IOWA STATE UNIVERSITY

Transmission Line Loading

Sag Calculations and High-Temperature Conductor Technologies

James Slegers

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Contents

Introduction

Transmission lines are physical structures, installed in the natural environment – an environment which subjects them to wind, rain, ice, snow, sunlight, and pollution. Beyond the natural environment, these structures exist in a human-developed environment. Structures must be designed to minimize damage to themselves, as well as preventing injury to humans and other structures. A successful design will be safe, reliable, and efficient. A few specific design criteria will be described below, which contribute to such a design.

Transmission lines will be designed to limit the distance that their conductors will sag, so that a minimum vertical clearance is maintained between the cables and the ground. This clearance must be guaranteed for a maximum static load. Guidelines for establishing a maximum static load are outlined by the National Electrical Safety Council (NESC) in the US and the International Electrotechnical Commission (IEC) throughout the world, and generally define this load in terms of the amount of ice accumulation that is likely to occur on a given line. Another form of static loading is wind displacement, where a steady wind will act on a conductor. Ice accumulation and high winds both occur during the same part of the year, so lines must be rated to withstand both phenomena simultaneously.

The NESC and IEC also provide specifications for the distances between individual conductors. Uncontrolled conductors which sway in strong winds may pass close to each other, causing arcing and short-circuit behavior. This is unacceptable. Many strategies are used to prevent this from occurring.

Vortex shedding by some cables can cause Aeolian vibration, a constant hum of the cable. This vibration causes significant conductor motion, and can shorten the lifespan of the cable and the support structures due to fatigue. Aeolian vibration is only significant for transmission lines built in a very specific scale of geometry, such that the frequency of vortex shedding behavior closely matches their natural frequency of vibration. It can be mitigated by adding conductor spacers, which change the natural frequency of the conductor. Vibration can also be mitigated by using unique conductors which dissipate mechanical energy or through conductors of unique geometries which spread the vortex-shedding behavior over a range of frequencies. In general, longer and heavier cables will be more resistant to vibration.

Ice shedding is a common event for lines which accumulate ice. When ice falls off of a conductor, it often comes off in large quantities. This sudden change in loading will cause the conductor to 'jump.' This displacement is mostly vertical, rather than horizontal. This phenomena will be analyzed for lines that may accumulate ice, to show that in the event of ice-shedding, phase conductors will not be brought close enough to induce arcing. This is typically remedied by increasing the vertical spacing between conductors.

In some circumstances, terrain, weather, and wind may come together to produce 'galloping.' Galloping is a violent motion of conductors which may cause displacements of cables by up to 10 feet in long spans. The displacement of galloping will typically be restricted to an elliptical zone around the static position of the line. Like Aeolian vibration, it may be reduced by adding phase-spacers. Slackening conductors may also reduce this behavior.

Generally, thicker (and thus heavier) conductors and longer spans will reduce the motion caused by wind or ice phenomena, at the cost of increased structural requirement at the suspension points and potentially greater sag.

1. Sag Calculation

The sag of a transmission cable is impacted by several phenomena – including changes in heating, changes in loading, and long-term creep. The distance that a cable will sag depends on the length of the conductor span, the weight of the conductor, its initial tension, and its material properties. The cable itself will have a unit weight, core cross-section and diameter, conductor cross-section and diameter, and stress-strain curves for both the core and the conductor. It will also have a coefficient of thermal elongation.

In any overhead transmission line, there will be multiple support structures. The distance between any two structures is called a span. The cable in a single span of a transmission line can be described by a set of hyperbolic functions which describe catenary curves [1]. For a cable with a span-length l, weight w, and horizontal tension H, the maximum sag distance S (the vertical distance between the point of attachment and the cable, at the lowest point in the span) is described by the hyperbolic function:

$$S = \frac{H}{w}\left[\cosh(\frac{wl}{2H}) - 1\right] \tag{1.1}$$

Where

S – Maximum sag distance, in ft.
H – Horizontal tension at each end, in lbs.
w – Weight per unit length, in lbs./ft.
l – Span length, in ft.

and $cosh$ is the hyperbolic cosine function. This function is nonlinear, and is not simple to work with for lines with multiple spans. For this reason, this function is often simplified by linearizing around $l = 0$.

$$S = S(0) + \frac{S'(0)}{1!}\, l + \frac{S''(0)}{2!} l^2 \ldots$$

$$S'(l) = \frac{\partial S}{\partial l} = \frac{1}{2}\sinh\left(\frac{wl}{2H}\right)$$

$$S''(l) = \frac{w}{4T}\cosh\left(\frac{wl}{2H}\right)$$

$$S = 0 + \frac{1}{2}(0)\, l + \frac{\frac{w}{4H}(1)}{2!}\, l^2 + \cdots \cong \frac{wl^2}{8H}$$

$$S = \frac{wl^2}{8H} \tag{1.1a}$$

The total length of the cable L is described by:

$$L = \frac{2H}{w}\sinh\left(\frac{wl}{2H}\right) \tag{1.2}$$

This function is often linearized around $l = 0$ as well:

$$L \cong l + \frac{w^2 l^3}{24H^2} \cong l + \frac{8S^2}{3\, l} \tag{1.2a}$$

$$\Delta L = L - l \cong \frac{w^2 l^3}{24H^2} \tag{1.3}$$

ΔL, the difference between L and l is referred to as the 'slack'.

A transmission line composed of multiple spans can be generalized using the principle of the *ruling span* [2]. In this generalization, a single span is formed which is representative of the entire transmission line. A span with these dimensions will have a sag which is equal to the sag that would be seen if the transmission line had equal spans, and the cable mounts could move freely. If the mounts are free to move, the horizontal tension from the cable at any point of attachment must be equal from both horizontal directions. For the ruling span itself, the tension at both ends is equal to the tension that would be found at each of the equal spans. This method is used in order to compare the behaviors of different conductor sizes and materials, throughout a single transmission line. The 'ruling span' S_R is the span length of this conductor. For a transmission line with n spans,

$$S_R = \sqrt{\frac{\sum S_i^3}{\sum S_i}} \tag{1.4}$$

In a real transmission line, conductors will be held in place by clamps attached to insulators, which may be stiff or free-hanging, but which will restrict the horizontal motion of the cable. Lines will also vary in elevation, which will change the distribution of weight of the conductors and thus affect the tension applied at the insulators.

Example 1: 1-mile of a transmission line is to be re-conductored, using Drake 795-kcmil ACSR conductor. The line has a ruling span of 400-ft. Drake ACSR has a rated tensile strength (RTS) of 31,500 lbs. , and a per-unit weight of 1093 lb/1000ft. The line will have an initial horizontal tension of 18% RTS. Find the initial sag distance and the slack for the ruling span of this line.

$$\boldsymbol{l = 400\,ft.}$$
$$\boldsymbol{H = 18\% \times 31500\,lbs. = 5670\,lbs.}$$
$$\boldsymbol{w = \frac{1093\,lbs.}{1000\,ft.} = 1.093\frac{lbs}{ft}}$$

First, find the sag, using the exact formula:

$$\boldsymbol{S = \frac{H}{w}\left[\cosh\left(\frac{wl}{2H}\right) - 1\right] = \frac{5670}{1.093}\left[\cosh\left(\frac{1.093 \times 400}{2 \times 5670}\right) - 1\right] = 3.856\,ft.}$$

Next, apply the approximate formula:

$$\boldsymbol{S \cong \frac{wl^2}{8T} = \frac{1.093 \times 400^2}{8 \times 5670} = 3.853\,ft.}$$

Now, calculate the slack, using the exact formula, and compare to the approximate formula:

$$\boldsymbol{\Delta L = \frac{2T}{w}\sinh\left(\frac{wl}{2H}\right) - l = 0.0991\,ft.}$$
$$\boldsymbol{\Delta L \cong \frac{w^2 l^3}{24H^2} = 0.0991\,ft.}$$

Example 1b: For the line in Example 1, if the conductor was instead replaced with Tern 795-kcmil ACSR, which has a tensile strength of 22,100 lbs. and weight of 895 lbs./1000ft., find the new initial sag.

