# Planning Tools

## 1.0 Introduction

There are some fundamental planning tools that are useful within the planning function such as power flow, optimal power flow, stability (transient, small signal, and voltage), and short-circuit analysis programs. These tools are also useful in an operations context and so are very broad-based in application. As a result, they are typically covered in other courses (at ISU, EE 456, EE 457, EE 553, and EE 554). We will not focus on these tools in these notes but rather three tools that are relatively specific to the planning function. These are

- Probabilistic reliability analysis
- Production costing
- Expansion planning

We begin this treatment with a section identifying the relation between the above three tools. Then we review the commercial offerings of which we are aware in relation to these tools. Next we describe some probability models for load and generation, important conceptual underpinnings of planning studies. We will conclude these notes with a description of the basics associated with production cost models.

## 2.0 Relation between tools

A simple statement of the generation expansion planning (GEP) problem is as follows:

$$\min \sum_{t=1}^{T} \overline{I}(t) - \overline{S}(t) + \overline{F}(t) + \overline{M}(t) + \overline{O}(t) \qquad (1)$$

where

- *I(t)* is total investment costs at year *t*
- *S(t)* is total salvage value of retired plants at year *t* (and for all plants still in operation at year *T*).
- *F(t)* is total fuel costs in year *t*.
- *M(t)* is total maintenance costs in year *t*.
- *O(t)* is the cost associated with outages.

and the overbar in (1) indicates the values must be present-worth values.

There are alternative formulations of the GEP problem, for example, one may remove outage costs from the objective function and then constrain an index reflecting reliability (such as loss-of-load probability, LOLP, or loss of load expectation, LOLE, or expected unserved energy EUE).

Regardless, it is clear that GEP is an optimization tool. It results in a particular selection of a generation expansion plan among various alternatives.

In contrast, probabilistic reliability analysis and production costing are evaluation tools. They require a particular plan, or scenario, in terms of generation (and transmission) facilities, and then they evaluate that particular plan to provide specific kinds of information.

The information provided by probabilistic reliability analysis is the outage costs, and/or the various reliability indices mentioned above.

The information provided by production costing includes the annual costs of operating the generation facilities, a cost that is dominated by the fuel costs but also affected by the maintenance costs. Production costing may also provide more time-granular estimates of fuel and maintenance costs, such as monthly, weekly, or hourly, from which it is then possible to obtain annual production costs.

Production cost programs are typically developed so that they simultaneously provide reliability information such as outage costs, LOLP, LOLE, or EUE. This does not eliminate the need for reliability evaluation programs which typically perform reliability assessment at a more refined level than production costing programs do.

We observe production costs programs and reliability evaluation programs provide *F(t)*, *M(t)*, and *O(t),* 3 of the 5 terms required by GEP problem in (1).

The other two terms in GEP, *I(t)* and *S(t)*, are typically estimated on a $/MW basis from experience, historical data, and data obtained from power plant manufacturers and developers. It is useful to think of production costs programs and reliability evaluation programs as "feed-ins" to the GEP problem. A flow-chart for addressing the GEP problem, shown in Fig. 1 [1], illustrates the need for investment information, production costs, and reliability evaluation.

