Quantitative Transient Voltage Dip
Assessment of Contingencies using
Trajectory Sensitivities

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Abstract—A new index, voltage critical clearing time (V-CCT), is presented for quantitative assessment of transient voltage dips subject to various contingencies. Calculation of V-CCT needs estimation using trajectory sensitivities of voltages with respect to fault clearing time. The V-CCT indices indicate the severity of fault-initiated contingencies by comprehensively evaluating transient voltage dips according to dynamic performance criteria. Using trajectory sensitivities to obtain V-CCT minimizes the computational effort by avoiding repetitive trial-and-error time-domain simulations. Performance of the proposed transient voltage dips assessment method has been tested on a 9-bus system, the New England 39-bus system and a 13000-bus system. For the small test systems, some selected contingencies are compared to verify the consistency of ranking using V-CCT with time-domain simulation analysis. The computational efficiency of the proposed assessment method is analyzed. Then the case studies on the large system build up the relation between analysis results and system operating condition. The results show that the assessment using V-CCT reflects the contingency severity in scope of transient voltage dips.

Index Terms—Transient voltage dip (TVD), trajectory sensitivity, contingency ranking.

I. INTRODUCTION

Transient voltage dip (TVD) refers to the short-term voltage magnitude reduction after faults or other disturbances, such as transformer energizing, large motor starting and heavy load switching [1], that result in extreme increase of currents. TVD is an important aspect of power quality. Severe TVD brings high consequences in various industry areas [2-5]. To avoid TVD, time-domain simulations must be done and preventive actions taken when unacceptable TVD is detected [6]. In this paper, we present a new index to facilitate fast TVD assessment after fault-initiated contingencies.

There is a significant body of literature on assessing TVD. In [7], the IEC and IEEE TVD standards and application areas were reviewed. Reference [8] presented various TVD indices relating voltage dip duration and energy variation. Reference [9] developed a TVD index considering compatibility between equipment and supply. The TVD duration assessment criteria were summarized in [10] from various industry resources. Some other TVD assessment standards include voltage dip window criterion [11] and economic cost [12]. In [13, 14], stochastic methods were presented for TVD assessment.

Inspired by critical clearing time (CCT), a familiar metric to indicate power system rotor angle stability [15, 16], this paper proposes an index called voltage critical clearing time (V-CCT). The system dynamic security subject to fault-initiated contingencies is quantified by fast estimation of V-CCT. To obtain V-CCT, voltage trajectory sensitivities with respect to fault clearing time are first calculated. The calculated trajectory sensitivity information is used to estimate V-CCT, which is defined as the maximum fault clearing time for which the limit of TVD dynamic security region is reached. V-CCT is a comprehensive index, because it considers multiple TVD dynamic performance criteria that define the TVD dynamic security region. Using trajectory sensitivities to calculate V-CCT avoids time-consuming repetitive trial-and-error time-domain simulations to obtain those critical values. The calculated V-CCT are used to rank the TVD severity of assessed contingencies.

The rest of the paper is organized as follows. Section II introduces the trajectory sensitivities with respect to fault clearing time, including the calculation, initial condition determination and computational efficiency analysis. Section III introduces the TVD dynamic performance criteria used by NERC/WECC to define the TVD dynamic security region and presents the concept of V-CCT. Then the procedure of using V-CCT for TVD assessment is described. Section IV introduces an index to quantify the estimation error. Section V provides the case study results from the tests on three benchmark systems. Section VI summarizes the contributions of this paper.

II. TRAJECTORY SENSITIVITIES WITH RESPECT TO FAULT CLEARING TIME

The dynamics of a power system considering switching actions can be described by a differential algebraic discrete model as in [17]. A special but common case is the model described by differential algebraic equations (DAEs)
\begin{align*}
  \dot{x} &= f(x, y, t, \lambda) \quad (1) \\
  0 &= \begin{cases} 
    g^-(x, y, t, \lambda) & s(x, y, t, \lambda) < 0 \\
    g^+(x, y, t, \lambda) & s(x, y, t, \lambda) > 0 
  \end{cases} \quad (2)
\end{align*}

where \( x \) are dynamic state variables, \( y \) are algebraic variables and \( \lambda \) are system parameters and initial conditions. Examples of system parameters are transmission line impedances, generation levels and load parameters. To calculate V-CCT, the system parameter \( \lambda \) considered in this paper is fault clearing time \( t_{cl} \). The discontinuity of the system is represented by switching between algebraic equations, denoted by superscripts “-” and “+”. A switching event occurs when the trigger function equals zero, i.e., \( s(x, y, t_{cl}) = 0 \). In this paper, \( g^- \) represents the period of time during the fault (the fault-on period), and \( g^+ \) the period of time after the fault (the post-fault period).