$$\boldsymbol{l = 400\,ft.}$$
$$\boldsymbol{H = 18\% \times 22100\,lbs. = 3978\,lbs.}$$
$$\boldsymbol{w = \frac{895\,lbs.}{1000\,ft.} = 0.895\,\frac{lbs}{ft}}$$

$$\boldsymbol{S \cong \frac{wl^2}{8H} = \frac{0.895 \times 400^2}{8 \times 3978} = 4.500\,ft.}$$

Example 2: Find the ruling span for a transmission line with spans of {320-ft., 400-ft., 420-ft., 400-ft., 400-ft., 350-ft., 420-ft.}

$$\boldsymbol{S_R = \sqrt{\frac{320^3 + 400^3 + 420^3 \ldots}{320 + 400 + 420 \ldots}} = 391.7\,ft.}$$

Environmental factors will impact the sag of a transmission line. In the design of transmission lines, two main environmental factors are often considered – temperature and ice loading.

a. Thermal Elongation

Heat causes conductors to expand. As a conductor expands, it becomes longer and sags lower. The distance that a particular conductor expands is often described by a linear temperature coefficient α_T. The length of a simple conductor, for temperatures T near an initial temperature T_0may be calculated as follows [3]:

$$L_T = \left(1 + \alpha_T \times (T - T_0)\right) L_{T_0} \qquad (1.5)$$

Where

L_T - Length of the cable at temperature T (°C)
L_{T_0} - Length of the cable at initial temperature T_0 (°C)
α_T - Coefficient of thermal expansion, $\frac{ft}{ft}\frac{10^{-6}}{°C}$

b. Stress-Strain Behavior

Conductor cables under tension will undergo deformation. Figure 1 shows a stress-strain diagram for a simple conductor. Strain (elongation) of the conductor is mostly linear at low stress (tension). This linear behavior is considered ‘elastic’. As tension increases past the yield stress, some of the strain becomes permanent. After this point, if the cable is relaxed, it will shrink linearly, but will retain some deformation permanently. This permanent deformation is ‘plastic’ deformation.

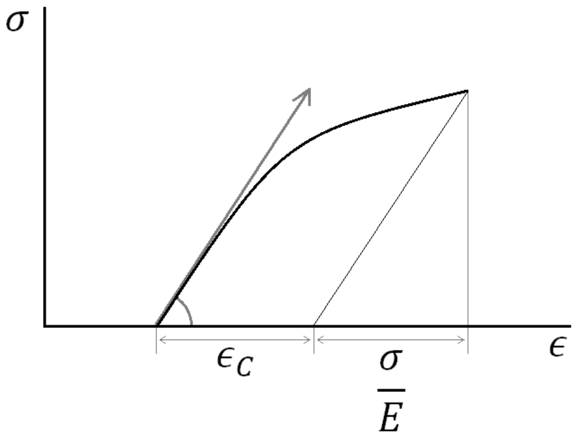


Figure 1: Stress-Strain Relationship

The length of a conductor in its range of elastic behavior, with respect to stress σ is represented by :

$$L_\sigma = L \times (1 + \epsilon_\sigma + \epsilon_C) \qquad (1.6)$$

$$\epsilon_\sigma = \frac{\sigma}{E} = \frac{H}{E\,A} \qquad (1.6a)$$

Where

L_σ – Length under stress σ, in ft
L – Length under no stress, in ft
ϵ_σ – Elastic strain, in $\frac{ft}{ft}$
σ – Stress, in $\frac{lbs}{in^2}$
E – Modulus of elasticity for the conductor, in $\frac{lbs}{in^2}$
A – Cross-sectional area of conductor, in in^2

H – Tension applied to the conductor, in lbs

ϵ_C – Plastic deformation of the cable, due to inelastic deformation and creep, in $\frac{ft}{ft}$

If a conductor is coated with a large enough amount of ice, it may be stretched past its yield stress. When the ice is eventually shed, the conductor will contract elastically, but will still bear some permanent deformation.

Every transmission line cable is under some tension. Over time, this tension will tend to permanently stretch the cable. This behavior is known as 'creep'. Creep has been modeled and parameterized for most types of cables. Transmission lines are long-term investments. They are typically used for 40 years or more, so it is important to design a line that will operate safely for many years in the future.

The elongation of a conductor under stress was described as simple and linear. In high-precision transmission design programs such as PLSS-CAD and SAGT, higher-dimension polynomials are used to express the load-strain curves, so that plastic deformations and creep can be calculated precisely.

c. Sag at High Temperature

When a conductor undergoes thermal elongation, the length L of the cable increases while the span l remains the same. This results in a decrease in tension in the conductor. So, to find the sag distance of a hot conductor, we must consider both thermal expansion and strain under tension. The tension of a conductor and the temperature at which the cable was strung will be known or specified. To find the sag, you must find a tension H at which the length of the elongated cable is equal to the catenary cable's length [2]:

$$L = L_0 \left(1 + \alpha_T \times (T - T_0)\right)\left(1 + \frac{H - H_0}{E\,A} + \epsilon_C\right) \tag{1.7}$$

Where

L_0 – Initial length, in ft.

L – Length at high-temperature conditions, in ft.

H_0 – Stringing (initial) tension, in lbs.

T_0 – Stringing temperature, in ℃

Substitute in the linear approximation of cable length, from (1.2a).

$$l + \frac{w^2 l^3}{24H^2} = \left(l + \frac{w^2 l^3}{24{H_0}^2}\right)\left(1 + \alpha_T \times (T - T_0)\right)\left(1 + \frac{H - H_0}{E\,A} + \epsilon_C\right)$$

Equation (1.8a) can be solved for horizontal tension H, which can be used to calculate the sag.

Example 3: A 400-ft span of Hawk 477-kcmil ACSR conductor is originally tensioned at 20% RTS, on a 60°F (15.5℃) day. The cable is rated at 75℃. Find the tension and sag of the cable at its original and rated temperatures. Assume no permanent elongation ($\epsilon_C = 0$). Hawk ACSR has the following properties:

$\boldsymbol{A = 0.435\ in^2}$
$\boldsymbol{\alpha_T = 19.3\ \times 10^{-6}/℃}$
$\boldsymbol{E = 11.5\ MPsi}$
$\boldsymbol{H_{RTS} = 19500\ lbs}$
$\boldsymbol{w = 0.656\ lbs/ft}$

$\boldsymbol{T = 75\ ℃}$
$\boldsymbol{T_0 = 15.5\ ℃}$
$\boldsymbol{H_0 = 20\%\ \times 19500\ lbs = 3900\ lbs}$

Multiply (1.8a) by H^2, rearrange as a polynomial, and solve for H:

$$\mathbf{0} = k_1 H^3 + k_2 H^2 + 0\,H - k_4$$

$$k_1 = \left(1 + \frac{w^2 l^2}{24 {H_0}^2}\right)\left(1 + \alpha_T\,(T - T_0)\right)\left(\frac{1}{E\,A}\right)$$

$$k_2 = \left(1 + \frac{w^2 l^2}{24 {H_0}^2}\right)\left(1 + \alpha_T\,(T - T_0)\right)\left(1 - \frac{H_0}{E\,A} + C\right) - 1$$

$$k_4 = \frac{w^2 l^2}{24}$$

Use MATLAB `roots()` command:

```
≫roots([k1 k2 0 –k4])

ans =

  -2277.2 + 1701.1i
  -2277.2 - 1701.1i
   1774.0
```

Only the positive-real root has physical meaning here. $H = 1774\ lbs.$ Now, compute the sag:

$$S \cong \frac{wl^2}{8H} = 7.40\,ft.$$

Example 3b: After 10 years, the transmission line in Example 3 has undergone ‘creep’, and now has a permanent elongation of 0.04% ($\epsilon_C = 0.0004$). Find the new tension and creep at 75℃.

