton R

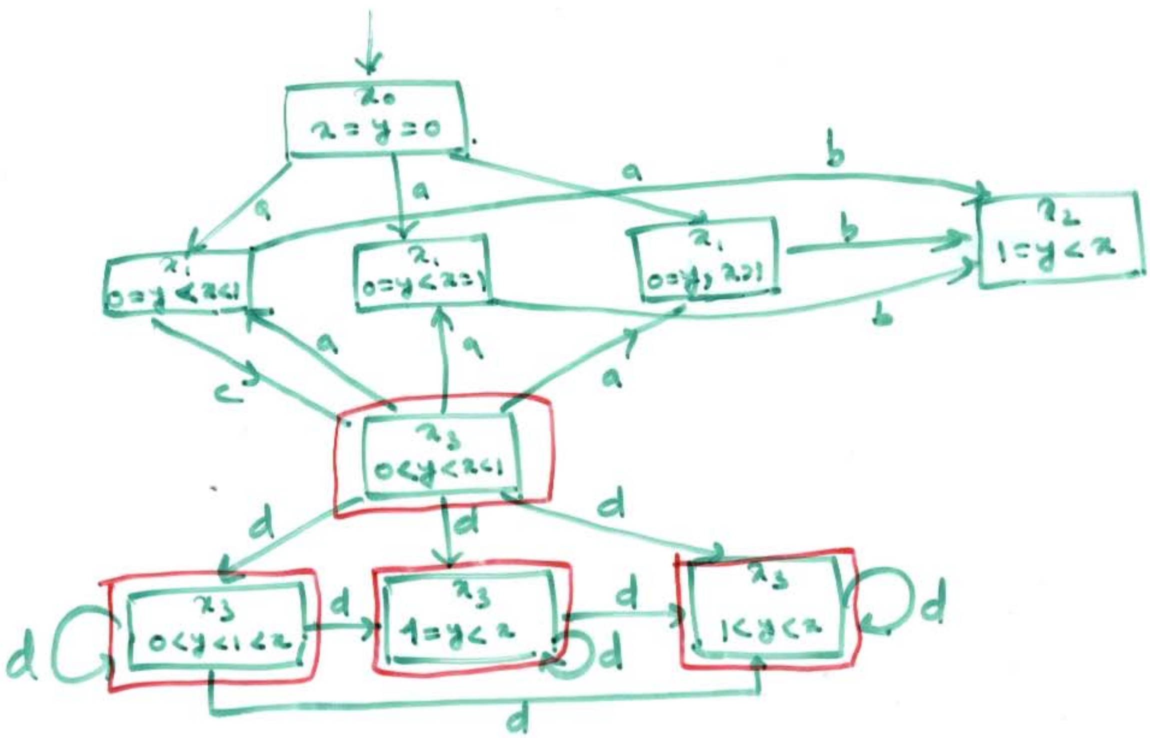


$C = \{x, y\}$

$n_x = n_y = 1$

18 regions

region automaton



$L_m(R) = \text{untime}(T_m(G)) = a(ca)^* c d^*$

Complexity: # of clock regions $= |C|!\ 2^{|C|} \prod_{x \in C} (2n_x + 2)$

$\forall x: \{x = c \mid c = 0, 1, \ldots, n_x\} \cup \{c-1 < x < c \mid c = 1, \ldots, n_x\} \cup \{x > c_x\} \to 2n_x + 2$

$\forall x_1, x_2$: $c-1 < x_1 < c$, $d-1 < x_2 < d$ whether $\text{fract}(x_1) \lesseqgtr \text{fract}(x_2)$

$\Rightarrow$ Emptiness PSPACE complete