To better understand the post-fault trajectory sensitivities with respect to \( t_{cl} \), the differential equations are also represented with a fault-on set \( f^- \) and a post-fault set \( f^+ \). Since \( f \) are continuous, \( f^- = f^+ \) is satisfied at \( t_{cl} \).

References \[18-24\] studied trajectory sensitivities with respect to both system parameters and initial conditions. Change of fault clearing time \( t_{cl} \) is a special case, because it results in the initial condition deviations of all variables for the post-fault period. Differentiating the post-fault part of (1)-(2) yields

\begin{align*}
  \frac{\partial^2 x}{\partial t^2 |_{t_{cl}}} &= \frac{\partial f^+}{\partial x} \frac{\partial x}{\partial t_{cl}} + \frac{\partial f^+}{\partial y} \frac{\partial y}{\partial t_{cl}} + \frac{\partial f^+}{\partial \lambda} \frac{\partial \lambda}{\partial t_{cl}} \\
  0 &= \frac{\partial g^+}{\partial x} \frac{\partial x}{\partial t_{cl}} + \frac{\partial g^+}{\partial y} \frac{\partial y}{\partial t_{cl}} + \frac{\partial g^+}{\partial \lambda} \frac{\partial \lambda}{\partial t_{cl}}. \quad (3)
\end{align*}

DAEs (1)-(2) describe a time-variant system, in which time \( t \) is an independent variable but not explicitly expressed, thus \( \frac{\partial f^+}{\partial t_{cl}} = 0 \) and \( \frac{\partial g^+}{\partial t_{cl}} = 0 \) are satisfied. Define

\[ \frac{\partial^2 x}{\partial t^2 |_{t_{cl}}} = \hat{x}_{cl}, \quad \frac{\partial x}{\partial t_{cl}} = x_{cl}, \quad \frac{\partial f^+}{\partial x} = f_{cl}^+ \quad \text{and} \quad \frac{\partial g^+}{\partial x} = g_{cl}^+ \].

The trajectory sensitivity equations (3)-(4) become

\begin{align*}
  \dot{\hat{x}}_{cl} &= f_{cl}^+ x_{cl} + f_{cl}^+ y_{cl} \quad (5) \\
  0 &= g_{cl}^+ x_{cl} + g_{cl}^+ y_{cl}. \quad (6)
\end{align*}

Both \( x(t, t_{cl}) \) and \( y(t, t_{cl}) \) vary with \( t_{cl} \). Given a small change in fault clearing time \( \Delta t_{cl} \), the resulting deviation of a variable trajectory is estimated through Taylor series expansion as

\begin{equation}
  \Delta \phi(t, t_{cl} + \Delta t_{cl}) = \frac{\partial \phi}{\partial t_{cl}} (t, t_{cl}) \Delta t_{cl} + \text{higher order terms}
\end{equation}

\begin{equation}
  \approx \frac{\partial \phi}{\partial t_{cl}} (t, t_{cl}) \Delta t_{cl} = \phi_{cl} (t, t_{cl}) \Delta t_{cl} \quad (t \geq t_{cl})
\end{equation}

where \( \phi = [x, y] \).