Recompute k_2, and solve for the tension.

```
≫roots([k1 k2 0 –k4])

  ans =
     -3776.3
     -2514.5
      1509.4
```

Again, only the positive-real root has physical meaning here. $H = 1509.4\ lbs.$ Now, compute the sag:

$$S \cong \frac{wl^2}{8H} = 8.692\,ft.$$

d. Sag with Ice Loading

The NESC provides guidelines for calculating the final (inelastic) sag of a transmission line. These vary by region, as ice accumulation is not significant everywhere. In general, the NESC guidelines require that strain be calculated as shown in (1.9) and demonstrated in Figure 2 [4]:

$$F = \sqrt{(w + w_i)^2 + f_w^2} + k_f \tag{a.}$$

Where

F – The resultant force of the weight of the conductor (with ice) and horizontal force from wind.
w – Weight of the conductor itself, in lbs/ft
w_i – Weight of the accumulated ice, in lbs/ft
f_w – Force from cross-winds, perpendicular to the conductor, in lbs/ft
k_f – A constant, added to the resultant, a kind of factor of safety, in lbs/ft

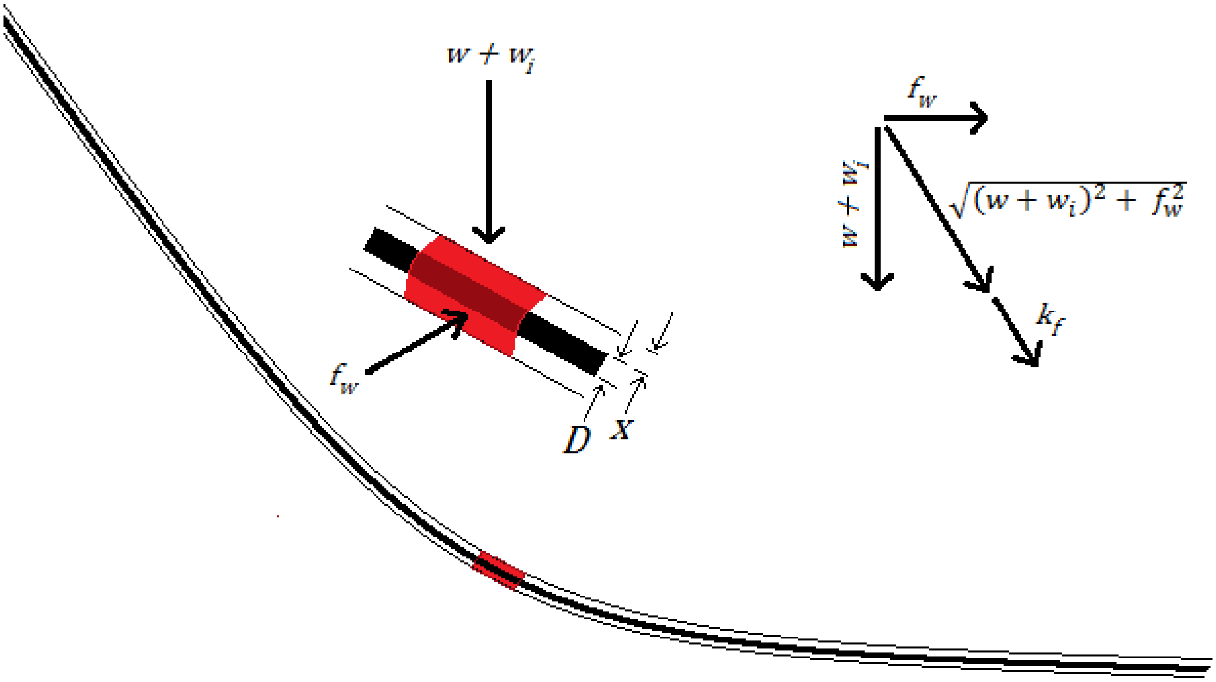


Figure 2: NESC Ice-Loading Methodology

Force f_w is calculated based on a constant pressure P_w applied to the cross-section of the ice-coated cable. f_w can be calculated from $f_w = P_w \times (\frac{D}{12} + \frac{2\,x}{12})$.

The weight of the conductor itself should be available from vendor documentation. Ice loading is usually described in terms of 'x inches of radial ice' – that is, a cylindrical layer of ice x inches thick, coating the conductor. The volume of x inches of ice is given:

$$v_i = \left[\left(\frac{D}{2} + \frac{x}{12}\right)^2 - \frac{D^2}{4}\right]\pi \qquad \text{b.}$$

Where

v_i – Volume of ice per unit length, in ft^3/ft
D – Diameter of the conductor, in ft
x – Thickness of ice coating the conductor, in in

The weight x inches of radial ice, and the total weight of the conductor with ice are, therefore:

$$w_i = v_i \times \rho_i \qquad \text{c.}$$

Where ρ_i is the density of ice (~ 57 lb/ft^3).

Force from cross-winds is calculated based on a fixed predetermined pressure applied to the exposed cross-sectional area of the cable. Table 1 lists the standard parameters required for NESC loading tests, by loading class. Figure 3 shows areas where those loading classes will be applicable [4].

NESC Loading Criteria	Zone 1 (Heavy)	Zone 2 (Medium)	Zone 3 (Light)
Radial Ice (in)	0.5	0.25	0

Horizontal wind pressure (lb/ft2)	4	4	9
Temperature (°C)	-20	-10	-1
Constant to be added to the Resultant (lb/ft)	0.3	0.2	0.05

Table 1: NESC Loading Criteria

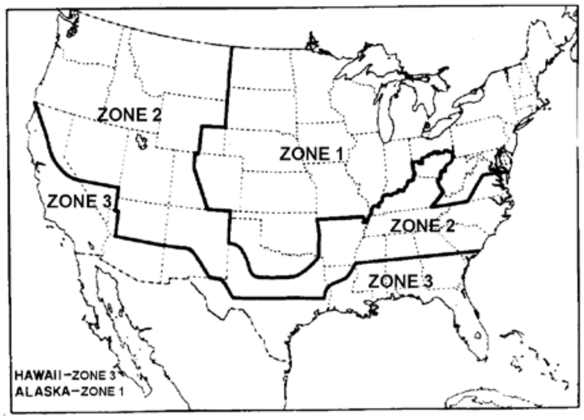


Figure 3: NESC Loading Zones

The elongation of an ice-loaded conductor is calculated similarly to calculation of thermally-induced elongation, but here the weight the cable is substituted with the force from ice-loading, so that the slack on the left hand of the equation is representative of the length of a catenary curve with the ice-load, and the right hand represents the elongation of the cable which was originally tensioned at H_0 at temp T_0.

$$l + \frac{\boldsymbol{F}^2 l^3}{24H^2} = \left(l + \frac{w^2 l^3}{24{H_0}^2}\right)\left(1 + \alpha_T \times (T - T_0)\right)\left(1 + \frac{H - H_0}{E\ A} + \epsilon_C\right) \qquad \text{d.}$$

e. Behavior of Layered Cables

Most conductors used in new transmission lines are composed of two or more materials. The most common – Aluminum Conductor, Steel Reinforced (ACSR) – has a stranded steel core surrounded by layers of strain-hardened aluminum. The steel core provides a great deal of strength, while the aluminum has very good conductive properties. The two materials utilized in this cable will expand at different rates due to temperature and tension. At low temperatures, ACSR can be approximated as a combination of the properties of both steel and aluminum. At higher temperatures, most of the tension will be imparted on the steel core, and it will elongate much like a regular steel cable. High temperatures impart slack to the cable, so cables operating at heightened temperatures will be under decreased tension.

To account for this combination, we must look at the stress-strain behavior of both materials, and show how they combine. Figure 4 shows the load-strain curves of aluminum and steel superimposed over each other [5]. “Initial” curves are the inelastic behavior of the layer under stress. “Final” curves represent the elastic behavior, after inelastic strain has occurred. The red curve is the composite elastic behavior of the conductor.

As you can see from the composite final curve, the elastic behavior of the cable depends on which layer is supporting the load. For cables at low tension, the steel cable carries all the tension, and the behavior is that of steel. At higher tension, the aluminum begins to stretch, and the behavior is a composite of both layers.

