Transmission Line Design Information

In these notes, I would like to provide you with some background information on AC transmission lines that may be useful to you in your project.

1. AC Transmission Line Impedance Parameters

AC transmission is done through 3-phase systems. Initial planning studies typically only consider balanced, steady-state operation. This simplifies modeling efforts greatly in that only the positive sequence, per-phase transmission line representation is necessary.

Essential Transmission line electrical data for balanced, steady-state operation includes:

- Line reactance
- Line resistance
- Line charging susceptance
- Current rating (ampacity)
- Surge impedance loading

Figure 1 below shows a *distributed parameter model* of a transmission line where $z=r+jx$ is the series impedance per unit length (ohms/unit length), and $y=jb$ is the shunt admittance per unit length (mhos/unit length).

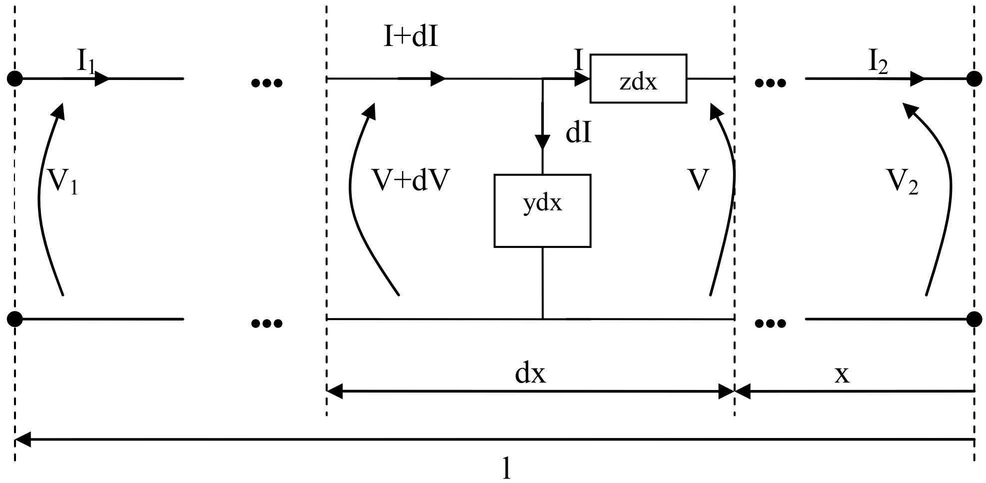


Fig. 1

I have notes posted under lecture 4, at www.ee.iastate.edu/~jdm/EE456/ee456schedule.htm, that derive the following model relating voltages and currents at either end of the line.

$$\boldsymbol{I(l)=I_1=I_2\cosh\gamma l+\frac{V_2}{Z_C}\sinh\gamma l} \qquad \text{(1a)}$$

$$\boldsymbol{V(l)=V_1=V_2\cosh\gamma l+Z_C I_2\sinh\gamma l} \qquad \text{(1b)}$$

where

- l is the line length,
- γ is the propagation constant, in general a complex number, given by

$$\gamma=\sqrt{zy} \text{ with units of 1/(unit length),} \qquad \text{(1c)}$$

where z and y are the per-unit length impedance and admittance, respectively, as defined previously.

- Z_C is the characteristic impedance, otherwise known as the surge impedance, given by

$$Z_C = \sqrt{\frac{z}{y}} \quad \text{with units of ohms.} \qquad (1d)$$

It is then possible to show that equations (1a, 1b) may be represented using the following pi-equivalent line model