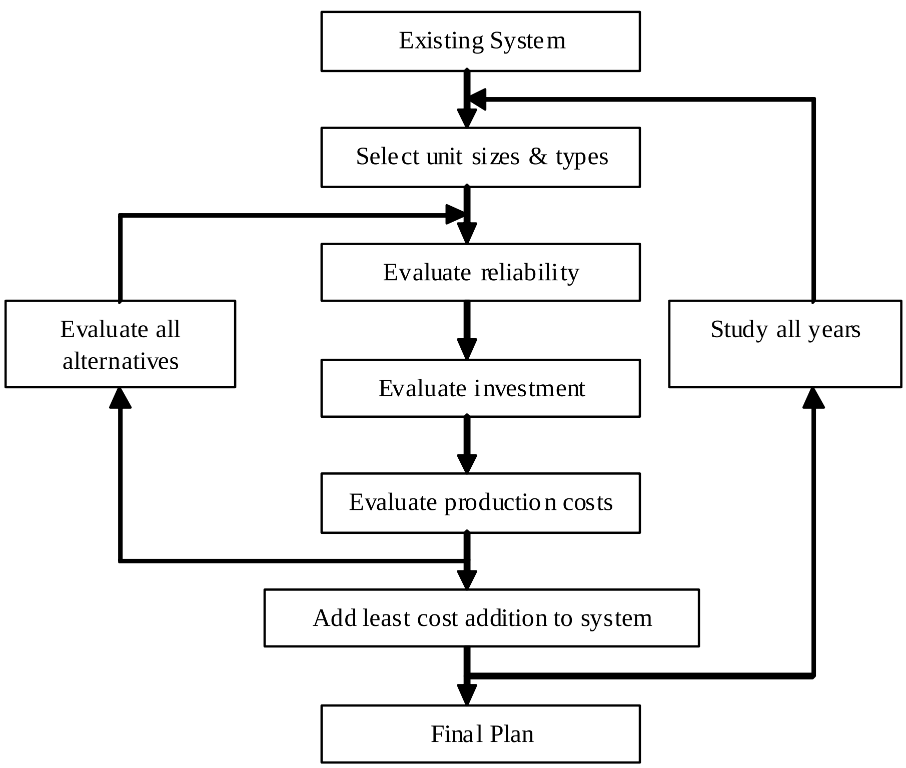


Fig. 1

# 3.0 Overview of commercial tools

In this section, we summarize commercially available evaluation tools. We do comment on some of the tools in terms of described functionality, but these comments should not be construed to indicate quality or desirability of one tool over another.

## 3.1 Reliability evaluation tools

Reliability evaluation has been segregated into hierarchical levels HL-I (generation only), HL-II (generation and transmission), and HL-III (generation, transmission, and distribution), where the last is normally addressed by assuming the generation and transmission sides are perfectly reliable. We will focus on HL-II in the remainder of this section.

Most HL-II evaluation procedures are characterized by two attributes:

- Method of representing stochastic nature of the operating conditions: By "operating conditions," we are referring to the basecase network configuration (topology and unit commitment) together with the loading and dispatch.
    - o Nonsequential: The nonsequential approach assumes a particular network configuration to be

evaluated. Then several loading conditions are selected based on their occurrence probability (as indicated by a load duration curve), and for each chosen loading condition, the dispatch is developed through an economic dispatch calculation (or an equivalent market-dispatch tool). The evaluation is performed once for each loading condition, and then indices are computed as a function of the loading probabilities.

o Sequential: The sequential approach assumes a particular network configuration to be evaluated together with an hourly or daily peak load forecast for an extended time period (e.g., year or several years). The method then steps through a series of sequential-in-time operating conditions, evaluating the reliability indices at each step, with final indices an accumulation of those evaluated at each step. Each sequential evaluation performed is called a trajectory. It is possible to compute indices based on a single trajectory or based on multiple trajectories. In the latter case, Monte-Carlo simulation may be used to select the trajectory.

The advantage to non-sequential simulation is it is typically faster than sequential simulation. The advantage to sequential simulation is that it captures the effects of inter-temporal effects such as hydro-scheduling, maintenance, and unit commitment.

- Method of representing stochastic nature of contingencies:
  - <u>Contingency enumeration</u>: Here, the "contingency states" corresponding to different numbers and combinations of outaged components are evaluated one by one, usually with some sort of intelligence to eliminate evaluation of some states.
  - <u>Monte-Carlo</u>: Here, the "contingency states" evaluated are chosen as a result of random draw where the chance of drawing a particular state is the same as the probability of that state.

The possible HL-II evaluation approaches are illustrated in Table 1.

