

Examples of Proofs in 1st. order logic

$$\bullet \forall x: (P(x) \rightarrow Q(x)), \forall x: P(x) \rightarrow \forall x: Q(x)$$

$$1. \forall x: P(x) \rightarrow Q(x)$$

Premise

$$2. \forall x: P(x)$$

Premise

$$3. P(a) \rightarrow Q(a)$$

\forall -elimination in 1

$$4. P(a)$$

\forall -elimination in 2

$$5. Q(a)$$

MP 3 & 4

$$6. \forall x: Q(x)$$

\forall -introduction in 5

("a" is arbitrary)

$$\bullet P(a), \forall x: (P(x) \rightarrow \neg Q(x)) \rightarrow \neg Q(a)$$

$$1. P(a)$$

premise

$$2. \forall x: (P(x) \rightarrow \neg Q(x))$$

premise

$$3. P(a) \rightarrow \neg Q(a)$$

\forall -elimination in 2

$$4. \neg Q(a)$$

MP 1 & 3.

("a" is arbitrary)

$$\bullet \forall x: (P(x) \rightarrow Q(x)), \exists x: P(x) \rightarrow \exists x: Q(x)$$

$$1. \forall x: (P(x) \rightarrow Q(x))$$

premise

$$2. \exists x: P(x)$$

premise

$$3. [P(a)]$$

assume $x = a$ in 2

$$4. P(a) \rightarrow Q(a)$$

\forall elimination in 1

$$5. Q(a)$$

MP 3 & 4

$$6. \exists x: Q(x)$$

\exists introduction in 5

("a" is assumed)