Sensitivities

1.0 Introduction

Operation of the Eastern Interconnection has become heavily reliant on using the so-called Interchange Distribution Calculator (IDC). This is an internet-accessed system that interfaces with OASIS and allows market participants and network operators to efficiently, but approximately, determine the change in MW flow on a *flowgate* given a set of changes in MW bus injections.

A flowgate is a circuit or set of circuits that interconnect different regions of a network that can be limiting under some condition.

The IDC does not represent buses but rather represents control areas, and there are 97 of them in the eastern interconnection. Therefore the flowgates often represent interconnections between these control areas; however, a flowgate may also be internal to a single control area as well.

For our purposes, a control area is a bus, and the flowgates are interconnections between the buses.

One of the most important uses of the IDC is in the coordination of Transmission Loading Relief (TLR) actions. TLR procedures are in place to guide operators in mitigating flows that exceed operational security limits. TLR *levels*, summarized in Table 1 [1] have been defined the correspond to different types of actions that may be taken. r which curtailments must be made. When a TLR level 5 is declared, all ongoing transactions including those with firm transmission service are subject to curtailment.

What we desire to obtain, then, is an expression for computing the change in flow on a branch in a network for a given change in MW bus injection.

Table 1: Summary of TLR Levels [1]

TLR Level	RELIABILITY COORDINATOR Action	Comments	
1	**Notify** RELIABILITY COORDINATORS of potential OPERATING SECURITY LIMIT violations		System Secure
2	**Hold** INTERCHANGE TRANSACTIONS at current levels to prevent OPERATING SECURITY LIMIT violations	Of those transactions at or above the CURTAILMENT THRESHOLD, only those under existing Transmission Service reservations will be allowed to continue, and only to the level existing at the time of the hold. Transactions using Firm Point-to-Point Transmission Service are not held. See Section B.1.	
3a	**Reallocation** Transactions using Non-firm Point-to-Point Transmission Service are curtailed to allow Transactions using higher priority Point-to-Point Transmission Service	Curtailment follows Transmission Service priorities. Higher priority transactions are enabled to start by the REALLOCATION process. See Section B.3.	
3b	**Curtail** Transactions using Non-firm Point-to-Point Transmission Service to mitigate Operating Security Limit Violation	Curtailment follows Transmission Service priorities. There are special considerations for handling Transactions using Firm Point-to-Point Transmission Service. See Section B.4.	Security Limit Violation
4	**Reconfigure** transmission system to allow Transactions using Firm Point-to-Point Transmission Service to continue	There may or may not be an OPERATING SECURITY LIMIT violation. There are special considerations for handling Transactions using Firm Point-to-Point Transmission Service. See Section B.5.	
5a	**Reallocation** Transactions using Firm Point-to-Point Transmission Service are curtailed (pro rata) to allow new Transactions using Firm Point-to-Point Transmission Service to begin (pro rata).	Attempts to accommodate all Transactions using Firm Point-to-Point Transmission Service, though at a reduced ("pro rata") level. Pro forma tariff also requires curtailment / REALLOCATION on pro rata basis with Network Integration Transmission Service and Native Load. See Section B.6.	System Secure
5b	**Curtail** Transactions using Firm Point-to-Point Transmission Service to mitigate Operating Security Limit Violation	Pro forma tariff requires curtailment on pro rata basis with Network Integration Transmission Service and Native Load. See Section B.7.	Security Limit Violation
6	**Emergency Action**	Could include demand-side management, re-dispatch, voltage reductions, interruptible and firm load shedding. See Section B.8.	
0	**TLR Concluded**	**Restore** transactions. See Section B.9.	System Secure

