EE 324 LAB 12

An Introduction to PI Controllers[1]

In this lab you will manually adjust a PI (Proportional Integral) controller used to control the angular velocity of an electric motor.

Prelab:

The transfer function of the motor (from input voltage to angular velocity), also referred to as a plant, is given by:

$$H(s) = \frac{10^7}{(s + 5 \times 10^2)(s + 2 \times 10^5)}$$

A classical continuous-time control system is represented in Figure 1, where $D(s)$ is the transfer function of the "controller".

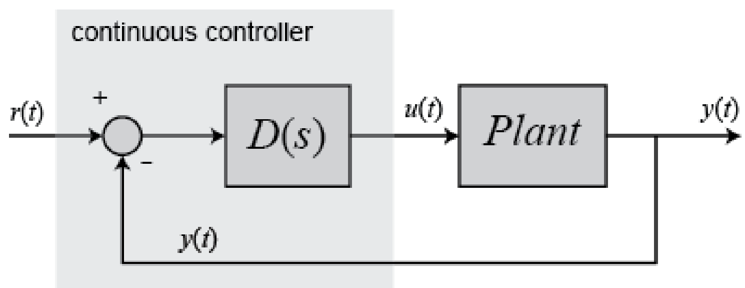

Figure 1 Continuous-time Control System

For our scenario, the purpose of the controller is to make sure that the angular frequency of the motor $y(t)$ remains close to the reference input signal $r(t)$. The analog controller can be replaced by a digital controller with transfer function $G(z)$, as in Figure 2.

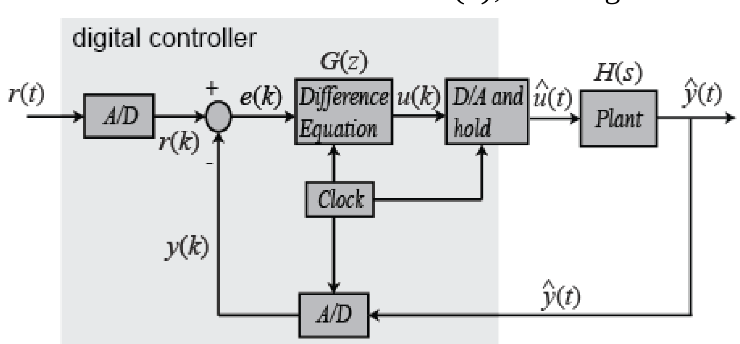

Figure 2 Discrete-time Control System

[1] The figures from this lab were copied from the Digital Control Tutorial available online at http://www.engin.umich.edu/group/ctm/digital/digital.html

Since $y(k)$ is just the discretized version of the system output $\hat{y}(t)$, we can consider the blocks encircled by the dashed line in Figure 3 as a new digital equivalent of the plant.

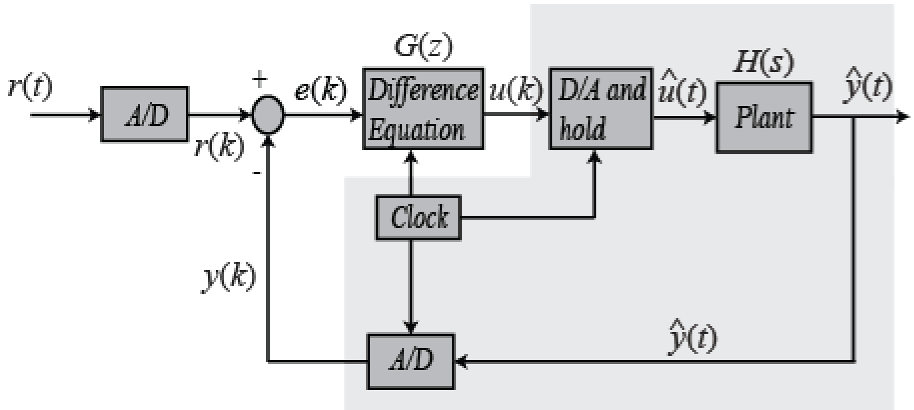

Figure 3 Equivalent Digital Representation of the Plant

The discretized plant block is isolated for clarity in Figure 4 and is denoted by $H_{zoh}(z)$ (as in "zero-order-hold").

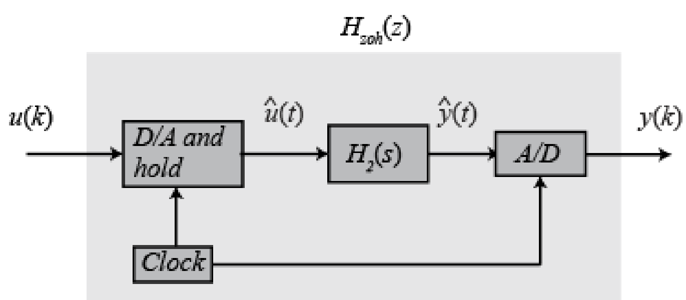

Figure 4 Equivalent Digital Representation of the Plant

For the digital controller $G(z)$ we shall use a digital Proportional-and-Integral (PI) controller with transfer function:

$$G(z) = K_p + K_i \frac{T_s}{2} \frac{z+1}{z-1}$$

where T_s is the sampling time of the A/D converters, and K_p and K_i are proportional and integral gains of the PI controller.

1. Represent the Bode plot of the motor and pick a suitable sampling time T_s, say 20 times faster than the Bode plot bandwidth (which is the largest pole value).

2. Compute the digital equivalent of the analog plant (motor) – remember the MATLAB function `c2d(H,`T_s`,'zoh')`, and write it down as $H_{zoh}(z)$.

3. Draw the block diagram of the overall digital control system, including the digital equivalent of the motor.

Laboratory Assignment:

1. Set $K_i = 0$.

2. Represent the z-domain root-locus of open-loop digital transfer function $G(z)|_{K_i=0} \times H_{zoh}(z) = K_p H_{zoh}(z)$ as K_p is varied, and pick a pole position, using `rlocfind`, which can provide good system performance: For stability and causality, the pole must lie inside the unit circle. Also, since for a z-domain pole $p = re^{j\Omega}$, $\frac{T_s}{|\ln(r)|}$ is its time-constant, $\frac{\Omega}{T_s}$ is its oscillation frequency, and $\frac{|\ln(r)|}{\sqrt{\ln^2 r + \Omega^2}}$ is its damping ratio, picking larger r and smaller Ω provides smaller time constant and larger damping. Note down the corresponding loop gain K_p.

3. Using this value of K_p, represent the Nyquist diagram for the open-loop transfer function $K_p H_{zoh}(z)$ and note down the phase and gain margins. (Use `H=tf(num, den,` T_s`)`and `nyquist(H)` commands.)

4. While keeping K_i equal to 0, next use the Simulink model of the closed-loop system to plot its step-response, and note down its settling-time (time it reaches with 5% of final dc-value), overshoot (maximum deviation over that dc-value), and rise-time (first time it attains that dc-value).

5. Keeping K_i equal to 0, modify the value of K_p until the step-response exhibits a good compromise between rise-time and overshoot. Note that increasing K_p may decrease the rise-time, but may increase the overshoot. So explore this trade-off in selecting K_p. Aim for no more than 10% overshoot.

6. Keeping K_p constant at the value found in the previous step, increase K_i from zero until the step-response becomes even more satisfactory. Note down this K_p, K_i pair of values, and the rise-time, overshoot, and settling-time.

7. Represent the Nyquist diagram of the new loop-gain $\left(K_p + K_i \frac{T_s}{2} \frac{z+1}{z-1}\right) \times H_{zoh}(z)$, and note down the phase and gain margins.