Table 1: HL-II Evaluation approaches

| Contingency selection | Operating Conditions | | |
|---|---|---|---|
| | Non-sequential | Sequential, single-trajectory | Sequential, multi-trajectory |
| Enumeration | Non-sequential, with contingency enumeration | Sequential, with contingency enumeration | Sequential, multi-trajectory w/ contingency enumeration |
| Monte-Carlo | Non-sequential, with Monte-Carlo contingency selection | Sequential, with Monte-Carlo contingency selection | Sequential, multi-trajectory w/ Monte-Carlo contingency selection |

A basic algorithm of HL-II evaluation that applies independent of the approach is given is Fig. 2.

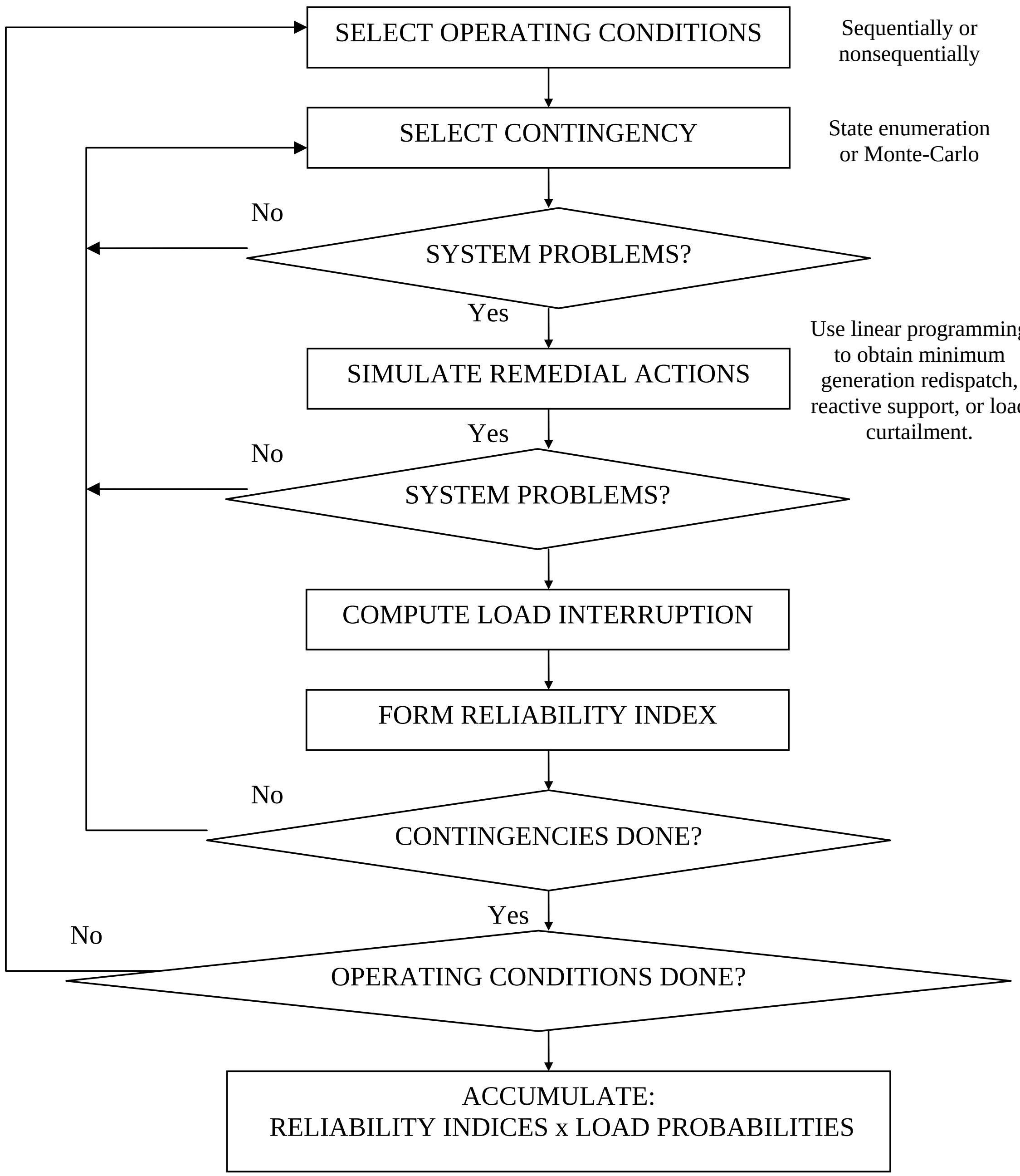


Fig. 2: Generic HL-II Evaluation Algorithm

Some commercially available tools include EPRI's Transmission Reliability Evaluation of Large Scale

System (TRELSS) and Composite Reliability Assessment by Monte-Carlo (CREAM); GE's Multi-Area Reliability Simulation (MARS) and Market Assessment and Portfolio Strategies (MAPS); PTI's Local Area Reliability Assessment (LARA) and Multi-Area Reliability AnaLysis (MAREL) and TPLAN, Powertech Lab's COMposite RELiability (COMREL) and Composite Reliability Using State Enumeration (CRUSE), and BC Hydro's Montecarlo Evaluation of COmposite system REliability (MECORE), ABB's NETREL, and General Reliability's TRANSREL.

Table 2 identifies some of these tools in terms of how they model the operating conditions, the contingency selection, and the network.

Table 2

| | Reliability modeling and evaluation tools | | | |
|---|---|---|---|---|
| Features | MARS | TRELSS | TPLAN | TRANSREL |
| Operating Conditions | Sequential | Non-sequential | Non-sequential | Non-sequential |
| Contingency selection | Monte-Carlo | Enumeration | Monte-Carlo | Enumeration |
| Network | Multi-area | Detailed | Detailed | ? |
| | | | | |

## 3.2 Production costing tools

We will describe in more detail the construction of production costing tools in Section 5. Here we simply mention some of the commercially available production costing tools via Table 4.

The Ventyx product Promod incorporates details in generating unit operating characteristics, transmission grid topology and constraints, unit commitment/ operating conditions, and market system operations. Promod can operate on nodal or zonal modes depending on the scope, timeframe, and simulation resolution of the problem. ProMod is not a forecasting model and does not consider the price and availability of other fuels.

