EE 438/538
Optoelectronic Devices & Applications

Part 1: Fundamentals of Optical Waves
Ray or Geometrical Optics

- Last time, we studied an extreme case of EM wave
- **Plane wave**: Its source must be infinitely big or infinitely far away

- The flat phase front and the parallel wavevector defines a “pencil of light” or a “ray”
- Its trace can be fully described using simple geometry

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Plane Wave: The subject of “Ray Optics” or “Geometrical Optics”
Beam Optics

- In reality, the no source is infinite or infinitely far away
- To describe EM waves from *finite* sources, the concept of “beam” was developed: It must be finite and diverging

Example of light from finite extent source

So, this a ray. Not feasible unless you are a Jedi.

This is a beam. It’s *finite* and *diverging*!

- More quantitative description of the beam concept:

  1. Divergence angle is small
     (< 15 degrees, \( \sin \theta \sim \theta \) must hold)

  2. Most energy concentrated near the propagation axis

**Paraxial approximation, Paraxial Beam**

Can we get a math formula for them? Not always. But yes at some special cases!
Step 1. Derive Paraxial Wave Eq. from Regular Wave Eq.

- Starting from Eq. 5, the wave equation
  \[ \frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2} + \frac{\partial^2 E}{\partial z^2} - \frac{1}{c^2} \frac{\partial E}{\partial t^2} = 0 \]

\[ E = E_H(x,y,z) \cdot e^{i\omega t} \]

- Helmholtz equation
  \[ \frac{\partial^2 E_H}{\partial x^2} + \frac{\partial^2 E_H}{\partial y^2} + \frac{\partial^2 E_H}{\partial z^2} + \frac{\omega^2}{c^2} E_H = 0 \]

- Paraxial wave equation
  \[ \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) E_p \cdot e^{i k z} + k^2 E_p \cdot e^{i k z} = 0 \]

\[ \rightarrow \left( \frac{\partial^2 E_p}{\partial x^2} + \frac{\partial^2 E_p}{\partial y^2} \right) E_p \cdot e^{i k z} + \left( \frac{\partial E_p}{\partial z} - 2 i k \frac{\partial E_p}{\partial z} - k^2 E_p \right) e^{i k z} + k^2 E_p \cdot e^{i k z} = 0 \]

Paraxial approximation
\[ \frac{\partial^2 E_p}{\partial x^2} + \frac{\partial^2 E_p}{\partial y^2} - 2 i k \frac{\partial E_p}{\partial z} = 0 \]

Paraxial approximation #1:
The divergence angle is small

(Caution) \( E_p \) is a function of \( z \) as well.
Step 2. Solve the Paraxial Wave Equation (skipped)

- Using cylindrical coordinate system, you will get:

\[ E_H = E_{H0} \cdot \frac{w_0}{w(z)} \cdot e^{-\frac{r^2}{w(z)^2}} \cdot e^{-i(Kz + \tan^{-1}\left(\frac{2r}{z_0}\right))} \cdot e^{-i\frac{Kz^2}{2R(z)}} \]

- **Magnitude**
  - Amplitude change along the direction of propagation
  - Amplitude change in transversal direction

- **Phase**
  - Phase change along the direction of propagation
  - Phase change in transversal direction

**Paraxial approximation #2**
(Most energy near the center) was also used in the solution process. So it is built-in.

**Important parameters defined during solution process**

\[ w^2(z) = w_0^2 \left(1 + \left(\frac{2r}{z_0}\right)^2\right), \quad R(z) = z \left(1 + \left(\frac{2r}{z_0}\right)^2\right), \quad \varepsilon_0 = \frac{\pi w_0^2}{\lambda} \]

Everything revolves around \( w_0 \)

What is it? Becomes obvious visually
**Solution of Paraxial Equation**

- Looks too complicated? It becomes much clearer visually.
- Basically, it is a finite width wave diverging from a circular spot in both directions.

