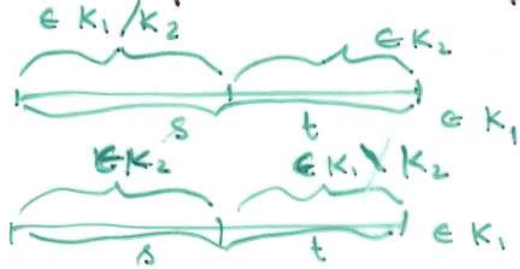
Operation on Languages

- Languages used for describing behavior of deterministic DESs.
- To describe complex systems in terms of simpler ones need operations on languages.



- Binary Operations:

Intersection $\quad K_1 \cap K_2$

difference $\quad K_1 - K_2$ $\qquad (K_1-K_2)-K_3 \neq K_1-(K_2-K_3)$

Choice $\quad K_1 + K_2 \quad (= K_1 \cup K_2)$

concatenation $\quad K_1 . K_2 \quad = \{s.t \in \Sigma^* \mid s \in K_1;\ t \in K_2\}$

- Quotient $\quad K_1/K_2 \quad = \{s \in \Sigma^* \mid \exists t \in K_2 \text{ s.t. } st \in K_1\}$
  "removes K_2 suffix from K_1"

after $\quad K_1 \backslash K_2 \quad = \{s \in \Sigma^* \mid \exists t \in K_2 \text{ s.t. } ts \in K_1\}$
  "removes K_2 prefix from K_1"

- Properties of Binary operations:

Operation	Notation	Commutative	Associative	Identity	Zero
Intersection	$\cap$	Yes	Yes	Σ^*	ϕ
Difference	$-$	no	no	none	none
choice	$+$	Yes	Yes	ϕ	none
concatenation	$.$	no	yes	$\{\varepsilon\}$	ϕ
Quotient	$/$	no	no	$\{\varepsilon\}$	ϕ
after	$\backslash$	no	no	$\{\varepsilon\}$	ϕ

Unary Operations on Languages

Complement $\quad K^c = \Sigma^* - K$

Kleene Closure $\quad K^* = \bigcup_{n \in \mathbb{N}} K^n \; ; \quad K^0 = \{\varepsilon\}$ and $K^{n+1} = K^n . K$

Prefix Closure $\quad pr(K) = \{ s \in \Sigma^* \mid \exists t \in K \text{ s.t. } s \leq t \}$

Extension Closure $\quad ext(K) = \{ s \in \Sigma^* \mid \exists t \in K \text{ s.t. } t \leq s \}$

Reverse $\quad K^R = \{ s^R \in \Sigma^* \mid s \in K \} ; \quad \varepsilon^R = \varepsilon$ and $(s\sigma)^R = \sigma s^R$

Projection on $\hat{\Sigma} \subseteq \Sigma \quad K \uparrow \hat{\Sigma} = \{ s \uparrow \hat{\Sigma} \mid s \in K \}$

$$\varepsilon \uparrow \hat{\Sigma} = \varepsilon \quad \text{and} \quad s\sigma \uparrow \hat{\Sigma} = \begin{cases} (s \uparrow \hat{\Sigma})\sigma & \text{if } \sigma \in \hat{\Sigma} \\ s \uparrow \hat{\Sigma} & \text{otherwise} \end{cases}$$

- K Kleene closed if $K^* = K$
  Prefix closed if $pr(K) = K$
  extension closed if $ext(K) = K$

- Properties:

operation	notation	idempotent	self-dual	monotone
complement	$(\cdot)^c$	no	yes	no
Kleene Closure	$(\cdot)^*$	yes	no	yes
prefix closure	$pr(\cdot)$	yes	no	yes
extension closure	$ext(\cdot)$	yes	no	yes
Reverse	$(\cdot)^R$	no	yes	yes
Projection	$(\cdot)\uparrow\hat{\Sigma}$	yes	no	yes

