Local Control

- **Def:** Supervisor called local if it can control only those events that it can also observe.

Observation mask: $M_{loc} : \Sigma \to \Sigma_{loc} \cup \{\varepsilon\}$ (projection type).

uncontrollable events for local supervisor: $\Sigma_{u,loc} = \Sigma_u \cup (\Sigma - \Sigma_{loc})$

- Stronger condition needed for existence of supervisors
- **Normality:** $M(s) = M(t),\ s \in pr(K),\ t \in L(G) \Rightarrow t \in pr(K)$.

equivalently, $M^{-1}M(pr(K)) \cap L(G) \subseteq pr(K)$

(HW) • **Fact:** Normality $\Rightarrow$ Observability (normality is stronger condition)

- **Theorem:** 1. K $\Sigma_{u,loc}$-controllable, M_{loc}-observable iff
  2. K Σ_u-controllable, M_{loc}-normal iff
  3. K $\Sigma_{u,loc}$ controllable, M_{loc}-normal.

$(1 \Rightarrow 2)$: Suffices to show M_{loc}-normality. Suppose for contradiction $M^{-1}M(pr(K)) \cap L(G) \not\subseteq pr(K)$. Let s smallest string in $M^{-1}M(pr(K)) \cap L(G) - pr(K)$.

Then $s \neq \varepsilon$, since $\varepsilon \in pr(K)$. So $s = \bar{s}\sigma$.
Since $\bar{s} \in pr(K)$, $s = \bar{s}\sigma \in L(G)$, and K $\Sigma_{u,loc}$-controllable $\Rightarrow \sigma \notin \Sigma_{u,loc}$

$\Rightarrow \sigma \in \Sigma_{loc} \Rightarrow M(\sigma) \neq \varepsilon$.

Since $s \in M^{-1}M(pr(K))$, there exists $t = \bar{t}\sigma \in pr(K)$ s.t. $M(t) = M(s) = M(\bar{s})\sigma$
contradiction to observability.

$(2 \Rightarrow 3)$: Suffices to show $\Sigma_{u,loc}$-controllability. Suppose for contradiction $pr(K)\Sigma_{u,loc} \cap L(G) \not\subseteq pr(K)$. Pick smallest $s \in pr(K)\Sigma_{u,loc} \cap L(G) - pr(K)$.
$s = \bar{s}\sigma$ with $\sigma \in \Sigma_{u,loc} - \Sigma_u \Rightarrow \sigma \notin \Sigma_{loc} \Rightarrow M(\sigma) = \varepsilon$. So $M(s) = M(\bar{s})$.
This contradicts normality since $\bar{s} \in pr(K)$, $s \in L(G) - pr(K)$.

$(3 \Rightarrow 1)$: obvious.

Test for Normality

- Need to test $M^{-1}M(pr(K)) \cap L(G) \subseteq pr(K)$
- Construct S_{NRM} by adding transitions in S s.t. $L(S_{NRM}) = M^{-1}M(pr(K))$
- (y_1, σ, y_2) a transition in $S \Rightarrow$ add (y_1, σ', y_2) where $M(\sigma') = M(\sigma)$

  $M(\sigma) = \varepsilon \Rightarrow$ add (y, σ, y) at every state y of S.

- Then K normal iff $L(S_{NRM}) \cap L(G) \cap L(\bar{S}^c) = \phi$

  computational complexity: $O(mn^2)$.

- Normality is preserved under union & intersection (for prefix-closed) $\Rightarrow$ sup $N(K)$ & inf $\overline{N}(K)$ exist.
- Acceptor for supremal normal sublanguage:
  - start with $\bar{S} \Rightarrow L_m(\bar{S}) = L_m(S) = K$, $L(\bar{S}) = \Sigma^*$
  - add transition is $\bar{S}$ as above to obtain $(\bar{S})_{NRM} \Rightarrow L_m(\bar{S}_{NRM}) = M^{-1}M(K)$, $L(\bar{S}_{NRM}) = \Sigma^*$
  - determinize $(\bar{S})_{NRM}$ to get $\hat{S} \Rightarrow L_m(\hat{S}) = M^{-1}M(pr(K))$, $L(\hat{S}) = \Sigma^*$.

- Consider $G \| \bar{S} \| \hat{S} \Rightarrow L_m(G\|\bar{S}\|\hat{S}) = K$, $L(G\|\bar{S}\|\hat{S}) = L(G)$.

  a typical state looks like $r = (x, y, \hat{Y})$, where $x \in X$, $y \in Y \cup \{y_d\} = \bar{Y}$, $\hat{Y} = \{y_1, \ldots, y_r\} \in 2^{\bar{Y}}$ with $y \in \hat{Y}$.

- r_1 and r_2 are called matching if $\hat{Y}_1 = \hat{Y}_2$.
- For each string leading to r_1, exists indistinguishable string leading to r_2

(a) $Z_0 := \{r \mid$ second coordinate is a dump state$\}$

(b) $Z' := Z_k \cup \{r \in Z - Z_k \mid \exists$ matching $r' \in Z_k\}$

$Z_{k+1} := Z' \cup \{r \in Z - Z' \mid r$ does not belong to trim component of $Z - Z'\}$

(c) Stop when $Z_k = Z_{k+1}$; else $k = k+1$, goto (b).

Maximally Permissive Supervison

- Consider $\underbrace{\sup P\left[\tilde{M}^{-1}\tilde{M}\left(pr(H)\right)\right]}_{f(H)} \cap L(G) \subseteq \underbrace{pr(H)}_{g(H)}$

- f monotone, not disjunctive; g monotone, not conjunctive
  $\sup O(K) := \sup\{H \subseteq K \mid H \text{ observable}\}$, $\inf \bar{O}(K) := \inf\{H \supseteq K \mid H \text{ observable}\}$
  need not exist.