\[
E_H = E_{H0} \cdot \frac{w_0}{w(z)} \cdot e^{-\frac{r^2}{w(z)^2}} \cdot e^{-i(Kz - \tan^{-1}\left(\frac{r}{w(z)}\right))} \cdot e^{-i \frac{K r^2}{2 R(z)}}
\]
Solution of Paraxial Equation

- Looks too complicated? It becomes much clearer visually
- Basically, it is a finite width wave diverging from a circular spot in both directions

This term shows that the intensity at the center of the beam (along the propagation axis) will decrease due to the divergence.

\[ E_H = E_{ho} \cdot \frac{w_0}{w(z)} \cdot e^{-\frac{r^2}{w^2(z)}} \cdot e^{-i(K^2 - \tan^{-1}(\frac{y}{w_0}))} \cdot e^{-\frac{kr^2}{2R(z)}} \]
Solution of Paraxial Equation

- Looks too complicated? It becomes much clearer visually.
- Basically, it is a finite width wave diverging from a circular spot in both directions.

\[ E_H = E_{ho} \cdot \frac{w_0}{w(z)} \cdot e^{-\frac{r^2}{w^2(z)}} \cdot e^{-i(Kz - \tan^{-1} \left( \frac{y}{z_0} \right))} \cdot e^{-j\frac{kr^2}{2R(z)}} \]

This term indicates that the transversal intensity profile is Gaussian: **Gaussian Beam**
Solution of Paraxial Equation

• Looks too complicated? It becomes much clearer visually
• Basically, it is a finite width wave diverging from a circular spot in both directions

\[ E \sim \left( \frac{r}{w_0} \right)^2 e^{-\frac{r^2}{w_0^2}} \]

This term indicates that the transversal intensity profile is Gaussian: **Gaussian Beam**

\[ E_H = E_{ho} \cdot \frac{w_0}{w(z)} \cdot e^{-\frac{r^2}{w^2(z)}} \cdot e^{-i(Kz - \tan^{-1}(\frac{r}{w_0}))} \cdot e^{-j\frac{kr^2}{2R(z)}} \]
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Solution of Paraxial Equation

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\[ E_H = E_{H0} \cdot \frac{w_0}{w(z)} \cdot e^{-\frac{r^2}{w(z)^2}} \]

\[ w^2(z) = w_0^2 \left( 1 + \frac{z}{z_0} \right)^2 \]

Linear increase in beam width when \( z >> z_0 \)
\( (w(z) \sim w_0 \cdot z) \)

This indicates that the extent of the beam (beam width) increases along the propagation
More specifically

$$\theta_{\text{div}} = \frac{\lambda}{\pi \cdot w_o}$$

$$z_o = \frac{\pi \cdot w_o^2}{\lambda}$$
Gaussian Beam

- Phase front

$z_0 < Z$: more and more like a spherical wave

$z_0 > Z$: more and more like a plane wave

\[ E_H = E_{ho} \cdot \frac{w_0}{w(z)} \cdot e^{-\frac{r^2}{w(z)^2}} \cdot e^{-i(Kz - \tan^{-1}\left(\frac{y}{z_0}\right))} \cdot e^{-i \frac{Kr^2}{2z}} \]
Gaussian Beam: The Importance

• #1: Most laser beams are Gaussian beams!
  – Lasing requires positive feedback
  – We need a resonator
  – The easiest way to realize an optical resonator is to place two mirrors face-to-face (called Fabry-Perot resonator)

Simple but too sensitive to alignment

Much more stable.
You can use any type of curve mirror. But the one with spherical curvature is the easiest to make (and cheapest too).
Gaussian Beam: The Importance

• #1: Most laser beams are Gaussian beams!

What’s so good about fitting the mirror surface and phase front? Envision it with “ray” view. Rays are perpendicular to the phase front. Fitting the mirror surface and phase front is equivalent to making all rays incident at normal angle, enabling them retrace their path exactly!
Gaussian Beam: The Importance

• #2: A plane wave focused by a lens becomes a Gaussian beam!

“Collimation”

Inverse Process

“Focusing”

“Depth of focus”

or

“Rayleigh Range”