EE 324 LAB 6

There and Back Again: Continuous to Discrete to Continuous

In this lab, you will learn how to study the frequency characteristics of discrete-time system and how these characteristics depend on the sampling time. You will also see the affect that different continuous-to-discrete (and reverse) approximation methods have on the frequency response.

Prelab:

Understanding the frequency behavior (Bode plot) of the system is not as intuitive as in the continuous-time case.

One approach is to transform $H_d(z)$ into a related continuous-time system $H(s)$ by using the bilinear transform (a.k.a. Tustin) approximation $s = \frac{2}{T}\frac{z-1}{z+1}$ relating the continuous-time Laplace-transform variable s with the discrete-time Z-transform variable z under the trapezoidal approximation introduced in the previous lab.

Note that the resulting $H(s)$ will be a function of T, the assumed sampling time, which may not be known to you.

1. For the discrete-time system of Problem 1 below, determine the continuous-time approximation for $T = .1$ and $T = 1$.

2. For the first continuous-time system $H_1(s)$ of Problem 2 below, compute its ZOH discrete-time approximation, $H_{ZOH}(z) \coloneqq (1 - z^{-1})\boldsymbol{Z}\left(\frac{H_1(s)}{s}\right)$. Note to find the $\boldsymbol{Z}$-transform of an s-domain quantity $G(s) \coloneqq \frac{H_1(s)}{s}$, first find the time-domain quantity $g(t)$ by taking the unilateral Laplace inverse of $G(s)$. Next sample $g(t)$ at $t = nT$ to get $g[n] \coloneqq g(nT)$. Finally, compute the $\boldsymbol{Z}$-transform of $g[n]$ to get $G(z) = \boldsymbol{Z}\left(\frac{H_1(s)}{s}\right)$. Then, $H_{ZOH}(z) = (1 - z^{-1})\boldsymbol{Z}\left(\frac{H_1(s)}{s}\right)$

Laboratory Assignment:

Problem 1:

We consider the discrete-time system defined by the following difference equation:

$$y[n] = 1.238y[n-1] - 0.3016y[n] + 0.03175x[n-2] - 0.03175x[n]$$

1. Compute the transfer function $H_d(z)$ of the system.

2. Compute the corresponding transfer functions $H_1(s)$ and $H_2(s)$ for $T = .1$ and $T = 1$, respectively, and then represent the two corresponding Bode plots (you may use the MATLAB function `bode`).

3. For describing the transfer function of the discrete-time system you may use either `zpk` or `tf` (make sure you specify the sampling time). Compare and report the step response of the discrete-time and the associated continuous-time systems, when $T = .1$ and $T = 1$. (Use the `step` command in MATLAB).

4. Verify your results using the MATLAB command `d2c(Hd,'tustin')`.

5. Draw your conclusions with respect to how the sampling time determines the discrete system's response.

Problem 2:

This section looks at Zero-Order-Hold (ZOH) discretization and the effects of the sampling time.

Consider the following two continuous-time systems described by their transfer functions:

$$H_1(s) = \frac{60}{s^2 + 2s + 60}; H_2(s) = \frac{600}{s^2 + 2s + 600}$$

For each $T \in \{1,0.1,0.01\}$ do the following:

1. Compute the Bode plots of $H_1(s)$ and $H_2(s)$ and compare them. Represent the poles of each system in the *s*-plane (you may use the MATLAB command `pzmap`).

2. Discretize each system using a ZOH with a sampling time *T*. You may use the MATLAB command `c2d(H,T,'zoh')`.

3. Compute and compare the step responses of H_1 and H_2 with those of their ZOH discretizations. For large values of *T* you should observe some interesting aliasing behavior due to the sampling time being not fast enough. It seems that the discretized system is "slower" than the actual one. This is the typical effect of under-sampling a system.

4. To figure out the ZOH discretization affects the frequency response of the systems, use the approach of Problem 1 to compute the associated continuous-time systems, $\overline{H}_1(s)$ and $\overline{H}_2(s)$, per the trapezoidal approximation method. Note that these systems should be different from H_1 and H_2.

5. Compute the Bode plots of $\overline{H}_1(s)$, $\overline{H}_2(s)$, and over-impose them on those of H_1 and H_2. What do you observe?

6. Describe the difference in terms of the poles and zeros, by over-imposing them on the same diagram that contains the poles of H_1 and H_2 derived in the first part.