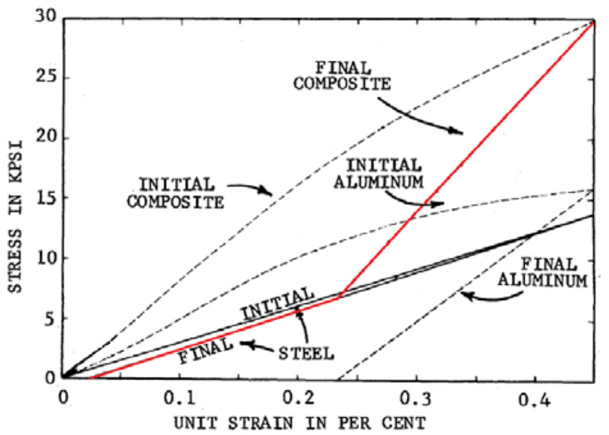


Figure 4: Composite Stress-Strain behavior of 24/7 636kcmil ACSR

The aluminum conductor layer and steel core will likely be at different temperatures. The layers have differing cross-sectional areas and different elastic moduli, as well as different thermal expansion coefficients. The creep behavior of each material is different as well. If the aluminum conductor layer exhibits more creep behavior over time, the relationship between these curves may also shift. Core materials, which are typically stronger than the conductor material, often exhibit very little creep. Figure 5 shows the effect of creep and thermal elongation on the composite load-strain behavior [6]. Curve 1 describes the aluminum, curve 2 describes the core, and curve 12 describes the composite behavior. Notice that the values of $\epsilon_{t1} + \epsilon_{c1}$ and $\epsilon_{t2} + \epsilon_{c2}$, which represent thermal elongation and creep, are unequal. The dotted line indicates the behavior of the aluminum strands under compression, which may also be modeled.

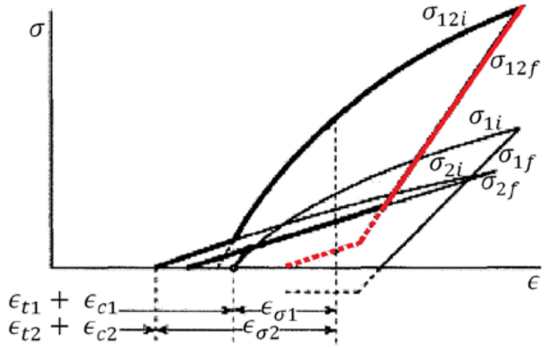


Figure 5: Composite Stress-Strain behavior with Creep and Thermal Elongation

As shown in Figure 4, there is often a transition point above which the behavior of the cable is dependent on both the core and conductive layer, and below which the conductive layer is in compression and the behavior is dependent solely on the core material. When a strung cable is heated, its thermal elongation causes excess sag and lower tension. The transition point to core-only behavior will be seen at a fixed temperature, which depends on the original stringing tension of the cable. High-temperature cables are often designed to shift the location of that transition point to a lower temperature, so that the whole load is applied to the core, which often has a lower coefficient of thermal elongation.

2. Ampacity of a Conductor

Ampacity is the current-carrying capacity of a cable. A cable will have a maximum operating temperature, which may be limited by the physical makeup of the cable, or may be limited by a maximum amount of allowable sag. High current in a cable will cause significant resistive heating. At the same time, direct sunlight will also heat the cable. The cable will be cooled by wind, through convective heat transfer. All of these factors impact the temperature of the cable, so to establish a thermal current-carrying limit, some operating conditions must be assumed.

a. Heat Balance Equation

The thermal behavior of a conductor can be calculated using a heat-balance equation. The simple steady-state model of a cable is described as follows [7]:

$$Q_C + Q_R = Q_S + Q_{EM} \tag{1.8}$$

Where

Q_R – Radiant heat loss per unit length, in W/m
Q_C – Convective heat loss per unit length, in W/m
Q_S – Heating from solar insolation per unit length, in W/m
Q_{EM} – Losses from AC current per unit length, including joule loss , in W/m

b. Q_R - Radiant Heat Loss

Radiant heat loss is thermal energy emitted by electromagnetic waves, due to the temperature difference between an object and its environment. Radiant heat loss can be estimated based on the geometry of a conductor, its temperature, and the ambient temperature of the environment around it, demonstrated in (2.1a):

$$Q_R = k_s k_e D\,\pi(T^4 - T_a^4)$$

Where

k_s – Stefan-Boltzmann constant, for black-box radiation = $5.6704 \times 10^{-8}\,\frac{W}{m^2{}^\circ C^4}$
k_e – Emission coefficient, near 0 for new cables, 0.5 to 1 for dirty or oxidized cables, typically around 0.6
D – Diameter of the cable, in m (So that $D\pi$ represents the surface area of the cable in $\frac{m^2}{m}$)
T – Cable temperature, in K
T_a – Ambient temperature, in K

c. Q_C - Convective Heat Loss

Convective heat loss is the effect of heat transfer due to fluid (in this case, air) passing in contact with an object (here, a metal conductor). Convective heat loss for conductor cables has been studied, and fitted to several different relationships. To compute convective loss, you must first compute the Nusselt number Nu, which itself will be based on the Reynolds number Re. In the IEEE Standard 738 [15], Q_C is calculated:

$$Re = \frac{v\,D\,\gamma}{\eta}$$

$$Nu_{\mathrm{lo}} = 0.32 + 0.43\,Re^{0.52} \text{ for low wind speeds}$$

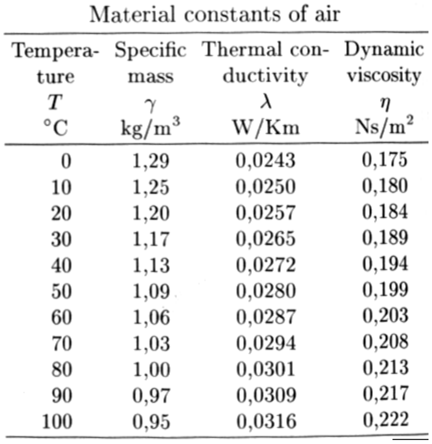
$$Nu_{\mathrm{hi}} = 0.24\,Re^{0.6} \quad \text{for high wind speeds}$$

$$Q_C = \pi\,\lambda\;Nu\;(T - T_a)$$

Where

v – Component of wind speed which is normal to the cable, in $\frac{m}{s}$
D – Diameter of the cable, in m
γ – Specific mass of air, in $\frac{kg}{m^3}$
η – Dynamic viscosity of air, in $\frac{N\,s}{m^2}$
λ – Thermal conductivity of air, in $\frac{W}{m\,^\circ C}$

Values of $\gamma, \eta,$ and λ are widely available, usually in fluid dynamics texts. A brief table of these values is given in Figure 6 [7]. Other models for Nu may be based on atmospheric pressure, humidity, or any other fitted relationship.



Material constants of air

Temperature T °C	Specific mass γ kg/m^3	Thermal conductivity λ W/Km	Dynamic viscosity η Ns/m^2
0	1,29	0,0243	0,175
10	1,25	0,0250	0,180
20	1,20	0,0257	0,184
30	1,17	0,0265	0,189
40	1,13	0,0272	0,194
50	1,09	0,0280	0,199
60	1,06	0,0287	0,203
70	1,03	0,0294	0,208
80	1,00	0,0301	0,213
90	0,97	0,0309	0,217
100	0,95	0,0316	0,222

Figure 6: Material Constants for Air

d. Q_S - Solar Radiation

Solar radiation is occurs over the total exposed area of a cable, and varies by location on earth and orientation of the cable with respect to the North-South axis. This value is regarded as a constant with respect to its environment, representing the most heat from solar radiation that a cable will be subjected to. This value can be approximated from:

$$Q_S = D \ k_a \, Q_{SH}$$

Where

D – Diameter of the cable, in m

k_a – Absorption coefficient, unitless. Usually, this value is around 0.5

Q_{SH} – Standard solar radiation, $\frac{W}{m^2}$. This value varies with geography from 850-1350 $\frac{W}{m^2}$.

e. Q_{EM} - AC Losses

AC current losses represent the resistive loss of a conductor due to AC current. This calculation uses the AC resistance of the cable, which represents not only the resistivity of the cable itself, but also the skin effect caused by alternating current. DC resistance increases nearly linearly with temperature. AC resistance follows this increase closely. The change of AC resistance with temperature can be approximated by a linearization around a reference temperature. Resistance and resistive losses are calculated:

$$R_{T,AC} = R_{20,AC} \times (1 + \alpha_R (T - 20))$$
$$Q_{EM} = {I_{RMS}}^2 \, R_{T,AC}$$

Where

I_{RMS} – RMS current flowing in a single conductor

$R_{T,20}$ – AC Resistance in the conductor, at 20 ℃, in Ω/m

$R_{T,AC}$ – AC Resistance in the conductor, at temperature T, in Ω/m

α_R – Temperature coefficient of resistance, in $\frac{1}{℃}$

T – Temperature of the conductor, in ℃

Values for $R_{T,20}$ and α_R can be found on spec-sheets for conductors.

f. I_{RATED} - Ampacity of a Conductor

Given Q_R, Q_C, Q_S, and Q_{EM}, for some operating temperature T, the ampacity of a cable can be calculated. Equation (2.1) is used, and (2.1g) is substituted in, resulting in:

$$Q_C + Q_R - Q_S = I_{RMS}{}^2 R_{T,AC}$$

Which can be reorganized as:

$$I_{RMS} = \sqrt{\frac{Q_C + Q_R - Q_S}{R_{T,AC}}} \qquad \text{g.}$$

Equation (2.2) will specify the rated steady-state current of a conductor for the environmental conditions used in the calculation of Q_C, Q_R, and Q_S. The conditions assumed for these calculation have typically been conservative assumptions about the windspeed and temperature during periods when the cable will run at or near its limit – for instance, a wind speed of 2 ft/s and an ambient temperature of 40℃. Limits may be specified for several distinct parts of the year – for instance, summer and winter months.