For a specific pre-fault operation condition, changing fault clearing time affects only fault-on trajectories and post-fault trajectories. The initial conditions of post-fault trajectory sensitivities for dynamic state variables are obtained using the fact that they are variable sensitivities to time \( t \) at \( t_{cl} \), i.e.,

\[ x_{cl} (t_{cl}, t_{cl}) = f^- (x, y, t_{cl}) \bigg|_{t=t_{cl}} = f^+ (x, y, t_{cl}) \bigg|_{t=t_{cl}}. \quad (8) \]

The initial conditions of post-fault trajectory sensitivities for algebraic variables are obtained from (6) as

\[ y_{cl} (t_{cl}, t_{cl}) = \left[- \left[g_{cl}^+ \right]^{-1} g_{cl}^+ x_{cl} \right]_{t=t_{cl}}. \quad (9) \]

Equation (9) requires that \( g_{cl}^+ \) be nonsingular along the post-fault trajectories. Otherwise, the inverse of \( g_{cl}^+ \) results in infinite sensitivity, a special case when sensitivity based estimation is not applicable \[19\].

Equations (1)-(2), along with their augmentation (5)-(6), give the solutions of post-fault trajectories and their trajectory sensitivities with respect to fault clearing time. An efficient calculation of trajectory sensitivities shown in \[19, 25\] is used in this paper: if an implicit method such as trapezoidal rule is used for integration of DAEs (1)-(2), and a Newton method is used to solve the nonlinear equations in each integration step \[26\], the Jacobian matrix factorization calculated in the solving process can be directly reused for solving the trajectory sensitivity equations (5)-(6). Since Jacobian matrix factorization is the most time-consuming part in the DAE solution \[27\], the additional computational effort of solving for the sensitivity equations is minimized.

### III. TVD Assessment

#### A. TVD Dynamic Performance Criteria and Dynamic Security Region

Calculation of V-CCT requires dynamic performance criteria to determine the TVD dynamic security region. Commonly used criteria consider both low voltage and high voltage limits during the oscillations and the time duration when a limit is violated, i.e., violation duration, as shown in Fig. 1. The criteria are used to evaluate the post-fault transient voltage trajectories and define the boundary of TVD dynamic security region. System performance subject to various disturbances can then be classified as acceptable or unacceptable in terms of TVD \[28, 29\]. Unacceptable cases need special attention to enhance the system dynamic security.

We use the criteria defined by NERC/WECC, stated as follows \[10\]:

- **N-1 contingencies**: Not to exceed 25% at load buses or 30% at non-load buses; not to exceed 20% for more than 20 cycles at load buses.
- **N-k (k ≥ 2) contingencies**: Not to exceed 30% at any bus; not to exceed 20% for more than 40 cycles at load buses.
Both N−1 contingencies and N−k (k ≥ 2) contingencies are considered in the NERC/WECC criteria. Transient voltages exceeding the defined magnitudes are treated as violations, as shown in Fig. 1. The violation regions include both voltage over-shooting region and voltage dip region. The violation duration is defined as the total time that a trajectory is out of the secure region. NERC/WECC criteria allow a few cycles of short periods exceeding certain voltage levels; this violation allowance is not included in the violation duration.

C. Procedures of the Proposed TVD Assessment Method

The procedure of TVD assessment using V-CCT is as follows:

Step 1: Select contingencies for TVD assessment.

Step 2: For contingency i, base-case time-domain simulation is performed, and voltage trajectory sensitivities with respect to fault clearing time are calculated.

Step 3: For contingency i, V-CCT is computed from (10)-(13) based on the base-case simulation and corresponding trajectory sensitivities obtained from step 2.

Step 4: i = i + 1 and repeat steps 2-3, until completing all contingencies, and then go to step 5.

Step 5: Rank the severity of all contingencies based on their V-CCT.

In the procedure described above, base-case time-domain simulation performed for each contingency will consume most of the analysis time. The effort of trajectory sensitivity calculation and the V-CCT computing process is negligible compared with time-domain simulation.

IV. ESTIMATION ERROR INDEX

Because power systems are nonlinear, using trajectory sensitivities to calculate V-CCT will cause estimation error, the degree of which depends on the magnitude of MAX(Δt_cl) and the system nonlinearity characteristics around t_cl^base. Detailed estimation nonlinearity analysis can be found in [20, 32]. In case of unacceptable estimation error analysis, reference [32] presented two strategies: second order trajectory sensitivities and switching operating states method, to reduce the estimation error. In [32], the test of the proposed two strategies to improve the estimation accuracy in case of large change in fault clearing time shows satisfactory results. To quantify the estimation performance in this paper, an estimation error index (E EI) is formulated as

\[
\text{E EI} = \frac{\left\| v'(k) - v^*(k) \right\|_2}{N} = \frac{\sqrt{\sum_{k=1}^{N} [v'(k) - v^*(k)]^2}}{N}
\]

where

- \( v'(\cdot) \) Estimated trajectory at estimated V-CCT
- \( v^*(\cdot) \) Real trajectory at V-CCT through trial-and-error simulations
- \( N \) Total points of a trajectory
- \( k \) k-th value along all the N points of a trajectory
- \( \| \cdot \|_2 \) 2-Norm operator

