

# 1st - Order Logic Inference Rules in PVS

1) skolemize (Eliminate  $\exists$  in premise,  $\forall$  in consequent)

$$\frac{\exists x: P(x)}{\beta} \quad \text{skolemize with } t \quad \frac{P(x \leftarrow t)}{\beta}$$

$$\frac{\alpha}{\forall x: P(x)} \quad \text{skolemize with } t \quad \frac{\alpha}{P(x \leftarrow t)}$$

2) instantiate (Eliminate  $\forall$  in premise,  $\exists$  in consequent)

$$\frac{\forall x: P(x)}{\beta} \quad \text{instantiate with } t \quad \frac{P(x \leftarrow t)}{\beta}$$

$$\frac{\alpha}{\exists x: P(x)} \quad \text{instantiate with } t \quad \frac{\alpha}{P(x \leftarrow t)}$$

• Termination (QED) in PVS occurs when,

$$\frac{P}{P} \quad (\text{one premise} = \text{one consequent})$$

$$\frac{}{T} \quad (\text{one consequent} = T)$$

$$\frac{}{F} \quad (\text{one premise} = F)$$

• 
$$\frac{\exists x: P(x) \vee \exists x: Q(x)}{\exists x: (P(x) \vee Q(x))} \quad \text{split} \quad \begin{array}{l} \frac{\exists x: P(x)}{\exists x: (P(x) \vee Q(x))} \rightarrow \\ \frac{\exists x: Q(x)}{\exists x: (P(x) \vee Q(x))} \rightarrow \end{array}$$

skolemize with c: 
$$\frac{P(c)}{\exists x: (P(x) \vee Q(x))} \quad \text{instantiate with } c \rightarrow \frac{P(c)}{P(c) \vee Q(c)} \equiv \frac{P(c)}{Q(c)}$$