A great deal of research has been done on the topic of Flexible AC Transmission Systems (FACTS). In many of these systems, the sag and temperature of one or all of the conductor spans in a transmission line will be monitored continuously, as will local weather conditions. Ampacity may be recalculated in real-time, based on present conditions. If these conditions are more favorable than the conservative conditions mentioned above (for instance, the temperature is below peak summer temperature, or there is significant wind), then the conductor may be rated at a higher ampacity during that time period. These systems could lead to better utilization of new or existing transmission lines.

Example 4: A new 161-kV transmission line is built, using Drake 795-kcmil ACSR conductors, one conductor per phase. The conductor temperature is limited to 75℃ in normal operation. Find the thermally-limited power rating of the line, when the ambient temperature is 40℃, and wind is blowing at $0.61\ m/s$ $(2\ ft/s)$. Drake ACSR has the following properties:

$$A = 0.7264\ in^2$$

$$D = 1.108\ in = 1.108\ in\ \frac{0.0254\ m}{1\ in} = 0.02814\ m$$

$$R_{75C} = 0.139\ \frac{\Omega}{mi} = 0.139\ \frac{\Omega}{mi}\ \frac{1\ mi}{1609\ m} = 86.3 \times 10^{-6}\ \frac{\Omega}{m}$$

$v = 0.61\ m/s$
$T_a = 75\ ℃ = 348\ K$
$T = 40\ ℃ = 313\ K$

Assume:
$Q_{SH} = 1000\ W/m^2$
$k_a = 0.5$
$k_e = 0.5$
$\eta = 0.211 \times 10^{-4}\ \frac{N\,s}{m^2}$ **(estimated from Figure 6)**
$\gamma = 1.015\ \frac{kg}{m^3}$ **(estimated from Figure 6)**
$\lambda = 0.02975\ \frac{W}{m\,K}$ **(estimated from Figure 6)**

$$Q_R = k_s k_e D\,\pi (T^4 - T_a^4) = (5.6704 \times 10^{-8}\ \frac{W}{m^2 ℃^4})(0.5)(0.02814\ m)\ \pi\ (348^4 K^4 - 313^4 K^4)$$

$$Q_R = 12.703\ \frac{W}{m}$$

$$Re = \frac{v\,D\,\gamma}{\eta} = \frac{\left(0.61\frac{m}{s}\right)(0.02814\ m)\left(1.015\frac{kg}{m^2}\right)}{0.211 \times 10^{-4}\frac{N\,s}{m^2}} = 825.7290$$

$$Nu = 0.32 + 0.43\ Re^{0.52} = 0.32 + 0.43\ (825.7)^{0.52} = 14.45$$

$$Q_C = \pi \lambda \; Nu \; (T - T_a) = \pi \left(0.02975 \frac{W}{m\,K}\right)(14.45)(348\,K - 313\,K)$$

$$Q_C = 47.269 \frac{W}{m}$$

$$Q_S = k_a \, D \, Q_{SH} = (0.5)(0.02814\,m)(1000 \frac{W}{m^2})$$

$$Q_S = 14.05 \frac{W}{m}$$

$$I_{RMS} = \sqrt{\frac{Q_C + Q_R - Q_S}{R_{T,AC}}} = \sqrt{\frac{47.269 + 12.70 - 14.05}{86.3 \times 10^{-6}}} = 729.4\,A$$

$$P_{RATED} = \sqrt{3}\, V_{ll} I = \sqrt{3}\,(161 \times 10^3\,V)(730\,A)$$

$$P_{RATED} = 203.4 \times 10^6\,W = 203.4\,MVA$$

3. High Temperature, Low Sag Transmission Technologies

Constructing new transmission lines is often difficult politically, and can be expensive. Transmission planners would like to maximize the carrying capacity of new and existing lines, to reduce the number of new lines that must be built.

Operating transmission lines at high current rates causes significant conductor heating. Heating of conductors can cause significant conductor sag, which will either limit the length of spans or require taller support structures. Conventional conductors may also be limited by their own maximum operating temperature, above which they will physically degrade.

One way to increase line capacity is to replace the conductors (‘reconductor’) with larger or stronger conductors. The scope of this upgrade will be limited by the size of the cables that the original support structures can hold, and the sag of the new cable. Engineers may also seek to reconductor a line due to mechanical problems such as vibration or galloping.

There are a variety of types of cable which have been developed which may perform better than conventional conductors. These cables are significantly more expensive, so they are not often used for new transmission lines, but they may present economical options for upgrading existing lines.

a. Conventional Conductors (AAC, AAAC, ACSR)

Most of the transmission lines in service today utilize aluminum (AAC), aluminum-alloy (AAAC) or steel-reinforced aluminum conductors (ACSR). Aluminum is utilized because of its high conductivity and low weight density. Steel is added in ACSR for extra strength, and for its resistance to sag.

The aluminum used in conventional conductors carries all or most of the tension in the cable. In order to provide adequate strength, the aluminum strands are ‘work hardened’ (or ‘cold worked’) to increase their physical strength. This increase in strength is due to dislocations in the crystal structure of the material which make it difficult for layers of atoms to slip past each other. These dislocations also slightly increase the electrical resistance of the conductor.

Heating a cold-worked conductor can cause it to anneal. When a material anneals, the dislocations in its crystal structure begin to release, reducing the material’s strength. Conductors should not be operated at temperatures which cause them to anneal. This is the basis of the operating temperature of most conductors.

Aluminum (AAC) cables are made entirely from extruded aluminum strands. They are simple and cheap. But, they are only as strong as the aluminum they are composed of, and they exhibit significant sag due to aluminum’s low elastic modulus. Some cables are made with an aluminum alloy (AAAC), which gives them higher tensile strength. Aluminum cables are not commonly used for new transmission projects, but many are still in use on older transmission lines.

Steel reinforced aluminum conductor (ACSR) cables are made with a steel cable at their core, surrounded by strands of aluminum. Both the steel and aluminum are cold-worked, so each provides some portion of the tensile strength of the cable. When heated, elongation is most closely related to the steel core, which stretches less than the aluminum.

AAC, AAAC, and ACSR cables are limited to operating temperatures of 90-100℃. Above that limit, the aluminum conductor will begin to anneal and lose strength. Often, transmission lines with these cables have been designed to operate below 60-75℃, to limit their sag.

b. Aluminum Conductor, Steel Supported (ACSS)

Another kind of cable, steel supported aluminum conductors (ACSS), known in the older literature as SSAC, and euphemized with the term "Sad SAC" is a sag-resistant steel-cored conductor [5]. Unlike ACSR, ACSS is almost entirely supported by its steel core. The aluminum strands are not cold-worked in manufacture, so they have the same properties as fully annealed aluminum. The steel core provides most of their tensile strength.

The behavior of ACSS is a composite of the steel core and annealed aluminum, though the aluminum carries little of the load, because of its low yield strength. The operating temperature of ACSS is not limited by the properties of aluminum, since the aluminum is already fully annealed. Instead, the temperature limitation comes from the properties of the steel core, which has an annealing temperature around 240℃ (though, the surface temperature may be significantly cooler). This is significantly higher than ACSR or AAAC. A higher temperature limit means that a greater amount of current can be passed without weakening the cable. Figure 7 shows the rated ampacity of equivalent ACSR and ACSS cables at rated operational temperatures (75 ℃ and 200 ℃, respectively), ambient temperature of 25 ℃ and wind at 2 ft/s [8].