The norm of trajectory difference is further normalized by the total number of value points (N) of a trajectory.

E EI evaluates voltage trajectory estimation of a single bus. For a power system, among the E EI for all buses, the largest one
is selected as the metric to quantify the estimation error. If a bus is tripped, its voltage is not considered.

EEI is used to evaluate the accuracy of V-CCT, but establishing this accuracy should be done off-line, because calculating \( v^*(\cdot) \) requires trial-and-error simulations.

V. CASE STUDIES

Three systems are used to evaluate the performance of the proposed assessment method: a 9-bus system [33], the New England 39-bus system [34] and a large 13000-bus system.

A. 9-Bus System

The one-line diagram of 9-bus system is shown in Fig. 2. The base-case contingencies are described in Table 1, all with fault clearing time 0.04s. It is assumed that a fault occur only at terminal of a transformer or one end of a transmission line. In the last column of Table 1, ‘F’ is followed by the number of bus closest to the faulted transmission line or transformer, and ‘O’ followed by outaged transmission line(s) or transformer(s) \( \{b_i, b_j\} \) connecting bus \( i \) and bus \( j \). Table 1 shows the estimated V-CCT based on trajectory sensitivities, actual V-CCT obtained from trial-and-error simulations, EEI and the rank. The rank \( r_1 \) is based on estimated V-CCT, and \( r_2 \) based on actual V-CCT.

Table 1

Voltage assessment results on the 9-bus System.

<table>
<thead>
<tr>
<th>#</th>
<th>Est./ Act. V-CCT</th>
<th>EEI (c-4)</th>
<th>Rank ( r_1 / r_2 )</th>
<th>Contingency Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.131 / 0.149</td>
<td>9.561</td>
<td>11 / 11</td>
<td>F-7; O-(7,8)</td>
</tr>
<tr>
<td>2</td>
<td>0.013 / 0.013</td>
<td>7.924</td>
<td>2 / 2</td>
<td>F-6; O-(6,9)</td>
</tr>
<tr>
<td>3</td>
<td>0.174 / 0.189</td>
<td>10.562</td>
<td>13 / 13</td>
<td>F-9; O-(8,9)</td>
</tr>
<tr>
<td>4</td>
<td>0.055 / 0.057</td>
<td>8.587</td>
<td>10 / 10</td>
<td>F-7; O-(5,7)</td>
</tr>
<tr>
<td>5</td>
<td>0.133 / 0.150</td>
<td>9.562</td>
<td>12 / 12</td>
<td>F-9; O-(6,9)</td>
</tr>
<tr>
<td>6</td>
<td>0.025 / 0.025</td>
<td>6.687</td>
<td>3 / 3</td>
<td>F-5; O-(5,7)-[4,5]</td>
</tr>
<tr>
<td>7</td>
<td>0.036 / 0.034</td>
<td>5.739</td>
<td>7 / 7</td>
<td>F-6; O-(4,6)</td>
</tr>
<tr>
<td>8</td>
<td>0.011 / 0.012</td>
<td>8.130</td>
<td>1 / 1</td>
<td>F-8; O-(8,9)-[7,8]</td>
</tr>
<tr>
<td>9</td>
<td>0.033 / 0.033</td>
<td>5.846</td>
<td>6 / 6</td>
<td>F-7; O-(5,7)-[4,5]</td>
</tr>
<tr>
<td>10</td>
<td>0.025 / 0.026</td>
<td>6.243</td>
<td>4 / 4</td>
<td>F-9; O-(6,9)-[3,4]</td>
</tr>
<tr>
<td>11</td>
<td>0.031 / 0.030</td>
<td>5.963</td>
<td>5 / 5</td>
<td>F-8; O-(8,9)</td>
</tr>
<tr>
<td>12</td>
<td>0.052 / 0.055</td>
<td>6.541</td>
<td>9 / 9</td>
<td>F-5; O-(5,7)</td>
</tr>
<tr>
<td>13</td>
<td>0.042 / 0.046</td>
<td>5.653</td>
<td>8 / 8</td>
<td>F-7; O-(8,9)-[7,8]</td>
</tr>
</tbody>
</table>

