

Resolution

$h = (f \vee p) \wedge (g \vee \neg p)$ satisfiable iff $(f \vee g)$ satisfiable.

(\Rightarrow) Suppose not true, i.e., both f and g not satisfiable.

Choose $p = \text{TRUE}$. Then $f \vee p = \text{TRUE}$, but $g \vee \neg p = \text{FALSE}$
So, $h = \text{FALSE}$

Choose $p = \text{FALSE}$. Then $g \vee \neg p = \text{TRUE}$, but $f \vee p = \text{FALSE}$
So, $h = \text{FALSE}$

Thus h not satisfiable, a contradiction.

(\Leftarrow) Suppose f is satisfiable, then $f \vee p$ satisfiable for any p .

Set $p = \text{FALSE}$; then $g \vee \neg p = \text{TRUE}$. So $h = (f \vee p) \wedge (g \vee \neg p)$ which is satisfiable.

Similarly if g is satisfiable, then $g \vee \neg p$ satisfiable for any p .

Choose $p = \text{TRUE}$, then $f \vee p = \text{TRUE}$. So $h = (f \vee p) \wedge (g \vee \neg p)$ which is satisfiable.

Example:

$\{p, \neg t\}$ $\{\neg p, \neg r\}$ $\{r, \neg t\}$ $\{q, r, t\}$ $\{q, \neg r, \neg t\}$ $\{r, \neg r, \neg t\}$
 $\{\neg r, s\}$ $\{r, \neg s\}$ $\{r, s, t\}$ $\{p, \neg s, t\}$ $\{p, s, \neg t\}$ $\{\neg p, \neg s, \neg t\}$

resolve w.r.t p:

$\{q, \neg q\}$ $\{q, s, t\}$ $\{q, \neg s, \neg t\}$ $\{r, \neg s, t\}$ $\{\neg q, s, \neg t\}$ $\{s, t, \neg s, t\}$
 $\{s, t, \neg t\}$ $\{\neg s, t, \neg s, \neg t\}$ $\{s, \neg t, \neg s, \neg t\}$ $\{\neg q, \neg t\}$ $\{q, \neg r, t\}$ $\{q, r, \neg t\}$
 $\{\neg r, \neg r, \neg t\}$ $\{\neg r, s\}$ $\{s, \neg s\}$

resolve w.r.t q:

$\{s, t, \neg s, t\}$ $\{s, t, s, \neg t\}$ $\{s, t, r, t\}$ $\{s, t, \neg r, \neg t\}$ $\{\neg s, \neg t, \neg s, t\}$ $\{\neg s, \neg t, \neg s, \neg t\}$
 $\{\neg s, \neg t, \neg r, t\}$ $\{\neg s, \neg t, r, \neg t\}$ $\{\neg s, t, r, t\}$ $\{\neg s, t, \neg r, \neg t\}$ $\{r, t, \neg r, t\}$ $\{r, t, \neg r, \neg t\}$

$\{s, \neg t, \neg r, t\}$ $\{s, \neg t, r, \neg t\}$ $\{\neg r, t, \neg r, \neg t\}$ $\{r, \neg t, \neg r, \neg t\}$ $\{\neg r, s\}$ $\{r, \neg s\}$

resolve w.r.t r:

$\{s, t, \neg s, \neg t\}$ $\{s, t, \neg s, t\}$ $\{s, t, s\}$ $\{\neg s, \neg t, \neg s\}$ $\{\neg s, t, \neg s\}$
 $\{\neg s, \neg t, \neg s, \neg t\}$ $\{\neg s, \neg t, \neg s, t\}$ $\{s, \neg t, s\}$

resolve w.r.t s:

$\{t, \neg t\}$ $\{t\}$ $\{\neg t\}$ $\{t, \neg t\}$

resolve w.r.t t:

$\{?\}$

So not satisfiable!