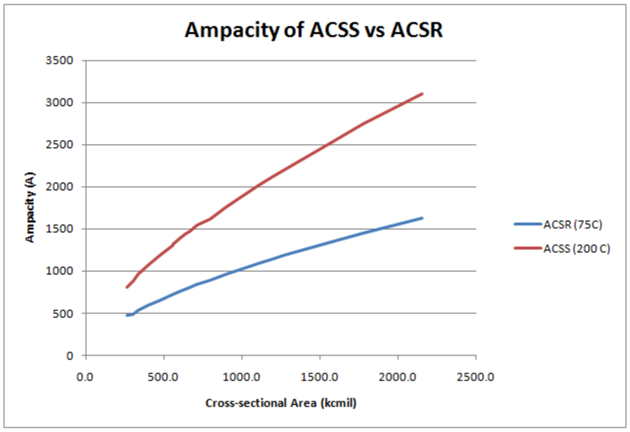


Figure 7: Ampacity Comparison - ACSS vs ACSR

ACSS and some other high temperature conductors are often built as compact conductors. These conductors are composed of trapezoidal wires which have a closer fit than round wires of the same cross-sectional area (see Figure 8) [10a]. Compact ('Trap Wire') conductors can replace conventional conductors of the same diameter, while increasing the cross-sectional area of the conductor. This will lower the resistance per-mile, and increase the ampacity of the cable. Figure 9 compares the amount of aluminum in ACSS conductors with round strands vs. those with trap wire. Resistance is inversely proportional to cross-sectional area. Figure 10 shows the ampacity of common sizes of ACSR, ACSS, and ACSS/TW cables of the same diameter [10].

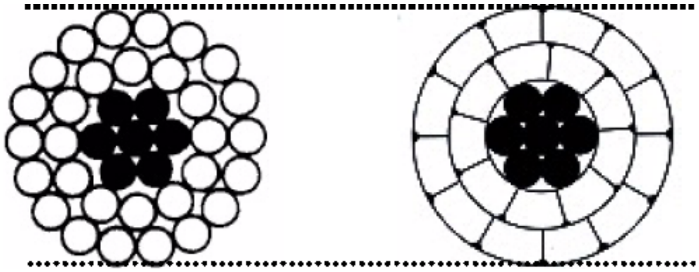


Figure 8: Round Wire vs. Trap Wire

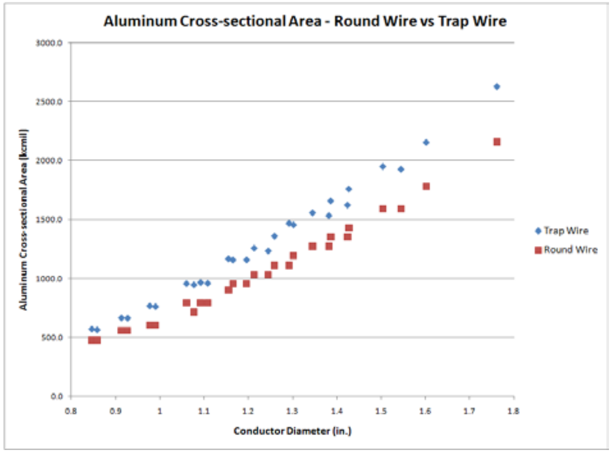


Figure 9: Aluminum Cross-sectional Area Comparison

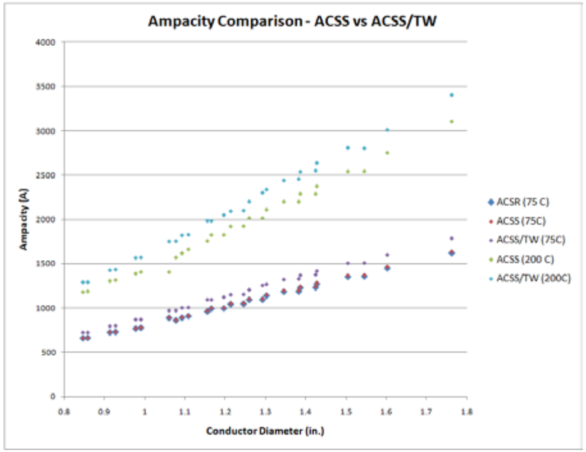


Figure 10: Ampacity Comparison

The high-temperature sag behavior of ACSS is generally better than that of ACSR. Often, the steel core will be pre-tensioned prior to installation, to prevent creep behavior. Plastic elongation of the annealed aluminum layer does not contribute significantly to final sag, since the slack is picked up by the elastic behavior of the steel core.

ACSS is among the cheapest high-temperature conductor technologies, with a bulk price 1.5-2x that of ACSR. It is composed of the same materials that make up ACSR, and is a commonly used to replace ACSR when uprating transmission lines.

c. (Super)Thermal-Resistant Aluminum Alloys – (Z)TACSR and(Z) TACIR

Aluminum alloy conductor strands have been developed that are resistant to annealing far above the normal temperatures of pure aluminum. These conductors are strain hardened and are alloyed with small amounts of other metals, such as Zr. The alloyed metals change the nature of the metallic crystal, increasing its annealing temperature. Alloys differ, depending on the desired operating temperature of the conductor. These alloys are often listed as Thermal-resistant and Super-thermal-resistant aluminum alloys (TAl and ZTAl). These alloys are designed to operate at 150°C and 210°C respectively [11].

(Super) Thermal-resistant Aluminum Alloy Cables, Steel Reinforced ((Z)TACSR) are formed similarly to ACSR, but utilize TAl or ZTAl rather than the typical work-hardened aluminum conductor stranding. TAl and ZTAl materials are also used in GTASCR and ACCR cables, as mentioned below.

TAl and ZTAl have slightly lower conductivities than are seen in regular aluminum, so larger cables may be required in order to achieve the ampacity of equivalent ACSR cables.

d. Composite Cores – ACCC and ACCR

Composite-cored cables have cores formed from fibers embedded in a matrix material. These materials tend to be very strong, and have small coefficients of thermal expansion. When the aluminum conductor layer elongates, it quickly imparts the whole load of the cable onto the core, which exhibits very little elongation with respect to heat. In high-temperature settings, these cables exhibit much less sag than ACSR or ACSS.

Figure 11: Composite-core Cables. Left: ACCC, Right: ACCR [12]

Aluminum Conductor, Composite Core (ACCC) , licensed by CTC Cable Corporation, utilizes a composite core made of stranded carbon-fiber and epoxy, and fully annealed aluminum as a conductor. The composite core is very strong, and has an extraordinarily small coefficient of thermal elongation . The operating temperature of ACCC is limited by its composite core, since the aluminum conductor is already fully annealed. Although the manufacturers rate the core at 180 ℃, independent testing has shown that above 150℃, the core will begin to permanently deform. Above 170℃, the core will begin to degrade, permanently losing strength [11]. This thermal operating range is lower than that of ACSS cables of a similar size. But, within its thermal operating range, the sag characteristics of ACCC are much better than ACSS.

3M sells a cable technology called Aluminum Conductor, Composite Reinforced (ACCR) which has a composite core made of Aluminum-Oxide strands embedded in aluminum, and conductor strands composed of a hardened heat-resistant Al-Zr alloy. The behavior of the core is similar to that of steel, but is significantly lighter, and is itself conductive. The alloy used in ACCR allows it to operate at 210℃ normally, and 240℃ in emergency [12]. The thermal expansion of ACCR is larger than that of ACCC, but still quite a bit less than that of ACSR or ACSS.

3M markets their product exclusively for uprating transmission lines through reconductoring. Their literature suggests that ACCR has costs 3-6 times those of comparable cables, but is cost-effective in situations where 30-40% of existing support structures would have to be replaced in order to uprate with ACSR.

Like ACSS, ACCC and ACCR cables are often sold as trap-wires. Due to the complexity of their composite cores, these cables are more expensive than ACSR or ACSS cables.

e. Invar Core –TACIR

Invar is the trade-name of 64FeNi, an alloy that has a low coefficient of thermal expansion at high temperatures. It can be used in place of a steel core to improve the sag behavior of a conductor. Invar and composite cores are usually paired with high-temperature aluminum conductor strands, in order to minimize the sag that is caused by operating at high temperature [13].