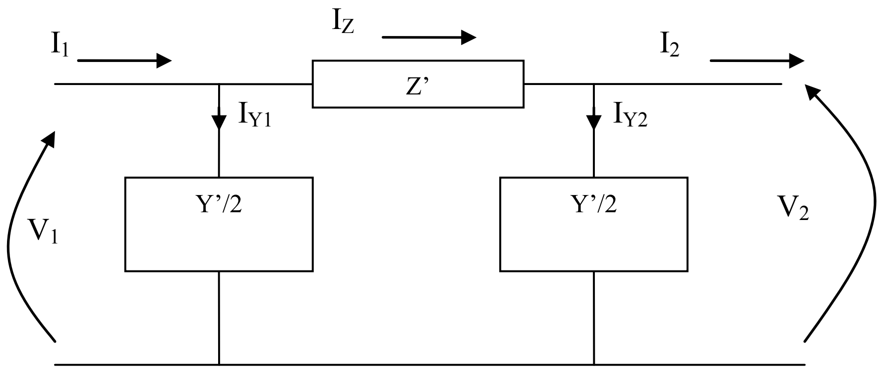


Fig. 2

where

$$Z' = Z\frac{\sinh \gamma l}{\gamma l} \qquad (2a)$$

$$Y' = Y\frac{\tanh(\gamma l/2)}{\gamma l/2} \qquad (2b)$$

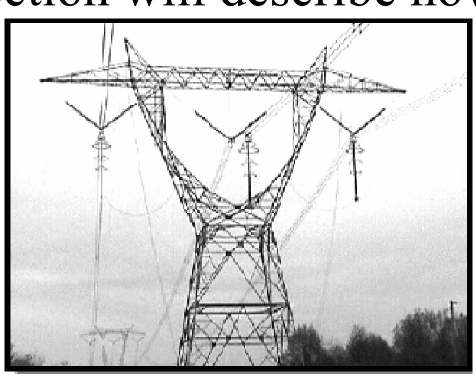
and $Z=zl$, $Y=yl$.

Two comments are necessary here:

1. Equations (2a, 2b) show that the impedance and admittance of a transmission line are not just the impedance per unit length and admittance per unit length multiplied by the line length, $Z=zl$ and $Y=yl$, respectively, but they are these values corrected by the factors

$$\frac{\sinh \gamma l}{\gamma l} \qquad \frac{\tanh(\gamma l/2)}{\gamma l/2}$$

   It is of interest to note that these two factors approach 1.0 (the first from above and the second from below) as γl becomes small. This fact has an important implication in that short lines (less than ~100 miles) are usually well approximated by $Z=zl$ and $Y=yl$, but longer lines are not and need to be multiplied by the "correction factors" listed above. The "correction" enables the lumped parameter model to exhibit the same characteristics as the distributed parameter device.
2. We may obtain all of what we need if we have z and y. The next section will describe how to obtain them.



2. Obtaining per-unit length parameters

In the 9/6 and 9/8 notes at

www.ee.iastate.edu/~jdm/EE456/ee456schedule.htm

I have derived expressions to compute per-unit length inductance and per-unit length capacitance of a transmission line given its geometry. These expressions are:

Inductance (h/m): $\boldsymbol{l_a = \frac{\mu_0}{2\pi}\ln\frac{D_m}{R_b}}$

- D_m is the GMD between phase positions:

$$\boldsymbol{D_m \equiv \left(d_{ab}^{(1)} d_{ab}^{(2)} d_{ab}^{(3)}\right)^{1/3}}$$

- R_b is the GMR of the bundle

$$
\begin{aligned}
R_b &= \left(r' d_{12}\right)^{1/2}, && \text{for 2 conductor bundle} \\
&= \left(r' d_{12} d_{13}\right)^{1/3}, && \text{for 3 conductor bundle} \\
&= \left(r' d_{12} d_{13} d_{14}\right)^{1/4}, && \text{for 4 conductor bundle} \\
&= \left(r' d_{12} d_{13} d_{14} d_{15} d_{16}\right)^{1/4}, && \text{for 6 conductor bundle}
\end{aligned}
$$

Capacitance (f/m): $\boldsymbol{\bar{c}_a = \frac{2\pi\varepsilon}{\ln(D_m / R_b^c)}}$

- D_m is the same as above.
- $\boldsymbol{R_b^c}$ is Capacitive GMR for the bundle:

$$
\begin{aligned}
R_b^c &= \left(r d_{12}\right)^{1/2}, && \text{for 2 conductor bundle} \\
&= \left(r d_{12} d_{13}\right)^{1/3}, && \text{for 3 conductor bundle} \\
&= \left(r d_{12} d_{13} d_{14}\right)^{1/4}, && \text{for 4 conductor bundle} \\
&= \left(r d_{12} d_{13} d_{14} d_{15} d_{16}\right)^{1/6}, && \text{for 6 conductor bundle}
\end{aligned}
$$

In the above, r is the radius of a single conductor, and r' is the Geometric Mean Radius (GMR) of an individual conductor, given by

$$r' = re^{-\frac{\mu_r}{4}} \tag{3}$$

2.1 Inductive reactance

The per-phase inductive reactance in Ω/m of a non-bundled transmission line is $2\pi f l_a$, where $l_a = \frac{\mu_0}{2\pi} \ln \frac{D_m}{R_b}$ Ω/m. Therefore, we can express the reactance in Ω/mile as

$$X_L = 2\pi f l_a = 2\pi f \left(\frac{\mu_0}{2\pi} \ln \frac{D_m}{R_b} \right) \frac{1609 \text{ meters}}{1 \text{ mile}}$$

$$= 2.022 \times 10^{-3} f \ln \frac{D_m}{R_b} \quad \Omega/\text{mile} \tag{4}$$