- Example:  $M(a) = M(b) \neq \varepsilon$

  G

  $K_1 = \{b\}$, $K_2 = \{aa\}$ $\Rightarrow$ both K_1, K_2 observable.

  $K = K_1 \cup K_2 = \{b, aa\}$ not observable since $b, a \in pr(K)$; $M(b) = M(a)$, $aa \in pr(K)$
  $ba \in L(G) - pr(K)$.

  $K_1 = \{b, aa, baa, aaa\}$, $K_4 = \{b, aa, ba\}$ $\Rightarrow$ both K_1, K_2 observable
  $K = K_3 \cap K_4 = \{b, aa\}$ not observable.

- Extremal prefix-closed and observable languages:

- Consider $\sup P\left[\tilde{M}^{-1}\hat{M}\left(pr(H)\right)\right] \cap L(G) \subseteq pr(H)$ and $pr(H) \subseteq H$ —(1)

  Equivalently, $\underbrace{\sup P\left[\tilde{M}^{-1}\tilde{M}\left(pr(H)\right)\right]}_{f(H)} \cap L(G) \subseteq \underbrace{H}_{g(H)}$ —(2)

  (1)⇒(2) obvious; (2) implies 1^{st} inequality of (1); for 2^{nd} inequality of (1) use:
  Also, $pr(H) \subseteq \sup P\left[\tilde{M}^{-1}\tilde{M}\left(pr(H)\right)\right] \cap L(G) \subseteq H$ from (2)

- f monotone, not disjunctive; g conjunctive
  $\sup PO(K)$ need not exist ; $\inf \overline{PO}(K)$ exists

Since f is idempotent,

$$\inf \overline{PO}(K) = K \cup \left(f(K) \cap L(G)\right) = \sup P\left[\tilde{M}^{-1}\tilde{M}\left(pr(K)\right)\right] \cap L(G).$$

- Since $\sup O(K)$, $\sup PO(K)$, $\sup PCO(K)$ do not exist, unique maximally permissive control under partial observation does not exist

- A unique maximally permissive supervisor under partial obs. does not exist.
- "Sub-optimal" solutions: $\sup PCN(K)$ or $\sup RCN(K)$
- Alternative: Find an observable sublanguage of K:

$$K_{po} = K - \left[\tilde{M}^{-1}\tilde{M}\left(\left(L(G) - pr(K)\right) \cap K\Sigma\right)\right]\Sigma^*.$$

- Thm: Suppose K is prefix-closed. Then

  1) $\sup PCN(K) \subseteq K_{po} \subseteq K$
  2) K_{po} is prefix-closed and observable
  3) K_{po} is controllable, whenever K is controllable

- If K is not prefix-closed, the replace K by $\sup P(K)$.
- Above computation is also useful in design of local supervisors.

$$\sup PC_{\Sigma_u} N_{M, ve}(K) = \left[\sup PC_{\Sigma_u, ve}(K)\right]_{PO_{M, ve}}.$$

I.e., a modular computation is possible.

Maximally Permissive local Supervision.

- Consider $\quad \underbrace{M^{-1} M\,(pr(H))}_{f(H)} \cap L(G) \subseteq \underbrace{pr(H)}_{g(H)}$
- f disjunctive, g monotone but not conjunctive; $\quad f^{-1}(H) = [M^{-1}M(H)]\Sigma^*$
- So $\sup N(K) := \{H \subseteq K \mid H \text{ normal}\}$ exists; $\inf \overline{N}(K) := \{H \supseteq K \mid H \text{ normal}\}$ need not exist
- Iterative computation of $\sup N(K)$:

$$K_0 := K \quad ; \quad K_{i+1} = K_i - f^{-1}(L(G) - g(K_i)) = K_i - [M^{-1}M\,(L(G) - pr(K_i))]\Sigma^*.$$

- Extremal prefix-closed and normal languages:

consider $(M^{-1}M\,(pr(H)) \cap L(G) \subseteq pr(H)] \wedge [pr(H) \subseteq H] \Leftrightarrow [\underbrace{M^{-1}M\,(pr(H))}_{f(H)} \cap L(G) \subseteq \underbrace{H}_{g(H)}]$.

- f disjunctive, g conjunctive $\Rightarrow$ $\sup PN(K)$ and $\inf \overline{PN}(K)$ exist.
- f idempotent, so

$$\sup PN(K) = K - f^{-1}(L(G) - K) = K - [M^{-1}M\,(L(G) - K)]\Sigma^*$$

$$\inf \overline{PN}(K) = K \cup [f(K) \cap L(G)] = M^{-1}M\,(pr(K)) \cap L(G).$$

- Extremal prefix-closed / relative-closed, controllable and normal languages:

- $\sup PCN(K)$ and $\inf \overline{PCN}(K)$ can be computed iteratively.
- A modular computation is possible when any pair of controllable and uncontrollable events whenever indistinguishable are both unobservable, i.e.,

$\sigma_1 \in \Sigma_u,\ \sigma_2 \in \Sigma - \Sigma_u,\ M(\sigma_1) = M(\sigma_2) \Rightarrow M(\sigma_1) = M(\sigma_2) = \epsilon$, equivalently,

$$M^{-1}[M(\Sigma_u) - \{\epsilon\}] \subseteq \Sigma_u$$

(A projection mask satisfies this condition)

- Under this condition: $\quad \sup PCN(K) = \sup N\,(\sup PC(K))$

$$\sup RCN(K) = \sup N\,(\sup RC(K))$$

($\sup N$ and $\inf \overline{N}$ computations preserve prefix closure and relative closure)