Though the estimated V-CCT deviate from the actual V-CCT to some degree, the rank \( r_1 \) and \( r_2 \) are consistent. To further validate the analysis results in Table 1, Fig. 3 compares the worst-case bus voltage in the rank 1 contingency CON #8 and the worst-case bus voltage in the rank 2 contingency CON #2.

- CON #8 is a three-phase fault on the bus 8 end of transmission line from bus 7 to bus 8. The fault is cleared by tripping the faulted line with inadvertent tripping of an adjacent bus 8.
- CON #2 is a three-phase fault that is applied on the bus 6 end of transmission line from bus 6 to bus 9. The faulted transmission line is cleared successfully after the fault.

CON #8 is an \( N-k \ (k \geq 2) \) inadvertent-tripping contingency. The worst-case bus is bus 5, which is a load bus. CON #2 is an \( N-1 \) contingency and the worst-case bus is bus 6, which is also a load bus. To compare these two contingencies, we set identical fault clearing time to be the typically used 5 cycles, i.e., 0.083s. The worst-case bus voltage trajectories are compared in Fig. 3, which indicates CON #8 is worse than CON #2, in accordance with the analysis result in Table 1.

Table 1 indicates that the estimated V-CCT are close to the actual V-CCT. To further analyze the estimation error, we plot in Fig. 4 the relation between EEI and \( \text{MAX}(\Delta_{t_1}) \).

In Fig. 4, the \( \text{MAX}(\Delta_{t_1}) \) are evenly distributed along the horizontal axis. Because of system nonlinearity, the estimation error increases with the absolute value of \( \text{MAX}(\Delta_{t_1}) \), denoted as \( abs[\text{MAX}(\Delta_{t_1})] \).

From Fig. 4, given fixed \( t_{cl}^{\text{base}} \), large \( abs[\text{MAX}(\Delta_{t_1})] \) is one reason of large EEI. This is because the nonlinearity is worse when V-CCT deviates more from \( t_{cl}^{\text{base}} \). Based on this feature, one strategy to reduce the effect caused by estimation error is to set the \( t_{cl}^{\text{base}} \) to a small value, to make it closer to V-CCT of more severe contingencies. Though larger EEI indicates worse estimation accuracy, the V-CCT is also larger, which suggests that corresponding contingencies are more secure and thus do
not need further attention.

CON #3 has the largest EEI. Compared with the actual V-CCT 0.189s, its estimated V-CCT is 0.174s, with EEI = 10.5621e-4. We take CON #3 as an example to observe the estimation accuracy, and relate the EEI value with direct observation of trajectories. CON #3 consists of a three-phase fault at the bus 9 end of the line from bus 8 to bus 9, cleared by tripping the faulted line. The worst-case bus is bus 6. The voltages at bus 6 obtained through (a) simulation at actual V-CCT, (b) estimation using (7) at estimated V-CCT, and (c) simulation at base-case clearing time (\( t_{base} = 0.04s \)) are shown in Fig. 5. Although this is for the case with the largest EEI, the estimation is still acceptable.

**Fig. 4** Comparison of EEI for different V-CCT deviations.

Test on the New England 39-bus system further confirms the analysis on the 9-bus system. The one-line diagram of the 39-bus system is in Fig. 6. The analysis results are in Table 2.

From Table 2, the estimated V-CCT are close to actual V-CCT. The rank \( r_1 \) and \( r_2 \) are consistent for most contingencies. Inconsistency occurs when a group of contingencies has very close actual V-CCT. This does not affect our analysis because this group of contingencies have almost the same severity, thus should receive equal attention of the operators. The relationship between EEI(e-4) and V-CCT deviation is setup for the 39-bus system in Fig. 7, which also shows the increase of EEI with \( \text{abs}[	ext{MAX}(\Delta t_{ij})] \), as in Fig. 4. Strict increase does not occur, because V-CCT is for the worst-case bus and the worst-case bus differs for different contingencies.