Let's expand the logarithm to get

$$X_L = \underbrace{2.022 \times 10^{-3} f \ln \frac{1}{R_b}}_{X_a} + \underbrace{2.022 \times 10^{-3} f \ln D_m}_{X_d} \quad \Omega/\text{mile} \tag{5}$$

where f=60 Hz. The first term is called the inductive reactance *at 1 foot spacing* because of the "1" in the numerator of the logarithm when R_b is given in feet.

Note: to get X_a, you need only to know R_b, which means you need only know the conductor used and the bundling.

(we assume). But you do *not* need to know the geometry of the phase positions.

But what is X_d? This is called the inductive reactance spacing factor. Note that it depends only on D_m, which is the GMD between phase positions. So you can get X_d by knowing only the distance between phases, i.e, you need not know anything about the conductor or the bundling.

<u>2.2 Capacitive reactance</u>

Similar thinking for capacitive reactance leads to

$$X_C = \underbrace{\frac{1}{f}\times 1.779\times 10^6 \ln\left(\frac{1}{r}\right)}_{X'_a} + \underbrace{\frac{1}{f}\times 1.779\times 10^6 \ln(D_m)}_{X'_d} \quad \Omega\text{-mile}$$

X'_a is the capacitive reactance at 1 foot spacing, and X'_d is the capacitive reactance spacing factor. Note the units are ohms-mile, instead of ohms/mile, so that when we invert, we will get mhos/mile, as desired.

3. **Example**

Let's compute the X_L and X_C for a 765 kV AC line, single circuit, with a 6 conductor bundle per phase, using conductor type Tern (795 kcmil). The bundles have 2.5' (30'') diameter, and the phases are separated by 45', as shown in Fig. 3. Assume the line is lossless.

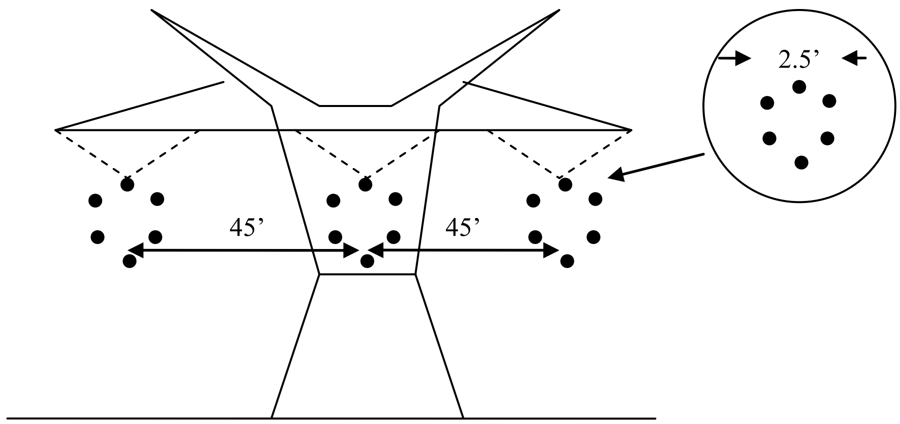


Fig. 3

We will use tables from [1], which I have copied out and placed on the website. We can see from the below table, obtained from [2] (and placed on the website), that this example focuses on AEP 3.

TABLE 2 LINE GEOMETRIES

Company/Country	Nominal Voltage (kV)	No. of Sub-conductors	Conductor Diameter (cm)	Phase Spacing (m)	Min. Conductor Heights* (m)
Hydro-Québec 1	735	4	3.50	15.3	15.3
Hydro-Québec 2	735	4	3.56	12.8	14.1
AEP 1	765	4	2.96	13.7	12.2
AEP 2	765	4	3.52	13.7	12.2/13.7
AEP 3	765	6	2.70	13.7	13.7
NYPA	765	4	3.52	15.2	15.5
Eskom	765	6	2.86	15.8	15.0
FURNAS	765	4	3.20	14.3	13
EDELCA 1 &2	765	4	3.33	15.0	14.7
EDELCA 3	765	4	3.33	13.2	13.7
KEPCO	765	6	3.042	See Note 1	19/28
POWERGRID	765	4	3.50	15.4	15
RUSSIA 1	750	5	2.24	17.5	12
RUSSIA 2	750	4	2.91	19	12
RUSSIA 3	1150	8	2.75	21.5-25	17.5
TEPCO	1000	8	3.42/3.84**	See Note 1	25/35

* Minimum heights in areas frequented by people including agricultural areas.
** Larger conductor used in populated areas; smaller conductor used in mountainous areas.
1. Double-circuit low reactance line

The tables show data for 24” and 36” 6-conductor bundles, but not 30”, and so we must interpolate.