TLR Lev	"Risk" Criteria		Transaction criteria	RELIABILITY COORD Action	Comments
	IMMINENCE	State			
1	Forsee possible condition resulting in violation	Secure		Notify	
2	Expected to approach, is approaching, SOL	Secure		Hold	Not > 30 minutes before going to higher levels so xactions may be made based on priority.
3a	Expected to approach is approaching, SOL	Secure	Some non-firm ptp at or above curtailment thres - holds, higher priority ptp reservation approved	Reallocate	Curtailments made at top of hour.
3b	Existing or imminent SOL violation or will occur on element removal	Insecure or about to be	Some non-firm ptp at or above their curtailment thresholds.	Hold and Curtail	Hold on nonfirm; Curtailments made immediately.
4	Existing or imminent SOL violation	Insecure or about to be		Hold and Reconfigur e	Hold on nonfirm.
5a	At SOL, no further reconfig possible	Secure	All non-firm ptp at or above curtailment thresholds curtailed; xaction request for previously arranged firm xmission service.	Reallocate	Curtailments made at top of upcoming hour.
5b	Existing or imminent SOL violation or one will occur on element removal, no further reconfig possible	Insecure or about to be	All non-firm ptp at or above curtailment thresholds curtailed.	Curtail	Curtailments made immediately.
6	Existing SOL violation or one will occur upon element removal	Insecure or about to be		Emergency Action	Could include redispatch, reconfiguration, voltage reductions, interruptible and firm load shedding.

2.0 Calculation of Generation Shift Factors

The desired quantity is referred to as the generation shift factor and will be denoted by $t_{b,k}$. It gives the fraction of a change in injection at bus k that appears on branch b. The Power Transfer Distribution Factor (PTDF) is a generalization of the generation shift factor.

This calculation of generation shift factors is relatively straightforward based on what we have done using the DC power flow model.

Recall the DC power flow equations and the corresponding matrix relation for flows across branches.

$$\underline{P} = \underline{B}'\underline{\theta} \tag{1}$$

$$\underline{P}_B = (\underline{D} \times \underline{A}) \times \underline{\theta} \tag{2}$$

Inverting eq (1) yields:

$$\underline{\theta} = [\underline{B}']^{-1}\underline{P} \tag{3}$$

Substitution of (3) into (2) yields:

$$\underline{P}_B = (\underline{D} \times \underline{A})[\underline{B}']^{-1}\underline{P} \tag{4}$$

Here, as we have previously defined in the notes on PowerFlow:

- $\underline{P}_B$ is the vector of branch flows. It has dimension of M x 1. Branches are ordered arbitrarily, but whatever order is chosen must also be used in $\underline{D}$ and $\underline{A}$.
- $\underline{D}$ is an M x M matrix having non-diagonal elements of zeros; the diagonal element in position row k, column k contains the negative of the susceptance of the k^{th} branch.
- $\underline{A}$ is the M x (N-1) *node-arc incidence matrix.*
- $\underline{B}$’ is the DC power flow matrix of dimension (N-1)x(N-1), where N is the number of buses in the network, obtained from the Y-bus as follows:

1. Replace diagonal element $\underline{B}'_{kk}$ with the sum of the non-diagonal elements in row k. Alternatively, subtract b_k (the shunt term) from B_{kk}, and multiply by -1.
2. Multiply all off-diagonals by -1.
3. Remove row 1 and column 1.

- $\underline{P}$ is the vector of nodal injections for buses 2, …, N

The calculation of eq. (4) provides the flows on all lines given the injections at all buses.

Bus this is not what we want. What we want is the fraction change in flow on all lines given a change in injections at one bus.

In other words, given a change in injection vector $\Delta\underline{P}$:

$$\Delta\underline{P} = \begin{bmatrix} P_2 \\ P_3 \\ \vdots \\ P_k \\ \vdots \\ P_N \end{bmatrix} - \begin{bmatrix} P_2^0 \\ P_3^0 \\ \vdots \\ P_k^0 \\ \vdots \\ P_N^0 \end{bmatrix} = \begin{bmatrix} \Delta P_2 \\ \Delta P_3 \\ \vdots \\ \Delta P_k \\ \vdots \\ \Delta P_N \end{bmatrix} = \underline{P} - \underline{P}^0 \tag{5}$$