Thermal Resistant Aluminum Alloy Conductor, Invar Reinforced (TACIR) is one type of cable which utilizes an Invar core. TACIR and ZTACIR (the Super-Thermal-resistant variety) utilize aluminum alloy conductor strands that can be operated at high temperatures without degrading their strength.

Invar steel has a coefficient of thermal elongation between those of composite cores and galvanized steel. It has been used extensively for new transmission lines in Japan and Korea, where right of way is extremely limited.

f. Gap-Type Conductors – Gap-Type (Super) Thermal-Resistant Aluminum Alloy Conductor, Steel Reinforced (G(Z)TACSR)

In a gap-type conductor, a stranded core is surrounded by a hollow cylinder of trap-wire, forming a gap between the core and the conductor which is filled with thermal-resistant grease (see Figure 12) [14]. When it is strung, special techniques will be used to impart the entire load on the core. In this way, the transition point between core-behavior and combined behavior can be controlled.

G(Z)TACSR utilizes a thermal-resistant aluminum alloy conductor, much like ACCR. The core is generally made of galvanized steel, the same as would be found in ACSS or ACSR. The gapped nature of this conductor makes it complicated to install. It may require specialized hardware, in order to pre-tension the core.

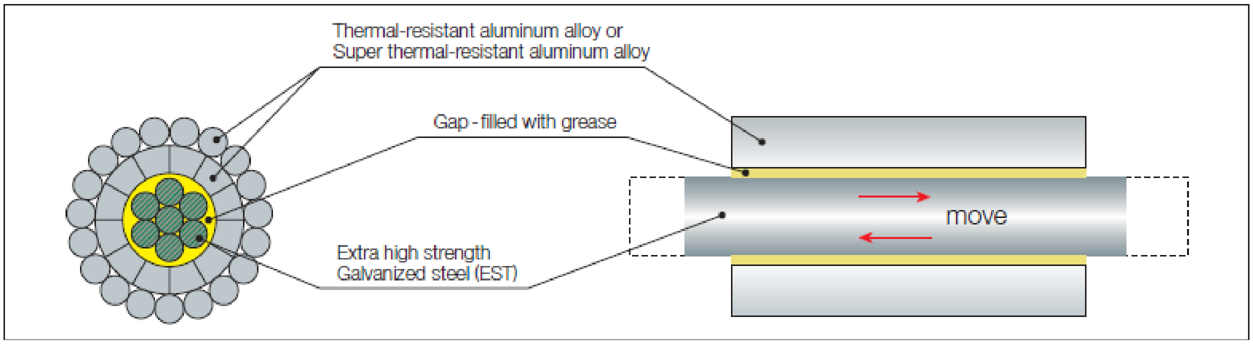


Figure 12: Gap-type Conductor Construction []

4. Comparison of High-Temperature Conductors

High temperature conductors are often used to uprate existing transmission lines, utilizing their decreased thermal expansion, higher allowable temperatures, or their higher cross-sectional area to increase ampacity without increasing final sag.

Increases in allowable operating temperature correspond to significant increases in ampacity. Figure 13 demonstrates the relationship between temperature and ampacity for 3 conductors of the same diameter. ‘Hawk’ – the ACSR cable – has the lowest operational temperature limit. The ‘Hawk ACSS’ cable has slightly less resistance, since it is made with annealed aluminum which is slightly more conductive than worked aluminum. This allows it to operate at a slightly higher current than the ACSR. The ACCC/TW conductor is a trap-wire, and the core takes up very little space. As a result, its conductive 0-worked aluminum has quite a bit more cross-sectional area than the ACSS cable, and it can operate at lower temperatures than ACSS. This comparison was done for environmental conditions identified below the figure.

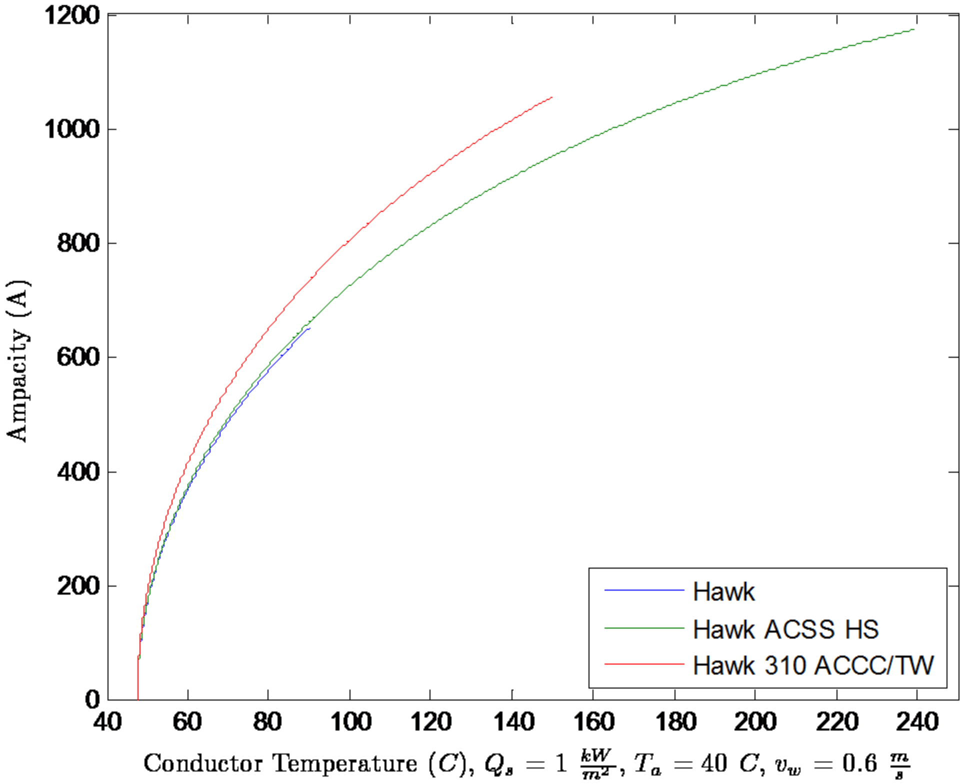


Figure 13: Comparison of Thermal Ampacity Limits for 3 Varieties of Conductor

Operating at higher current will incur greater $I^2R(T)$ losses. R increases with temperature and current, so losses will increase at slightly greater than the square of current. This relationship is visualized in Figure 14 for the same 3 conductors as before. It should be clear that the significant increases in ampacity allowed by the high-temperature conductors come at the cost of significantly higher joule loss.

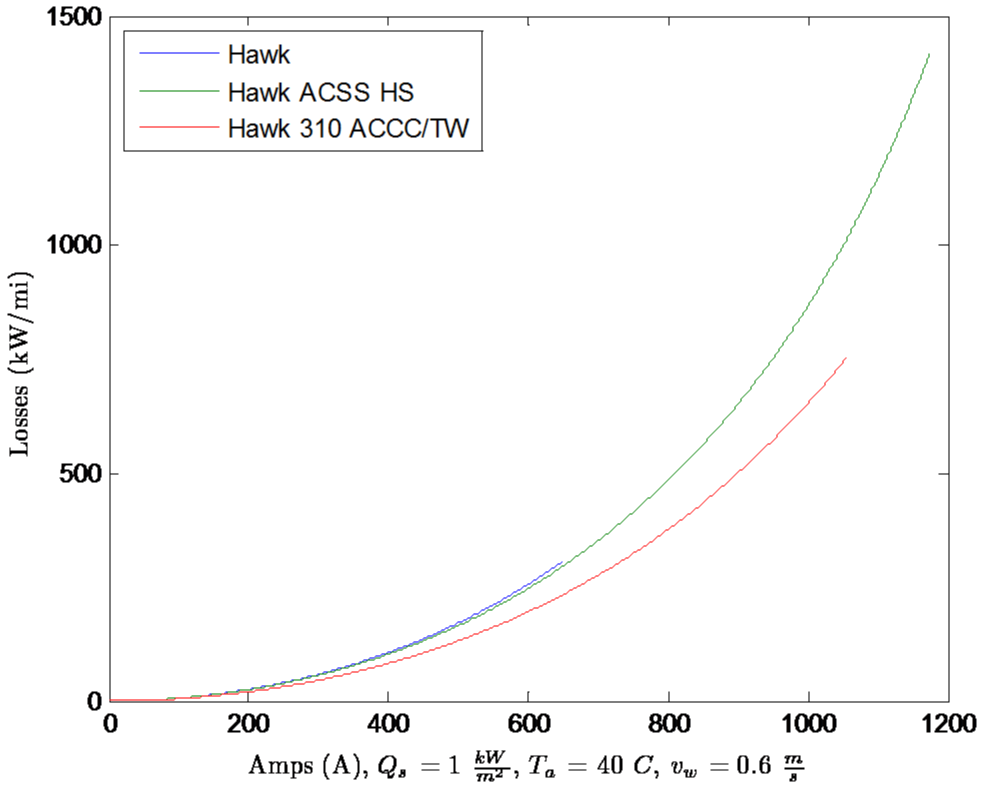


Figure 14: Real Power Loss (kW/mile) for 3 Varieties of Conductor

Losses decrease nearly linearly with resistance. Resistance in conductors is proportional to cross-sectional area of a conductor. Ampacity also increases with added cross-sectional area. Thus, one way to improve ampacity and decrease losses is to use cables of the same type with larger cross-sectional areas. This has been the primary strategy when uprating lines, prior to the emergence of high-temperature alternatives.