Get per-unit length inductive reactance:
From Table 3.3.1, we find
24" bundle: 0.031
36" bundle: -0.011
30" bundle: interpolation results in X_a=0.0105.

From Table 3.3.12, we find
45' phase spacing: X_d=0.4619

And so $X_L=X_a+X_d$=0.0105+0.4619=0.4724 ohms/mile.
Now get per-unit length capacitive reactance.

From Table 3.3.2, we find
24" bundle: 0.065
36" bundle: -0.0035
30" bundle: interpolation results in X'_a=0.0307.

From Table 3.3.13, we find
45' phase spacing: X'_d=0.1128

And so $X_C=X'_a+X'_d$=0.0307+0.1128=0.1435Mohms-mile.
Note the units of X_C are ohms-mile$\times 10^6$.

So $z=jX_L=j0.4724$ Ohms/mile, and this is the number in Table 1 of the project for the 6 bdl, 765 kV circuit.
And $y=1/-jX_C=1/-j(0.1435\times 10^6)=j6.9686\times 10^{-6}$ Mhos/mile

Now compute the propagation constant, γ,

$$\gamma = \sqrt{zy} = \sqrt{j0.4724 \times j6.9686 \times 10^{-6}}$$

$$= \sqrt{-3.292 \times 10^{-6}} = j0.0018 / mile$$

Recalling (2a, 2b)

$$Z' = Z \frac{\sinh \gamma l}{\gamma l} \qquad (2a)$$

$$Y' = Y \frac{\tanh(\gamma l/2)}{\gamma l/2} \qquad (2b)$$

Let's do two calculations:

- The circuit is 100 miles in length. Then l=100, and

  $$Z = j.4724 ohms / mile * 100 miles = j47.24\, ohms$$

  $$Y = j6.986 \times 10^{-6} mhos / mile * 100 miles = j0.0006986 mhos$$

  $$\gamma l = \frac{j0.0018}{mile}(100 miles) = j0.18$$

  Convert Z and Y to per-unit, V_b=765kV, S_b=100 MVA

  Z_b=$(765\times10^3)^2/100\times10^6$=5852.3ohms,

  Y_b=1/5852.3=0.00017087mhos

  Z_{pu}=j47.24/5852.3=j0.0081pu,

  Y_{pu}=j0.0006986/.00017087=j4.0885pu

  $$Z' = Z \frac{\sinh \gamma l}{\gamma l} = j0.0081 \frac{\sinh(j.18)}{j.18} = j0.0081 \frac{j.179}{j.18} = j0.00806$$

  $$Y' = Y \frac{\tanh(\gamma l/2)}{\gamma l/2} = j4.0885 \frac{\tanh(j.18/2)}{j.18/2} = j4.0885 \frac{j0.0902}{j.09} = j4.0976$$

- The circuit is 500 miles in length. Then l=500, and
  $$Z = j.4724 ohms/mile * 500 miles = j236.2\ ohms$$
- $$Y = j6.986\times10^{-6} mhos/mile * 500 miles = j0.0035\ mhos$$

  $$\gamma l = \frac{j0.0018}{mile}(500 miles) = j0.90$$

Convert Z and Y to per-unit, V_b=765kV, S_b=100 MVA

Z_{pu}=j236.2/5852.3=j0.0404pu,

Y_{pu}=j0.0035/.00017087=j20.4834pu

$$Z' = Z\frac{\sinh\gamma l}{\gamma l} = j.0404\frac{\sinh(j.90)}{j.90} = j.0404\frac{j.7833}{j.90} = j.0352$$

$$Y' = Y\frac{\tanh(\gamma l/2)}{\gamma l/2} = j20.4834\frac{\tanh(j.90/2)}{j.90/2} = j20.4834\frac{j0.4831}{j.45} = j21.99$$

It is of interest to calculate the surge impedance for this circuit. From eq. (1d), we have

$$Z_C = \sqrt{\frac{z}{y}} = \sqrt{\frac{\text{j.4724}}{\text{j6.9686}\times 10\text{-}6}} = 260.3647\text{ohms}$$

Then the surge impedance loading is given by

$$P_{SIL} = \frac{V_{LL}^2}{Z_C} = \frac{\left(765\times10^3\right)^2}{260.3647} = 2.2477\text{e}+009$$

The SIL for this circuit is 2247 MW. We can estimate line loadability from Fig. 4 below as a function of line length.

