More on Quantifiers

- Quantifiers are **shorthands** for disjunction ($\exists$) & conjunction ($\forall$)

Consider predicate $p : X \to \mathbb{B}$ over finite domain $X = \{x_1, \dots, x_n\}$

Then $\forall x \in X : p(x) \equiv p(x_1) \wedge p(x_2) \wedge \dots \wedge p(x_n)$

$\exists x \in X : p(x) \equiv p(x_1) \vee p(x_2) \vee \dots \vee p(x_n)$

- So all laws of propositional logic are applicable to formulae with quant.

DeMorgan's law: $\neg(\forall x \in X : p(x)) \equiv \neg(p(x_1) \wedge \dots \wedge p(x_n))$

$\equiv \neg p(x_1) \vee \dots \vee \neg p(x_n)$

$\equiv \exists x \in X : \neg p(x)$

Similarly, $\neg(\exists x \in X : p(x)) \equiv \forall x \in X : \neg p(x)$

Example: It's not the case that someone has a toy.

$\equiv$ Everyone does not have a toy.

- Care must be taken when quantification is over empty set.

$\forall x \in \emptyset : p(x)$ is a tautology (equivalent to TRUE)

$\exists x \in \emptyset : p(x)$ is a contradiction (equivalent to FALSE)

Example: "All unicorns can swim" is TRUE (since no unicorns exist).

"Exist unicorn can swim" is FALSE (— ").

- Formula with free variable: $\exists y \in \mathbb{R} : y^2 \stackrel{?}{=} x$

Here y is bound, whereas x is free. So this is a predicate over x. What is that predicate?

- Formula with multiple quantifier: $\underbrace{\forall x \in \mathbb{N} \; \underbrace{\exists y \in \mathbb{R} : y^2 = x}_{\substack{y \text{ is bound, } x \text{ is free} \\ \Rightarrow \text{predicate over } x}}}_{\substack{\text{no free variable (called sentence)} \\ \Rightarrow \text{Either TRUE or FALSE.}}}$