Satisfiability

- One key question in prop. logic is whether a given formula f satisfiable?
  (Recall, satisfiable $\equiv$ not a contradiction) (f not satisfiable iff $f \equiv$ FALSE)
- f in DNF $\Rightarrow$ satisfiability trivial
- Suppose f in CNF-form : $f \equiv c_1 \wedge c_2 \wedge \dots \wedge c_n \equiv \{c_1, \dots, c_n\}$

$c_i \equiv l_{i1} \vee l_{i2} \dots \vee l_{in_i} \equiv \{l_{i1}, \dots, l_{in_i}\}$

$l_{ij} \equiv$ proposition or neg. of proposition

(c_i called clause; l_{ij} : called literal)

- Resolution is a method employed in determining satisfiability.
  The idea is to eliminate a prop. variable, thereby transforming the formula but preserving the property of satisfiability.

WLOG, any clause does not contain a prop. & its negation as literal
(If such a clause exists, then it is a tautology and can be removed from CNF-form yielding an equivalent formula.

Consider pair of clauses containing a prop. & it's negation respectively

$$\frac{(f_i \vee p) \wedge (g_j \vee \neg p)}{(f_i \vee g_j)} \quad \begin{array}{c}\text{satisfiable}\\ \text{iff}\\ \text{satisfiable}\end{array}$$

Resolution

- A seq. of resolution on f will result in
  either ~~single~~ clause $\Leftrightarrow$ f satisfiable,
  or empty clause $\Leftrightarrow$ f not satisfiable

DAVIS-PUTNAM ALGO
DP Algo (1958)

- Example:

$$p\ \frac{\{\neg p, q\} \quad \{p\}}{\{q\}}$$

$$\begin{array}{l} \\ p \\ \\ q \\ \\ r \\ \\ s \end{array}\begin{array}{c} \{\neg p, q\} \ \{\neg q, \neg r, s\} \ \{p\} \ \{r\} \ \{\neg s\} \\ \hline \{q\} \ \{\neg q, \neg r, s\} \ \{r\} \ \{\neg s\} \\ \hline \{\neg r, s\} \ \{r\} \ \{\neg s\} \\ \hline \{s\} \ \{\neg s\} \\ \hline \{\} \end{array}$$

- Checking equivalence by satisfiability: $(f \equiv g) \Leftrightarrow (f \wedge \neg g \equiv g \wedge \neg f \equiv \text{FALSE})$