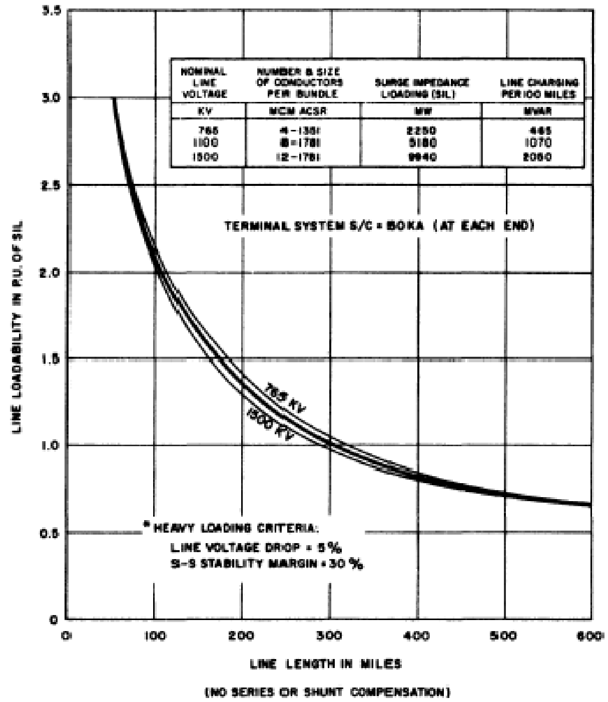


Fig. 4

100 mile long line: P_{max}=2.1(2247)=4719 MW.

500 mile long line: P_{max}=0.75(2247)=1685 MW.

4. Conductor ampacity

A conductor expands when heated, and this expansion causes it to sag. Conductor surface temperatures are a function of the following:

a) Conductor material properties

b) Conductor diameter

c) Conductor surface conditions

d) Ambient weather conditions

e) Conductor electrical current

IEEE Standard 738-2006 (IEEE Standard for Calculating Current–Temperature Relationship of Bare Overhead Conductors) [3] provides an analytic model for computing conductor temperature based on the above influences.

In addition, this same model is used to compute the conductor current necessary to cause a "maximum allowable conductor temperature" under "assumed conditions."

- Maximum allowable conductor temperature: This temperature is normally selected so as to limit either conductor loss of strength due to the annealing of aluminum or to maintain adequate ground clearance, as required by the National Electric Safety Code. This temperature varies widely according to engineering practice and judgment (temperatures of 50 °C to 180 °C are in use for ACSR) [3], with 100 °C being not uncommon.
- Assumed conditions: It is good practice to select "conservative" weather conditions such as 0.6 m/s to 1.2 m/s wind speed (2ft/sec-4ft/sec), 30 °C to 45 °C for summer conditions.

Given this information, the corresponding conductor current (I) that produced the maximum allowable conductor temperature under these weather conditions can be found from the steady-state heat balance equation [3].

For example, the Tern conductor used in the 6 bundle 765kV line (see example above) is computed to have an ampacity of about 860 amperes at 75 °C conductor temperature, 25 °C ambient temperature, and 2 ft/sec wind speed. At 6 conductors per phase, this allows for 6×860=5160 amperes, which would correspond to a power transfer of √3 * 765000 * 5160=6837 MVA.

Recalling the SIL for this line was 2247 MW. Figure 4 indicates the short-line power handling capability of this circuit should be about 3(2247)=6741 MW.

➔Short-line limitations are thermal-constrained.

For purposes of the project, where you will be considering relatively long lines, you will not need to be too concerned about ampacity. Limitations of SIL or lower will be more appropriate to use for these long lines.

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[1] Electric Power Research Institute (EPRI), "Transmission Line Reference Book: 345 kV and Above," second edition, revised, 1987.
[2] R. Lings, "Overview of Transmission Lines Above 700 kV," IEEE PES 2005 Conference and Exposition in Africa, Durban, South Africa, 11-15 July 2005.
[3] IEEE Standard 738-2006, "IEEE Standard for Calculating the Current–Temperature Relationship of Bare Overhead Conductors," IEEE, 2006.