Predicate Logic (1^{st}-order logic)

- Prop. logic limited in scope (Two constants, Boolean-valued variables)
  Useful for digital hardware but software deal with real variables
- Predicate logic allows arbitrary variables and constants
  Predicate $\equiv$ Relation
- <u>Language of Logic</u>: $\mathcal{L} = C \cup F \cup R$ (sets of consts, fns, relations)

  <u>Interpretation</u> of $\mathcal{L}$ on set $\mathcal{U}$ is a map I over $\mathcal{L}$ s.t.
  $\forall c \in C$, $I(c) \in \mathcal{U}$ is an element of $\mathcal{U}$
  $\forall f \in F$, $I(f)$ is n-ary fn. over $\mathcal{U}$ for n-ary fn. symbol f
  $\forall r \in R$, $I(r)$ is n-ary relation over $\mathcal{U}$ for n-ary rel. symbol r
  (interprétation assigns meaning to symbols in $\mathcal{L}$)
  <u>$\mathcal{L}$-structure or $\mathcal{L}$-algebra</u>: $(\mathcal{U}, I(\mathcal{L}))$

  <u>Examples</u>: Boolean-algebra: $C = \{T, F\}$, $F = \{\neg, \wedge, \vee\}$, $S = \mathbb{B}$
  Number system algebra: $(\mathbb{N}, 0, 1, +, \cdot)$
  Integer system algebra: $(\mathbb{Z}, 0, 1, +, \cdot, -)$
  Linear algebra (of matrices): $(M(\mathbb{C}), O, I, +, \cdot, -)$
  matrices with complex entries zero matrix

- <u>Terms of $\mathcal{L}$-algebra over variables set X</u>
  Each constant $c \in C$ is a term
  Each variable $x \in X$ is a term
  $t_1, \ldots, t_n$ terms, $f \in F$ n-ary fn. $\Rightarrow f(t_1, \ldots, t_n)$ term

  <u>Example</u> (terms of Boolean logic):
  ~~Constants~~ : T, F
  Prop. variables: p, q, r
  f, f_1, f_2 terms $\Rightarrow \neg f$, $f_1 \wedge f_2$, $f_1 \vee f_2$ terms