Therefore,

$$\begin{aligned} \Delta\underline{P}_B &= \underline{P}_B - \underline{P}_B^0 \\ &= (\underline{D}\times\underline{A})\left[\underline{B}'\right]^{-1}\underline{P} - (\underline{D}\times\underline{A})\left[\underline{B}'\right]^{-1}\underline{P}^0 \\ &= (\underline{D}\times\underline{A})\left[\underline{B}'\right]^{-1}\left(\underline{P} - \underline{P}^0\right) \\ &= (\underline{D}\times\underline{A})\left[\underline{B}'\right]^{-1}\Delta\underline{P} \end{aligned} \tag{6}$$

Now let the $\Delta\underline{P}$ vector be all zeros except for the element corresponding to the k^th bus, and assign this bus an injection change of 1.

$$\Delta\underline{P} = \begin{bmatrix} \Delta P_2 \\ \Delta P_3 \\ \vdots \\ \Delta P_k \\ \vdots \\ \Delta P_N \end{bmatrix} = \begin{bmatrix} \Delta P_2 \\ \Delta P_3 \\ \vdots \\ \Delta P_k \\ \vdots \\ \Delta P_N \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{bmatrix} \tag{7}$$

Then

$$\Delta\underline{P}_B = \begin{bmatrix} \Delta P_{B1} \\ \Delta P_{B2} \\ \vdots \\ \Delta P_{Bb} \\ \vdots \\ \Delta P_{BM} \end{bmatrix} = \begin{bmatrix} t_{1,k} \\ t_{2,k} \\ \vdots \\ t_{b,k} \\ \vdots \\ t_{M,k} \end{bmatrix} = (\underline{D} \times \underline{A})\left[\underline{B}'\right]^{-1} \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{bmatrix} \tag{8}$$

Question: Does the above equation imply that the injection is changed at only one bus? Explain.

Example 1:

Consider the example that we started in the "PowerFlow" notes and continued using in the LOPF notes. Compute the generation shift factors for all branches corresponding to an increase in bus 2 injection and a decrease in bus 3 injection.

$$\begin{bmatrix} t_{1,23} \\ t_{2,23} \\ t_{3,23} \\ t_{4,23} \\ t_{5,23} \end{bmatrix} = \begin{bmatrix} 10 & 0 & 0 & 0 & 0 \\ 0 & 10 & 0 & 0 & 0 \\ 0 & 0 & 10 & 0 & 0 \\ 0 & 0 & 0 & 10 & 0 \\ 0 & 0 & 0 & 0 & 10 \end{bmatrix} \begin{bmatrix} 0 & 0 & -1 \\ -1 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & -1 & 1 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} 20 & -10 & 0 \\ -10 & 30 & -10 \\ 0 & -10 & 20 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & -10 \\ -10 & 0 & 0 \\ 10 & -10 & 0 \\ 0 & -10 & 10 \\ 0 & -10 & 0 \end{bmatrix} \begin{bmatrix} 0.0625 & 0.025 & 0.0125 \\ 0.025 & 0.05 & 0.025 \\ 0.0125 & 0.025 & 0.0625 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & -10 \\ -10 & 0 & 0 \\ 10 & -10 & 0 \\ 0 & -10 & 10 \\ 0 & -10 & 0 \end{bmatrix} \begin{bmatrix} 0.0375 \\ -0.025 \\ -0.0125 \end{bmatrix} = \begin{bmatrix} 0.125 \\ -0.375 \\ -0.2125 \\ 0.125 \\ 0.25 \end{bmatrix}$$

Note that the above generation shift factors are for a "double shift."

You can think of it like this. A generation shift factor for branch b, bus k would be $t_{b,k}$ and another generation shift factor for branch b, bus j would be $t_{b2,j}$. If we have an injection increase at bus k of ΔP_k and an injection increase at bus j of ΔP_j, then

$$\Delta P_b = t_{b,k}\Delta P_k + t_{b,j}\Delta P_j \qquad (9)$$

Increase P_k, Decrease P_1

Decrease P_j, Increase P_1

Therefore, if $\Delta P_k = -\Delta P_j$, then

$$\Delta P_b = \left(t_{b,k} - t_{b,j}\right)\Delta P_k \tag{10}$$

3.0 Generation Shift Factors with Distributed Slack

Equation (8) shows how to compute the generation shift factors for the case when a single specified slack bus corresponds to bus 1.