**Fig. 6** New England 39-bus test system.

<table>
<thead>
<tr>
<th>#</th>
<th>Est. / Act. V-CCT</th>
<th>EEI (e-4)</th>
<th>Rank ( r_1 / r_2 )</th>
<th>Contingency Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.630 / 0.667</td>
<td>6.734</td>
<td>25 / 25</td>
<td>F-2; O-[230]</td>
</tr>
<tr>
<td>2</td>
<td>0.864 / 0.872</td>
<td>7.384</td>
<td>26 / 26</td>
<td>F-10; O-[1032]</td>
</tr>
<tr>
<td>3</td>
<td>0.192 / 0.200</td>
<td>4.094</td>
<td>7 / 9</td>
<td>F-19; O-[1933]</td>
</tr>
<tr>
<td>4</td>
<td>0.403 / 0.412</td>
<td>5.925</td>
<td>19 / 19</td>
<td>F-20; O-[2034]</td>
</tr>
<tr>
<td>5</td>
<td>0.255 / 0.259</td>
<td>4.924</td>
<td>11 / 11</td>
<td>F-22; O-[2235]</td>
</tr>
<tr>
<td>6</td>
<td>0.542 / 0.554</td>
<td>6.674</td>
<td>24 / 24</td>
<td>F-23; O-[2324]</td>
</tr>
<tr>
<td>7</td>
<td>0.509 / 0.519</td>
<td>6.637</td>
<td>23 / 23</td>
<td>F-25; O-[2537]</td>
</tr>
<tr>
<td>8</td>
<td>1.092 / 1.214</td>
<td>7.375</td>
<td>27 / 27</td>
<td>F-10; O-[1013]</td>
</tr>
<tr>
<td>9</td>
<td>0.456 / 0.461</td>
<td>5.837</td>
<td>21 / 21</td>
<td>F-39; O-[39]-[939]</td>
</tr>
<tr>
<td>10</td>
<td>0.193 / 0.199</td>
<td>4.109</td>
<td>8 / 8</td>
<td>F-17; O-[1727]</td>
</tr>
<tr>
<td>11</td>
<td>0.330 / 0.290</td>
<td>5.826</td>
<td>15 / 14</td>
<td>F-18; O-[18]-[1718]</td>
</tr>
<tr>
<td>12</td>
<td>0.249 / 0.245</td>
<td>4.937</td>
<td>10 / 10</td>
<td>F-27; O-[27]-[2627]</td>
</tr>
<tr>
<td>13</td>
<td>0.336 / 0.331</td>
<td>5.803</td>
<td>17 / 17</td>
<td>F-3; O-[34]</td>
</tr>
<tr>
<td>14</td>
<td>0.266 / 0.262</td>
<td>4.962</td>
<td>12 / 12</td>
<td>F-13; O-[13]-[1314]</td>
</tr>
<tr>
<td>15</td>
<td>0.154 / 0.156</td>
<td>4.836</td>
<td>4 / 4</td>
<td>F-16; O-[1619]</td>
</tr>
<tr>
<td>16</td>
<td>0.031 / 0.032</td>
<td>2.984</td>
<td>2 / 2</td>
<td>F-29; O-[29]-[2629]</td>
</tr>
<tr>
<td>17</td>
<td>0.184 / 0.186</td>
<td>3.869</td>
<td>5 / 6</td>
<td>F-24; O-[1624]</td>
</tr>
<tr>
<td>18</td>
<td>0.335 / 0.330</td>
<td>5.884</td>
<td>16 / 16</td>
<td>F-4; O-[4]-[44]</td>
</tr>
<tr>
<td>19</td>
<td>0.282 / 0.279</td>
<td>4.951</td>
<td>13 / 13</td>
<td>F-13; O-[1013]</td>
</tr>
<tr>
<td>20</td>
<td>0.194 / 0.196</td>
<td>4.213</td>
<td>9 / 7</td>
<td>F-17; O-[1617]-[1727]</td>
</tr>
<tr>
<td>21</td>
<td>0.102 / 0.104</td>
<td>3.926</td>
<td>3 / 3</td>
<td>F-16; O-[1619]-[1621]</td>
</tr>
<tr>
<td>22</td>
<td>0.455 / 0.460</td>
<td>5.837</td>
<td>20 / 20</td>
<td>F-1; O-[139]</td>
</tr>
<tr>
<td>23</td>
<td>0.368 / 0.361</td>
<td>5.826</td>
<td>18 / 18</td>
<td>F-6; O-[4]-[44]</td>
</tr>
<tr>
<td>24</td>
<td>0.508 / 0.516</td>
<td>6.443</td>
<td>22 / 22</td>
<td>F-8; O-[78]</td>
</tr>
<tr>
<td>25</td>
<td>0.329 / 0.326</td>
<td>5.826</td>
<td>14 / 15</td>
<td>F-18; O-[318]</td>
</tr>
<tr>
<td>26</td>
<td>0.187 / 0.185</td>
<td>3.907</td>
<td>6 / 5</td>
<td>F-16; O-[1516]-[1617]</td>
</tr>
<tr>
<td>27</td>
<td>0.020 / 0.021</td>
<td>2.987</td>
<td>1 / 1</td>
<td>F-24; O-[1624]-[2324]</td>
</tr>
</tbody>
</table>
Two contingencies are selected for comparison. The fault clearing time is 5 cycles for both contingencies. Since the worst contingency CON #27 involves very fast voltage collapse immediately after the fault is cleared, the rank 2 contingency CON #16 and rank 3 contingency CON #21 are analyzed, and the voltage trajectories of all buses are shown in Fig. 8.

