

Deduction in Propositional Logic

(11)

- Deduction or inferencing requires
 - i) lang. of logic to express prior knowledge (axioms)
 - ii) rules to infer new knowledge (theorems)
- Establishing theorem can proceed in one of 3 ways:
 - i) Truth-table based
 - ii) Substitution (over prop. laws) based
 - iii) Inferencing based upon rules of inference
- Example: $((p \rightarrow q) \wedge \neg q) \rightarrow \neg p$

P	q	$p \rightarrow q$	$\neg q$	$(p \rightarrow q) \wedge \neg q$	$\neg p$	$((p \rightarrow q) \wedge \neg q) \rightarrow \neg p$
T	T	T	F	F	F	T
T	F	F	T	F	F	T
F	T	T	F	F	T	T
F	F	T	T	T	T	T

truth-table based

$$((p \rightarrow q) \wedge \neg q) \rightarrow \neg p \equiv \neg((p \rightarrow q) \wedge \neg q) \vee \neg p$$

$$\equiv (\neg(p \rightarrow q) \vee q) \vee \neg p$$

$$\equiv (\neg(p \vee q) \vee q) \vee \neg p$$

$$\equiv ((\neg p \wedge \neg q) \vee q) \vee \neg p$$

$$\equiv ((\neg p \vee q) \wedge (\underbrace{\neg q \vee q}_{\text{TRUE}})) \vee \neg p$$

$$\equiv (p \vee q) \vee \neg p \equiv \text{TRUE}$$

Substitution-based

To see deduction based on rules of inference, we need to first enlist those.