The example above shows how to compute the generation shift factors for the case when a single specified slack bus corresponds to some other bus in the network (not the bus corresponding to the reference by way of omission from its corresponding row and column in the B' matrix).

What we are interested in here is computation of generation shift factors for the case when we would like to distribute the slack, or the compensation, throughout the network. The key criterion to guide this is that the elements in the nodal injection vector should correspond to the percentage of desired compensation for each bus.

This criterion is illustrated below:

$$\Delta \underline{P}_B = \begin{bmatrix} \Delta P_{B1} \\ \Delta P_{B2} \\ \vdots \\ \Delta P_{Bb} \\ \vdots \\ \Delta P_{BM} \end{bmatrix} = \begin{bmatrix} t_{1,k} \\ t_{2,k} \\ \vdots \\ t_{b,k} \\ \vdots \\ t_{M,k} \end{bmatrix} = (\underline{D} \times \underline{A})\left[\underline{B}'\right]^{-1} \begin{bmatrix} c_2 \\ c_3 \\ \vdots \\ 1 \\ \vdots \\ c_N \end{bmatrix} \tag{11}$$

where

$$c_1 = \sum_{i=2}^{N} c_i = 1 + \sum_{\substack{i=2 \\ i \neq k}}^{N} c_i \tag{12}$$

is the allocation desired for the reference bus.

One way to distribute the slack is to distribute equally to all buses. In this case,

$$c_i = \frac{-1}{N-1} \tag{13}$$

where we use *N-1* in the denominator because one bus, bus *k*, is the bus for which the computation is being made (and therefore $c_k=1$).

Example 2:
Using the system from the example above, compute generation shift factors for all branches corresponding to an increase in bus 2 injection, when the slack is equally distributed to all buses.

$$\begin{bmatrix} t_{1,2all} \\ t_{2,2all} \\ t_{3,2all} \\ t_{4,2all} \\ t_{5,2all} \end{bmatrix} = \begin{bmatrix} 10 & 0 & 0 & 0 & 0 \\ 0 & 10 & 0 & 0 & 0 \\ 0 & 0 & 10 & 0 & 0 \\ 0 & 0 & 0 & 10 & 0 \\ 0 & 0 & 0 & 0 & 10 \end{bmatrix} \begin{bmatrix} 0 & 0 & -1 \\ -1 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & -1 & 1 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} 20 & -10 & 0 \\ -10 & 30 & -10 \\ 0 & -10 & 20 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ -0.333 \\ -0.333 \end{bmatrix}$$

$$
= \begin{bmatrix} 0 & 0 & -10 \\ -10 & 0 & 0 \\ 10 & -10 & 0 \\ 0 & -10 & 10 \\ 0 & -10 & 0 \end{bmatrix} \begin{bmatrix} 0.0625 & 0.025 & 0.0125 \\ 0.025 & 0.05 & 0.025 \\ 0.0125 & 0.025 & 0.0625 \end{bmatrix} \begin{bmatrix} 1 \\ -0.333 \\ -0.333 \end{bmatrix}
$$

$$
= \begin{bmatrix} 0 & 0 & -10 \\ -10 & 0 & 0 \\ 10 & -10 & 0 \\ 0 & -10 & 10 \\ 0 & -10 & 0 \end{bmatrix} \begin{bmatrix} 0.05 \\ 0 \\ -0.0166 \end{bmatrix} = \begin{bmatrix} 0.1664 \\ -0.5001 \\ 0.4999 \\ -0.1666 \\ -0.0002 \end{bmatrix}
$$

It is of interest to compare the answer from the example where the slack was distributed entirely to bus 3 and the example where the slack was distributed to all buses.

$$
\begin{bmatrix} t_{1,23} \\ t_{2,23} \\ t_{3,23} \\ t_{4,23} \\ t_{5,23} \end{bmatrix} = \begin{bmatrix} 0.125 \\ -0.375 \\ -0.2125 \\ 0.125 \\ 0.25 \end{bmatrix} \qquad \begin{bmatrix} t_{1,2all} \\ t_{2,2all} \\ t_{3,2all} \\ t_{4,2all} \\ t_{5,2all} \end{bmatrix} = \begin{bmatrix} 0.1664 \\ -0.5001 \\ 0.4999 \\ -0.1666 \\ -0.0002 \end{bmatrix}
$$

Clearly the assumption on slack distribution is important!

