Ft propositional logic system

- Efforts toward developing "smallest possible prop. logic sys"

"smallest" ⇒ small no. of (i) connectives, (ii) axioms to define them, (iii) rules to infer further tautologies

- Ft system is due to Łukasiewicz developed in 1928

- Propositional logic formula $f = \mathbf{T} \mid \mathbf{F} \mid p \mid \neg f \mid f \to g$ (syntax)

- Axioms (definitions of $\neg$ and $\to$): A1: $f \to (g \to f)$

  (semantics of $\neg$ and $\to$)

  A2: $(f \to (g \to h)) \to ((f \to g) \to (f \to h))$

  A3: $(\neg f \to \neg g) \to (g \to f)$

- Rule of inference: Modus ponens : $\dfrac{f,\ f \to g}{g}$

- Ft system is Sound & Complete prop. logic system.

- Sound: Only tautology can be proved.
- complete: All tautology can be proved.
- Gödel proved that exist logic systems that are sound & incomplete. (1930)

Robbin's Conjecture (1933):

$$x \vee y = y \vee x$$
$$(x \vee y) \vee z = x \vee (y \vee z)$$
$$\neg(x \vee y) \vee (x \vee \neg y) = \neg x$$

- Above 3 eqs are enough to axiomatize propositional logic.

- Conjecture studied by noted logicians, remained open for over 60 years.

- 1996, McCune of Argonne National Lab, submitted a proof of conjecture found by an automated theorem prover (EQP) he had written.
- See Dec 10, 1996 NY Times Article