Section 1.1
Propositional Logic

Some applications:
Design of digital electronic circuits.
Expressing conditions in programs.
Queries to databases & search engines

proposition : true = T (or 1) or false = F (or 0) (binary logic)

- ‘the moon is made of green cheese’
- ‘ go to town!’ X - imperative
- ‘What time is it?’ X - interrogative

propositional variables: P, Q, R, S, . . .

New Propositions from old: calculus of propositions - relate new propositions to old using

TRUTH TABLES

logical operators: unary, binary

Unary

Negation

‘not’
Symbol: ¬

Example:

P: I am going to town

¬ P:
I am not going to town;
It is not the case that I am going to town;
I ain't goin'.

Truth Table:

P	¬ P
F (0)	T (1)
T (1)	F (0)

Binary

Conjunction

'and'
Symbol:

Example:

P - 'I am going to town'
Q - 'It is going to rain'

P Q: 'I am going to town and it is going to rain.'

Truth Table:

P	Q	P Q
0	0	0
0	1	0
1	0	0
1	1	1

Note: Both P and Q must be true!!!!!

Disjunction

inclusive 'or'

Symbol:

Example:

P - 'I am going to town'

Q - 'It is going to rain'

P Q: 'I am going to town or it is going to rain.'

Truth Table:

P	Q	P Q
0	0	0
0	1	1
1	0	1
1	1	1

Note: Only one of P, Q need be true. Hence, the *inclusive* nature.

Exclusive OR

Symbol:

Example:

P - ‘I am going to town’
Q - ‘It is going to rain’

P Q: ‘Either I am going to town or it is going to rain.’

Truth Table:

P	Q	P Q
0	0	0
0	1	1
1	0	1
1	1	0

Note: Only one of P and Q must be true.

Implication

‘If…then…’
Symbol: $\rightarrow$

Example:

P - ‘I am going to town’
Q - ‘It is going to rain’

$P \rightarrow Q$: ‘If I am going to town then it is going to rain.’

Truth Table:

P	Q	$P \rightarrow Q$
0	0	1
0	1	1
1	0	0
1	1	1

Equivalent forms:

If P, then Q
P implies Q
If P, Q
P only if Q
P is a sufficient condition for Q
Q if P
Q whenever P
Q is a necessary condition for P

Note: The implication is false only when P is true and Q is false!

There is no causality implied here!

'If the moon is made of green cheese then I have more money than Bill Gates' (T)

'If the moon is made of green cheese then I'm on welfare' (T)

'If 1+1=3 then your grandma wears combat boots' (T)

‘If I’m wealthy then the moon is not made of green cheese.’ (T)

‘If I’m not wealthy then the moon is not made of green cheese.’ (T)

Terminology:

P = premise, hypothesis, antecedent
Q = conclusion, consequence

More terminology:

Q → P is the **CONVERSE** of P → Q

$\neg Q \to \neg P$ is the **CONTRAPOSITIVE** of P → Q

Example:

Find the converse and contrapositive of the following statement:

R: ‘Raining tomorrow is a sufficient condition for my not going to town.’

Step 1: Assign propositional variables to component propositions

P: It will rain tomorrow
Q: I will not go to town

Step 2: Symbolize the assertion

$$R: P \rightarrow Q$$

Step 3: Symbolize the converse

$$Q \rightarrow P$$

Step 4: Convert the symbols back into words

‘If I don’t go to town then it will rain tomorrow’

or

‘Raining tomorrow is a *necessary condition* for my not going to town.’

or

‘My not going to town is a *sufficient condition* for it raining tomorrow.’

Biconditional

‘if and only if’, ‘iff’
Symbol: $\leftrightarrow$

Example: P - ‘I am going to town’, Q - ‘It is going to rain’

$P \leftrightarrow Q$: ‘I am going to town if and only if it is going to rain.’

Truth Table:

P	Q	P ↔ Q
0	0	1
0	1	0
1	0	0
1	1	1

Note: Both P and Q must have the <u>same</u> truth value.

Others: NAND (|) Sheffer Stroke; NOR (↓) Peirce Arrow (see problems)

Breaking assertions into component propositions - look for the logical operators!

Example:

‘If I go to Harry’s or go to the country I will not go shopping.’

P: I go to Harry’s
Q: I go to the country
R: I will go shopping

If……P……or…..Q…..then….not…..R

$$(P \quad Q) \rightarrow \neg R$$

Constructing a truth table:

- one column for each propositional variable

- one for the compound proposition

- count in binary

- n propositional variables = 2^n rows

You may find it easier to include columns for propositions which themselves are component propositions.

Truth Table:

P	Q	R	$(P \quad Q) \rightarrow \neg R$
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	0

Question:

How many different propositions can be constructed from n propositional variables?