There are other ways to distribute the slack. For example, we may distribute the slack equally to all generation buses. Or we may distribute the slack equally to all load buses.

4.0 Generation Shift Factor Matrix

Given a specified slack distribution, we may compute a matrix of generation shift factors according to

$$\underline{T} = \underbrace{\begin{bmatrix} t_{1,1} & t_{1,2} & \cdots & t_{1,k} & \cdots & t_{1,N} \\ t_{2,1} & t_{2,2} & \cdots & t_{2,k} & \cdots & t_{2,N} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ t_{b,1} & t_{b,2} & \cdots & t_{b,k} & \cdots & t_{b,N} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ t_{M,1} & t_{M,2} & \cdots & t_{M,k} & \cdots & t_{M,N} \end{bmatrix}}_{M \times N}$$

$$= (\underbrace{\underline{D}}_{M \times M} \times \underbrace{\underline{A}}_{M \times (N-1)}) \underbrace{\left[\underline{B}'\right]^{-1}}_{(N-1)\times(N-1)} \underbrace{\begin{bmatrix} c & 1 & c & c & c & c & c \\ c & c & 1 & c & c & c & c \\ c & c & c & 1 & c & c & c \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ c & c & c & c & c & \vdots & c \\ c & c & c & c & c & \cdots & 1 \end{bmatrix}}_{(N-1)\times N}$$

The above assumes that we desire generation shift factors for every branch (a row of $\underline{T}$) and every bus (a column of $\underline{T}$). Note that the first column of $\underline{T}$ is for a shift at the bus 1, which is the one assumed to be deleted from the $\underline{B}$*’* matrix.

However, we need not include every branch. There may be some branches that we know from experience will never overload, or there may be policy that requires a particularly application to only monitor certain branches. The latter is the case for NERC’s IDC described at the beginning of this document.

Example 3: Let's compute the T-matrix for our previous example. We assume a distributed slack bus, where, c=-1/3. Therefore

$$\underline{T} = \begin{bmatrix} 10 & 0 & 0 & 0 & 0 \\ 0 & 10 & 0 & 0 & 0 \\ 0 & 0 & 10 & 0 & 0 \\ 0 & 0 & 0 & 10 & 0 \\ 0 & 0 & 0 & 0 & 10 \end{bmatrix} \begin{bmatrix} 0 & 0 & -1 \\ -1 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & -1 & 1 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} 20 & -10 & 0 \\ -10 & 30 & -10 \\ 0 & -10 & 20 \end{bmatrix}^{-1} \begin{bmatrix} -0.333 & 1 & -0.333 & -0.333 \\ -0.333 & -0.333 & 1 & -0.333 \\ -0.333 & -0.333 & -0.333 & 1 \end{bmatrix}$$

$$\underline{T} = \begin{bmatrix} 0.3333 & 0.1667 & 0 & -0.5 \\ 0.3333 & -0.5 & 0 & 0.1667 \\ 0 & 0.5 & -0.3333 & -0.1667 \\ 0 & -0.1667 & -0.3333 & 0.5 \\ 0.3333 & 0 & -0.3333 & 0 \end{bmatrix}$$

Remember: each column is the set of shift factors for a unit increase in injection (generation) at a certain bus. Column 1 is when the injection at bus 1 is increased (there is no "1" in that column because that is the one corresponding to the bus that was deleted in the B' matrix). Column 2 is when the injection at bus 2 is increased, and so on.

References:

[1] North American Electric Reliability Council (NERC) Operating Manual, Appendix 9C1, May, 2004, available at www.nerc.com.