State Machine : Operations

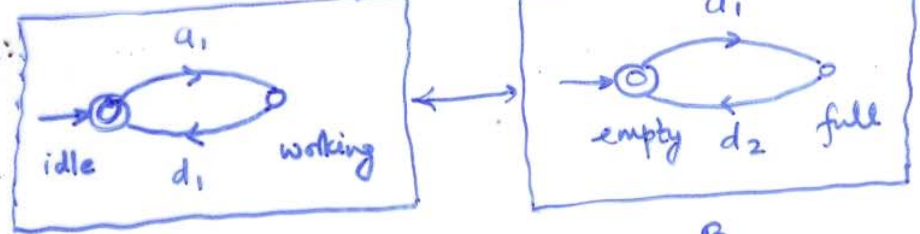
Synchronous composition : $G_i = (X_i, \Sigma_i, \alpha_i, x_{0,i}, X_{m,i})$; $i = 1, 2$.

$$G_1 || G_2 := (X, \Sigma, \alpha, x_0, X_m)$$

$X := X_1 \times X_2$; $\Sigma := \Sigma_1 \cup \Sigma_2$; $x_0 := (x_{0,1}, x_{0,2})$; $X_m := X_{m,1} \times X_{m,2}$

$$\underset{x=(x_1,x_2)}{\alpha(x,\sigma)} = \begin{cases} (\alpha_1(x_1,\sigma), \alpha_2(x_2,\sigma)) & \text{if } \alpha_1(x_1,\sigma), \alpha_2(x_2,\sigma) \text{ defined} ; \sigma \in \Sigma_1 \cap \Sigma_2 \\ (\alpha_1(x_1,\sigma), x_2) & \text{if } \alpha_1(x_1,\sigma) \text{ defined} ; \sigma \in \Sigma_1 - \Sigma_2 \\ (x_1, \alpha_2(x_2,\sigma)) & \text{if } \alpha_2(x_2,\sigma) \text{ defined} ; \sigma \in \Sigma_2 - \Sigma_1 \\ \text{undefined} & \text{otherwise} \end{cases}$$

- Common events occur synchronously ; others asynchronously.
- Example:

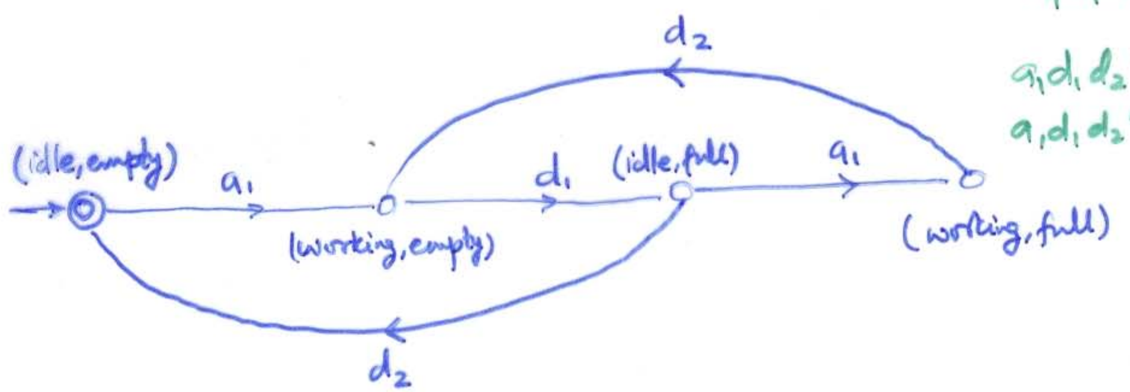


M B

$a_1 d_1 d_2 \in L_m(M||B)$

$a_1 d_1 d_2 \uparrow \Sigma_1 = a_1 d_1 \in L_m(M)$

$a_1 d_1 d_2 \uparrow \Sigma_2 = d_1 d_2 \in L_m(B)$



M || B

verify using software
compose the two SM's shown in Problems 2.26 of Chapter 1.
ftp://kumar.ee.engr.uky.edu/pub/HTTP/index.html

HW: ① Prove these

- $L_m(G_1||G_2) = \{s \in \Sigma^* \mid s \uparrow \Sigma_1 \in L_m(G_1), s \uparrow \Sigma_2 \in L_m(G_2)\}$.