The primary feature of all the cable technologies above is their improved sag behavior. Figure 15 & Figure 16 show the results of a study by an Irish utility [15], which compared the theoretical performance of a variety of high-temperature conductors in order to find which would be most appropriate for re-conductoring an existing 220kV transmission line.

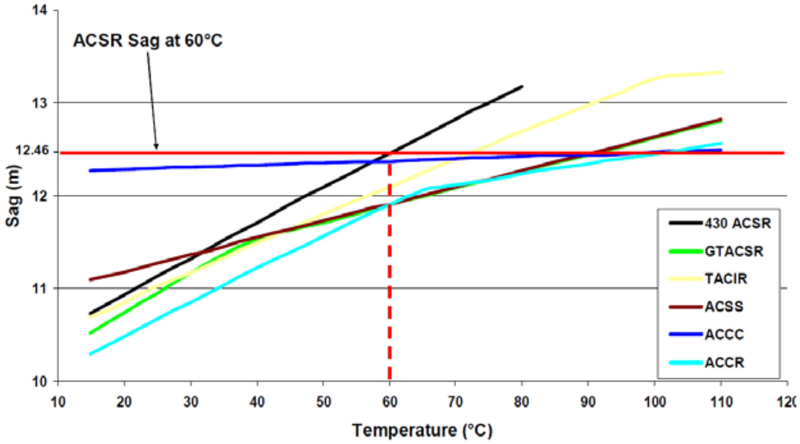


Figure 15: Sag vs. Temperature for Each Conductor Type

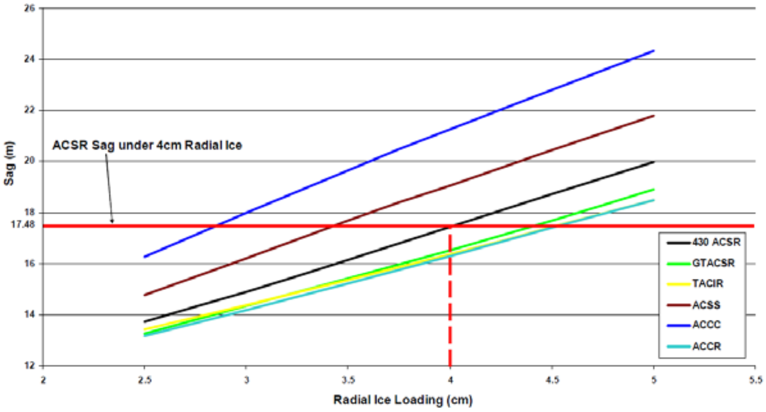


Figure 16: Sag vs. Radial Ice Loading for Each Conductor Type

All 5 of the high-temperature conductors outperform the ACSR alternative under high-temperature conditions. It is clear where some of the cables transition all of their load to their cores. For ACCR, the transition point is around 65 °C. For GTACSR, it is even lower, since most of the tension should already be applied to the core at its initial stringing temperature, which is around 20°C. Other conductor technologies such as ACSR and TACIR have very high transition temperatures, which will not be reached under normal operating conditions.

Figure 16, however, shows that some conductors may be inappropriate for regions with heavy ice-loading. The performance of ACCC and ACSS is significantly worse than that of ACSR. These two conductors have fully annealed aluminum conductor strands, which require their cores to carry the entire tension load. The core of ACCC has a particularly low elastic modulus, and small cross-sectional area, which makes the ice-loaded sag substantially worse.

High-temperature conductors have higher material costs than standard conductors, ranging from 1.3-6x the cost of equivalent ACSR cables. They also operate at high temperatures, incurring significant losses, and requiring specialized high-temperature hardware. However, their improved sag capabilities allow these cables to be used in longer spans, or at lower sags, reducing the number of towers that would have to be built or replaced.

Figure 17 comes from the same report as above, and shows the relative present-value cost of uprating a 220kV transmission line. Construction costs were added to losses (capitalized over 25 years), for a range of peak loading values. Curves for individual conductor technologies are terminated where that cable would meet its physical limitations. They were compared against a scenario in which the existing transmission line was reconductored with ACSR cable, to be capable of providing 150MVA of capacity. The cost at "0 MVA" represents construction costs alone, since losses will be zero.

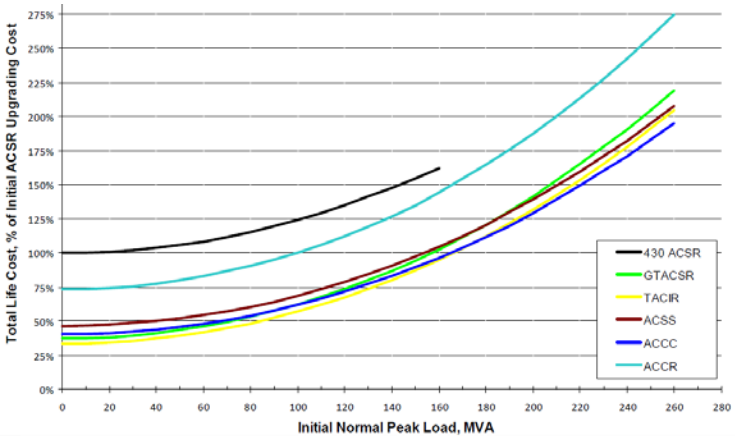


Figure 17: Cost of Uprating, based on Normal Peak Loading

It is evident that the construction cost of uprating with any of the high-temperature technologies will be significantly lower than uprating with ACSR. Uprating with ACSR would likely require many of the support structures to be raised or replaced, in order to meet existing sag requirements at a higher temperature. The high-temperature conductors would not require as many structural upgrades, evidently.

The cost difference between ACSR and the other conductors varies. ACSS, which is physically very similar to ACSR, has a cost of 1.5-2x that of ACSR. According to 3M, the cost of ACCR is 3-6x that of ACSR. The rest of the cable technologies appear to lie within that range of 1.5-6x.

Appendix A:

Material Properties for HTLS Cables []:

Aluminum Types for HTLS Conductors			
Types of Aluminum	Conductivity (%IACS)	Minimum Tensile Strength (MPa)	Max. Operating Temperature(°C)
Hard Drawn (1350-H19)	61.2	159-200	80-90
Fully Annealed (1350-H0)	63	59-97	200-250
Thermal Resistant (TAl)	60	159-176	150
Super Thermal Resistant (ZTAl)	60	159-176	210

Core Types for HTLS Conductors			
Types of Core	Tensile Strength (MPa)	Modulus of Elasticity (GPa)	Thermal Expansion (10^-6/°C)
Galvanized Steel			
HS	1230 - 1320	207	11.5
EHS	1725 - 1825	207	11.5
UHS	1725 - 1965	207	11.5
Aluminum Clad Steel (ACS)			
20EHSA	1515 - 1620	162	13
14EHSA	1725 - 1825	174	11.8
Galvanized Invar Steel	1030 - 1080	162	2.8-3.6
ACCC (Polymer Core)	c. 2200	117	1.6
ACCR (Metal Matrix)	c. 1300	216	6.3

Mechanical and Electrical Properties for several common cable sizes []:

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