# Predicate Logic (1st order logic)

- In 0-order prop. logic variables are binary valued
  ⇒ not suitable for reasoning about programs which allow reals/integers
  1st order logic allows arbitrary valued variables

- Also in 1st-order logic predicates or relations play a central role
  Predicate (equivalently relation) is a binary valued map:
  $r : \mathcal{U} \to \mathbb{B}$, here $\mathcal{U}$ = domain set and $\mathbb{B} = \{T, F\}$.

- Examples of relations/predicates:
  Unary (over 1 variable) Vowel : Alphabet $\to \mathbb{B}$ Vowel(a) = T, Vowel(b) = F
  Binary (over 2 variables) $> : \mathbb{R}^2 \to \mathbb{B}$ $[5 > 3] = T$, $[4 > 6] = F$.
  In general n-ary relations/predicates may be defined

- Quantifiers: Used for quantifying over elements of domain $\mathcal{U}$ of relations
  Universal quantifier, Forall ($\forall$), quantifies over all elements of $\mathcal{U}$
  Existential quantifier, Exists ($\exists$), quantifies over some elements of $\mathcal{U}$

- Examples of quantifiers: $\forall x \in \mathbb{R} : [x^2 > 0]$
  Here $[x^2 > 0]$ is a unary predicate, whose variable x is universally quantified

  $\forall x \in \mathbb{R}\ \exists y \in \mathbb{R} : [y > x]$

  Here $[x > y]$ is binary predicate, both of whose variables have been quantified

- In the language of 1st order logic,
  - predicate is called formula
  - formula with all variables quantified is called sentence
  - unquantified variables of formula are called free variables
  - quantified variables of formula are called bound variables