$L(G_1||G_2) = \{s \in \Sigma^* \mid s \uparrow \Sigma_1 \in L(G_1), s \uparrow \Sigma_2 \in L(G_2)\}$.

② Compute comp. of machines in 2,6 [illegible]

- $\Sigma_1 = \Sigma_2 \Rightarrow L_m(G_1||G_2) = L_m(G_1) \cap L_m(G_2)$; $L(G_1||G_2) = L(G_1) \cap L(G_2)$.

State machine : Unary Operation

- Complementation : $G^c := (X, \Sigma, \alpha, x_0, X - X_m)$

$$G \text{ DSM} \Rightarrow L_m(G^c) = L(G) - L_m(G) \quad ; \quad L(G^c) = L(G)$$

HW: $L_m(G^c) = L(G) - L_m(G)$ may not hold for NSM. Give example. Prove this

- Completion : $\bar{G} := (\bar{X}, \Sigma, \bar{\alpha}, x_0, X_m)$

$$\bar{X} := X \cup \{x_D\} ; \quad \bar{\alpha}(\bar{x}, \sigma) := \begin{cases} \alpha(\bar{x}, \sigma) & \text{if } \bar{x} \in X, \alpha(\bar{x}, \sigma) \text{ defined} \\ x_D & \text{otherwise} \end{cases}$$

$$L_m(\bar{G}) = L_m(G) \quad ; \quad L(\bar{G}) = \Sigma^*.$$

HW: Prove this

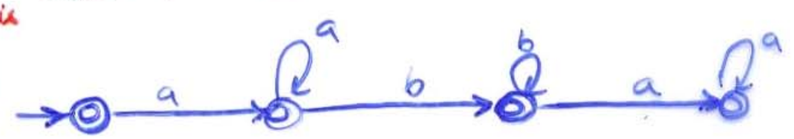
- Reverse : $G^R := (X \cup \{x_0^R\}, \Sigma, \alpha^R, x_0^R, \{x_0\})$

$$\alpha^R(x, \sigma) = \begin{cases} \{x' \in X \mid \alpha(x', \sigma) = x\} & \text{if } x \in X, \sigma \in \Sigma \\ X_m & \text{if } x = x_0^R, \sigma = \varepsilon \\ \phi & \text{otherwise} \end{cases}$$

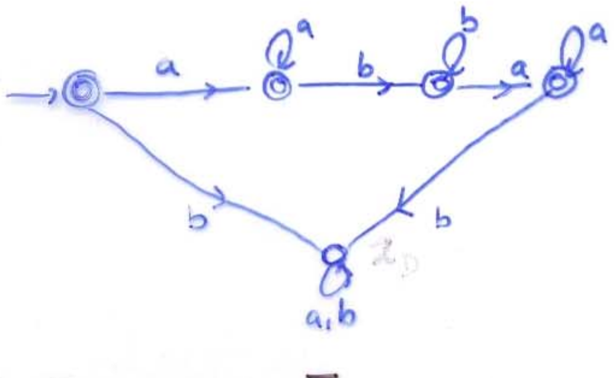
$$L_m(G^R) = (L_m(G))^R$$

HW: Prove this

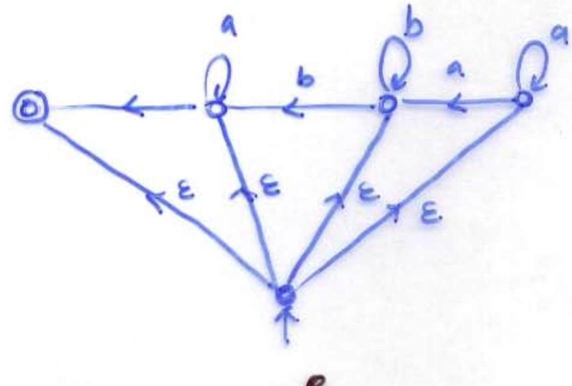
Example:



G



$\bar{G}$



G^R