EE 324 LAB 4

The Financial Problems

In this lab, you will model and simulate two, real-life systems.

Prelab:

Problem 1:

If we purchase a house, or a car, and take a loan of *d* dollars with a fixed interest rate of *R* percent per year (r = R/12 percentage per month), then the loan is paid back through the process known in economics as amortization. Using simple logic, it is not hard to conclude that the outstanding principle, $y[k]$, at $k + 1$ discrete-time instant (month), is given by the recursive formula (difference equation):

$$y[k+1] = y[k] + ry[k] - x[k+1] = (1+r)y[k] - x[k+1]$$

where $x[k+1]$ stands for the payment made in the $(k+1)st$ discrete-time month.

1. Build the block diagram of the system in Simulink using the delay block z^{-1}. Note that $Z\{y[k+1]\} = zZ\{y[k]\}$. The z^{-1} block in discrete-time systems functions as a “memory cell”, much like the $\frac{1}{s} = s^{-1}$ block in continuous-time systems. Also note that the initial condition $y[-1]$ can be specified in the delay block parameters window. This will represent the initial outstanding principle.

Problem 2:

The national income is governed by the following set of difference equations:

$$y[k] = c[k] + i[k] + x[k]$$

$$c[k] = \alpha y[k-1]$$

$$i[k] = \beta(c[k] - c[k-1])$$

where α and β are positive constants, $y[k]$ is the national income, $c[k]$ represents consumer expenditures, $i[k]$ is induced private investment, and $x[k]$ represents government expenditures.

1. Derive the input/output equation, using only x (input) and y (output) terms.

2. Build the block diagram of the system in Simulink using the delay block z^{-1}.

Laboratory Assignment:

Problem 1:

Continuing with the first problem:

2. Assume that a student purchases a car and takes a loan of \$10,000 with an interest rate $R = 5\%$ per year, for a period of 4 years (i.e. 48 months). Find out the required minimum monthly payment *p*.

   This problem can be solved analytically. However, in this lab, we want to use the simulations to help us find the right value. Set the initial condition $y[-1] = 10{,}000$ and plot the response of the system to a step of size p. (Hint: Set monthly payment input $x[k] = pu[k]$, ie, discrete-time step of size p so that out $y[k] = 0, @k = 48$.

Pick different values of p and compute the number of months it takes to finish the loan (i.e. to get to $y = 0$), until you find the minimum value of p for which the number of months is less than 48.

3. For this value of p, find the total amount of money the student pays for the car.

4. How much would the student save, if he/she could find a better rate of $R = 4\%$?

5. How much should the student pay per month, if he/she wants to repay the loan in 36 months instead of 48 at $R = 4\%$?

Problem 2:

1. Using the derived difference equation, derive the transfer function of the system from x to y.

2. Simulate the impulse response and the step response for three sets of parameters:
$(i)\ \alpha = \beta = 0.5\ (complex\ poles)$
$(ii)\ \alpha = \beta = 1\ (repeated\ poles)$
$(iii)\ \alpha = \beta = 2\ (real\ poles)$
and two sets of initial conditions:
$(i)\ y[-1] = y[-2] = 0\ (forced\ response)$
$(ii) y[-1] = 10, y[-2] = 0\ (forced + natural\ response)$

For your report, analyze all of your results and comment on how each system works and how different kinds of poles and initial conditions effect the behavior.

Lab 4 Tips:

- Define variables in MATLAB workspace to use them in block parameters (p, r, R, alpha, beta, etc.).
- **For all inputs set Sample time = 1**. *This allows you to consider simulation time as # samples.*

Problem 1:

1. Recommended simulation stop time = 50 (allows you to see up to k = 50 months).
2. Input is a step with parameters: **Step time = 1 (default), Final value = p (monthly payment in $)**
3. Change values of *p* until you get y[48] <= 0. Find the minimum *p*-value.
4. Total amount paid, *T*, after *k* months: $T = pk$
5. Change *R* (and *r*) – Repeat procedure to find minimum *p*-value.
   Find difference between *T* at *R* = 5% and at *R* = 4%.
6. At *R* = 5% (original annual interest rate), find minimum *p*-value for y[36] <= 0.

Problem 2:

1. Recommended simulation stop time = 10.
2. Input is a step with parameters: **Step time = 1, Final value = 1 (defaults)**
3. Mathematically derive difference equation (discrete-time) – Already done in prelab.
4. Derive transfer function (z-domain) – Similar to in CT (Take z-Transform and rearrange.)
5. Simulate at 3 different *alpha* and *beta* combinations, 2 different inputs (step & impulse), 2 sets of initial conditions **(3x2x2 = 12 plots)**.

Report Requirements:

Problem 1:

1. Block diagram
2. Plot for paying off loan by the 48th month
   a. Min *p*-value for *R* = 5%, y[48] <= 0
   b. Total amount paid under those conditions
3. Amount saved at min *p*-value for *R* = 4%, y[48] <= 0 vs. at min *p*-value for *R* = 4%
4. Plot for paying off loan by the 36th month
   a. Min *p*-value for *R* = 5%, y[36] <=0

Problem 2:

1. Difference equation (in DT, aka in terms of k)
2. Transfer function, T(z)
3. Block diagram (only one needed)
4. 12 plots – Specify what parameters (*alpha*, *beta*, initial conditions) and inputs used!

5. Mathematical analysis – Calculate exact poles for the 3 *alpha* and *beta* combos.
    a. Discuss how the pole location (w.r.t. unit circle) impacts stability and use it to describe your figures.
    b. Do not leave the plots unexplained. Connect your mathematical analysis to the plots/concepts.