

## Example involving while loop

- Bag containing white & black balls  
While at least two balls in bag, pick two randomly  
different color  $\Rightarrow$  Put white back, throw black away  
same color  $\Rightarrow$  Throw both; Add black (from outside pile)

- Question: Color of last ball?

Before we answer this, need to determine if there is ever a last ball?  
(i.e., termination must also be established)

- $w$ : no. of white balls,  $b$ : no. of black balls.

$$w \leq W$$

( $W$ : initial no. of white balls)

$$b \leq B$$

( $B$ : initial no. of black balls)

while  $w+b \geq 2$  do

$$S1) w, b \leftarrow w, b-1$$

(different color)

$$S2) w, b \leftarrow w-2, b+1$$

(both white)

$$S3) w, b \leftarrow w, b-1$$

(both black)

} randomly one selected  
and executed while  
 $w+b \geq 2$

- Termination: For each assignment it holds that

$$0 \leq w+b = T \quad \{ s_i \} \quad 0 \leq w+b \leq T$$

Total =  $T$  before assignment  $\Rightarrow$  Total less than  $T$  ( $= T-1$ ) afterwards

So in  $w+B-1$  iterations termination occurs

- $w+b$ : decreases monotonically and bounded below, called loop-variant  
loop-variant used for proving termination of while-loop.

- Color of last ball?

To answer this, need to guess appropriate loop-invariant.