- CON #16 is a three-phase fault on the bus 29 end of transmission line from bus 26 to bus 29, cleared by tripping the faulted line.
- CON #21 is a three-phase fault on the bus 16 end of the transmission line from bus 16 to bus 19, followed by tripping that line and also inadvertent tripping of an adjacent transmission line bus from 16 to bus 21. This is an N-2 contingency.

Fig. 8 shows that, CON #16 results in voltage collapse after about 1s; whereas CON #21 is not so severe.

To test the computational efficiency of the analysis, the proposed assessment method has been implemented on a MATLAB based time-domain simulator PSAT [35]. PSAT has built-in trapezoidal method with Newton method for integration. The generator 4th order model is used for simulation [27]. The trajectory sensitivity is calculated using the method in [19]. The modified PSAT performs (a) time-domain simulation, (b) trajectory sensitivity calculation and (c) solving (10)-(12) to calculate V-CCT for the two benchmark systems.

The simulation has been conducted in Windows environment, with an Intel Core 2 Duo CPU (2.10Hz) and 4.00 GB RAM. Fig. 9 shows the average analysis time of single contingency for the test cases.

From Fig. 9, V-CCT calculation costs negligible time compared with the whole analysis process. Since the augmented DAEs (5)-(6) are linear equations, the solution can be obtained through one iteration of Newton method at each time step, and the involved Jacobian matrix factorization can be obtained while solving the original DAEs (1)-(2). With the size increase of the test systems, the portion of iteration and matrix factorization in the whole simulation process has also increased, which provides more time saving in calculating trajectory sensitivities, as observed in Fig. 9.

C. 13000-Bus System

A 13000-bus system representing the PJM system is used to test V-CCT for large real systems. This test system has 430 generators, 13402 buses and 12488 branches. The total generation is 4.8 GW. Since PSAT is not capable of handling this large system, PSS/E is used for simulation. All generators are represented with GENROU machine model, IEEET1 exciter model and TGOV1 governor model [36]. Since it is difficult to list the results for all credible contingencies in the paper, only 10 contingencies within the same zone are selected. Six of the contingencies are three-phase faults on generator terminals followed by tripping the faulted generators. The rest contingencies are three-phase faults on transmission lines (138kV–500kV) followed by tripping the faulted lines.

