Timed-Automata : Model for "real-time" systems

- Event occurrence time information needed for analyzing timed qualitative properties : "message be received within 10 s of sending"

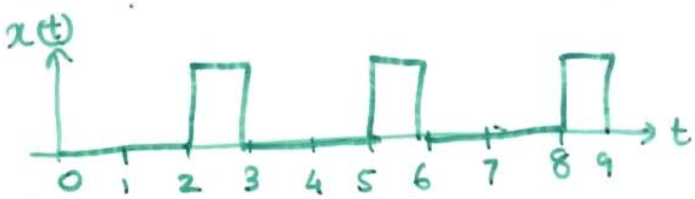
- Timed behavior of deterministic DES's described by timed-languages $\subseteq (\Sigma \times \mathbb{R}_+)^*$ consisting of

  timed-traces $\in (\Sigma \times \mathbb{R}_+)^*$

  $$(\sigma_0, t_0)\,(\sigma_1, t_1)\,(\sigma_2, t_2) \ldots \quad \begin{cases} t_0 \le t_1 \le t_2 \ldots \\ \text{and any finite interval} \\ \text{contains finite events} \end{cases}$$

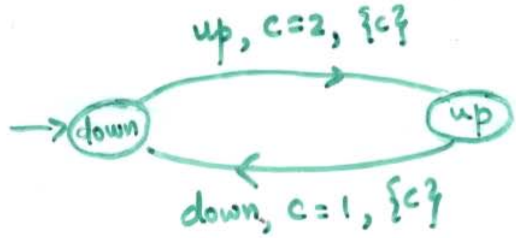
- Timed-automata (automata with clocks) accept timed languages

- Example :



clock signal

G



$T(G) = \{(up, 2), (down, 3)(up, 5) \ldots\}$

Timed automaton $G = (X, \Sigma, C, x_0, X_m, E)$

X: activity states; C: clocks; E: transitions

$$E \subseteq \underbrace{X}_{\text{start activity state}} \times \underbrace{\Sigma}_{\text{event label}} \times \underbrace{\Phi(C)}_{\text{clock condition}} \times \underbrace{2^C}_{\text{reset clocks}} \times \underbrace{X}_{\text{end state}}$$

transition set

$$\phi := c \le K \mid c \ge K \mid \phi_1 \wedge \phi_2 \mid \neg \phi$$

clock condition

$$e = (x, \sigma, \phi, \hat{C}, x')$$

transition

Timed traces accepted by a timed automaton

- state of G = $(x, \vec{v}) \in X \times \mathbb{R}_+^{|C|}$

  initial state = $(x_0, \vec{0})$

- <u>Notation</u>: $\vec{v} + t$, $t \in \mathbb{R}_+$, denotes clock vector after time t

  $\vec{v}\downarrow_{\hat{c}}$, $\hat{c} \subseteq C$, denotes clock vector with clocks in $\hat{c}$ reset to 0.

- <u>state transition</u>:

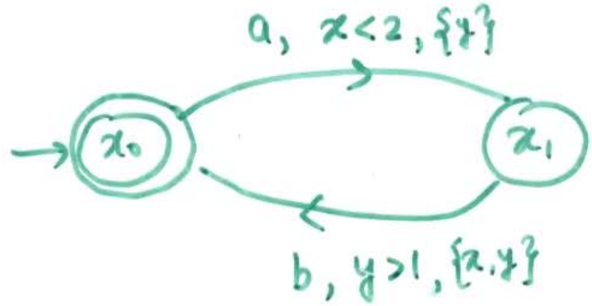
$$\begin{array}{c}(x, \vec{v}) \\ \Big\downarrow (\sigma, t) \\ (x', \vec{v'})\end{array} \iff \exists\, e = (x, \sigma, \phi, \hat{c}, x') \in E$$

such that

$\vec{v} + t$ satisfies ϕ (e enabled)

and $(\vec{v}+t)\downarrow_{\hat{c}} = \vec{v'}$

- <u>Example</u>:



$$(x_0, [0,0]) \xrightarrow[t_0<2]{a,\, t_0} (x_1, [t_0, 0]) \xrightarrow[t_1 - t_0 > 1]{b,\, t_1} (x_0, [0,0])$$

$$T_m(G) = \left\{ (a, t_0)(b, t_1)(a, t_2)(b, t_3) \ldots \;\middle|\; \begin{array}{l} t_{2i} - t_{2i-1} < 2 \text{ , and} \\ t_{2i+1} - t_{2i} > 1 \end{array} \right\}$$