Section 4.4
Recursive Algorithms

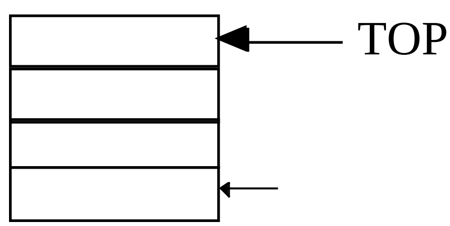
A recursive algorithm is one which calls itself to solve "smaller" versions of an input problem.

How it works:

• The current status of the algorithm is placed on a *stack*.

A *stack* is a data structure from which entries can be added and deleted only from one end.

- like the plates in a cafeteria:



PUSH: put a 'plate' on the stack.

POP: take a 'plate' off the stack.

When an algorithm calls itself, the current *activation* is suspended in time and its parameters are **PUSH**ed on a stack.

The set of parameters need to restore the algorithm to its current activation is called an *activation record*.

Example:

```
procedure factorial (n)
/* make the procedure idiot proof */
    if n < 0 return 'error'
    if n = 0 then return 1
else
    return n factorial (n-1)
```

The operating system supplies all the necessary facilities to produce:

factorial (3): **PUSH** 3 on stack and call

factorial (2): **PUSH** 2 on stack and call

factorial (1): **PUSH** 1 on stack and call

factorial (0): return 1

POP 1 from stack and return (1) (1)

POP 2 from the stack and return (2) [(1) (1)]

POP 3 from the stack and return (3) [(2) [(1) (1)]]

Complexity:

Let f(n) be the number of multiplications required to compute factorial (n).

$f(0) = 0$: the *initial condition*

$f(n) = 1 + f(n-1)$: the *recurrence equation*

Example:

A recursive procedure to find the max of a nonvoid list.

Assume we have a built-in functions called

- Length which returns the number of elements in a list
- Max which returns the larger of two values
- Listhead which returns the first element in a list

Max requires one comparison.

procedure *maxlist* (list)
/* strip off head of list and pass the remainder */

if Length(list) = 1 **then**
    return Listhead(list)
        else
    return Max(Listhead(list), *maxlist*
        (remainder of list))

The recurrence equation for the number of comparisons required for a list of length n, f(n), is

• f(1) = 0
• f(n) = 1 + f(n-1)

Example:

If we assume the length is a power of 2:

• We divide the list in half and find the maximum of each half.

• Then find the Max of the maximum of the two halves.

procedure *maxlist2* (list)
/* a *divide and conquer* approach */

if Length (list) = 1 **then**
return Listhead(list)
else
a = *maxlist* (fist half of list)
b = *maxlist* (second half of list)
return Max{a, b}

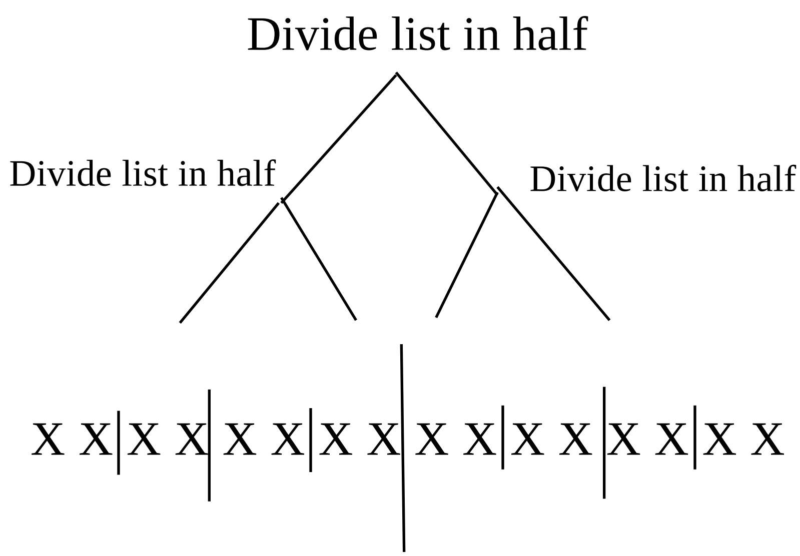
Recurrence equation for the number of comparisons required for a list of length n, f(n), is

• f(1) = 0
• f(n) = 2 f(n/2) + 1

• There are two calls to *maxlist* each of which requires f(n/2) operations to find the max.

• There is one comparison required by the Max function.

If n = 16:



$f(16) = 2\ f(8) + 1$
$f(8) = 2\ f(4) + 1$
$f(4) = 2\ f(2) + 1$
$f(2) = 2\ f(1) + 1$

So

$f(2) = 1$,
$f(4) = 2\ (1) + 1 = 3$
$f(8) = 2\ (3) + 1 = 7$
$f(16) = 2(7) + 1 = 15$
$f(n)$?