Calculating trajectory sensitivities using the method in section II requires information that is not available from PSS/E, such as Jacobian matrix. Thus, the method introduced in [37] is used to approximate trajectory sensitivities. Base-case
simulation and a second run with infinitesimal increment of fault clearing time approximate the trajectory sensitivity as

\[
\begin{align*}
\frac{\partial v}{\partial t_{cl}}(t) &= \lim_{\varepsilon \to 0^+} \frac{v(t, t_{cl} + \varepsilon) - v(t, t_{cl})}{\varepsilon} \\
&\approx \frac{v(t, t_{cl} + \tau) - v(t, t_{cl})}{\tau}
\end{align*}
\]

(15)

where \( \tau(>0) \) is an infinitesimal increment. Trajectory sensitivity approximation using (15) simulates a contingency twice, which takes more time than the numerical solution method in section II but is still more efficient than the trial-and-error method based on repetitive time-domain simulations. The analysis results are shown in Table 3.

Table 3

<table>
<thead>
<tr>
<th>#</th>
<th>Est. / Act.</th>
<th>Rank</th>
<th>Remarks (trippered elements)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.165 / 0.161</td>
<td>2 / 2</td>
<td>Gen. (137.1 MW; 305.4 MVar)</td>
</tr>
<tr>
<td>2</td>
<td>0.193 / 0.200</td>
<td>5 / 5</td>
<td>Gen. (62.0 MW; 120.5 MVar)</td>
</tr>
<tr>
<td>3</td>
<td>0.197 / 0.201</td>
<td>6 / 6</td>
<td>Gen. (33.7 MW; 98.5 MVar)</td>
</tr>
<tr>
<td>4</td>
<td>0.165 / 0.171</td>
<td>5 / 3</td>
<td>Gen. (116.9 MW; 203.5 MVar)</td>
</tr>
<tr>
<td>5</td>
<td>0.176 / 0.179</td>
<td>4 / 4</td>
<td>Gen. (63.5 MW; 139.5 MVar)</td>
</tr>
<tr>
<td>6</td>
<td>0.139 / 0.135</td>
<td>1 / 1</td>
<td>Gen. (293.9 MW; 349.2 MVar)</td>
</tr>
<tr>
<td>7</td>
<td>0.315 / 0.320</td>
<td>10 / 10</td>
<td>Line (44.6 MW; 0.2 MVar)</td>
</tr>
<tr>
<td>8</td>
<td>0.260 / 0.270</td>
<td>8 / 8</td>
<td>Line (104.1 MW; 92.6 MVar)</td>
</tr>
<tr>
<td>9</td>
<td>0.216 / 0.219</td>
<td>7 / 7</td>
<td>Line (380.7 MW; 381.9 MVar)</td>
</tr>
<tr>
<td>10</td>
<td>0.269 / 0.277</td>
<td>9 / 9</td>
<td>Line (88.4 MW; 13.9 MVar)</td>
</tr>
</tbody>
</table>

Table 3 shows the estimated and actual V-CCT, and the corresponding ranking. The estimated V-CCT are close to actual V-CCT. Rank r1 and r2 are consistent. For generator-tripping contingencies, the tripped generation is listed in the remarks column of the table. For the line-tripping contingencies, the power transfer on the lines is listed in the remarks column.

Table 3 indicates that the V-CCT of generator-tripping contingencies are relatively smaller than V-CCT of the line-tripping contingencies. The severity of TVD is associated with the generation of the tripped generators. For line-tripping contingencies, taking out a line with more power transfer results in lower V-CCT and therefore higher severity in TVD.

VI. CONCLUSIONS

This paper has presented a new index, i.e., V-CCT, for power system TVD quantitative assessment. V-CCT applies the concept of CCT to TVD assessment. It indicates the severity of fault-initiated contingencies based on TVD dynamic performance criteria. Obtaining V-CCT is computationally efficient, due to the use of trajectory sensitivities to estimate V-CCT.

The proposed index has been tested on a 9-bus system, the New England 39-bus system and a 13000-bus system. Computational efficiency has also been analyzed to show that trajectory sensitivity calculation and V-CCT calculation consume much less time compared with the time used for base-case time-domain simulation. Ranking the contingencies provides a priority list for further actions to prevent the potential unacceptable voltage quality.

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REFERENCES