The ADICA product GTMAX, developed by Argonne National Lab, can be employed to perform regional power grade or national power development analysis. GTMax will evaluate system operation, determine optimal location of power sources, and assess the benefits of new transmission lines. GTMax can simulate complex electric market and operating issues, for both regulated and deregulated market.

The PowerCost, Inc. product GenTrader employs economic unit dispatch logic to analyze economics, uncertainty, and risk associated with individual generation resources and portfolios. GenTrader does not represent the network.

PROSYM is a multi-area electric energy production simulation model developed by Henwood energy Inc. It is an hourly simulation engine for least-cost optimal production dispatch based on the resources' marginal costs, with full representation of generating unit characteristics, network area topology and electrical loads. PROSYM also considers and respects operational and chronological constraints; such as minimum up and down times, random forced outages and transmission capacity. It is designed to determine the station generation, emissions and economic transactions between interconnected areas for each hour in the simulation period.

## 3.3 Expansion planning tools

Some commercially available expansion planning tools include EGEAS, GEM, STRATEGIST, and PLEXOS.

The New Zealand Electricity Commission developed GEM as a long range generation capacity planning model. It is formulated as a mixed integer programming problem (MIP). The computer code is written using the GAMS optimization software and the model is solved with CPLEX.

The EPRI product EGEAS is a modular production cost and generation expansion software package which employs dynamic programming algorithm to form candidate portfolios from identified alternatives meeting a capacity planning constraint. It also has modules which accommodate demand-side management options and facilitate development of environmental compliance plans. Some of the key functions of EGEAS are asset retirement evaluation, emission evaluation from new plants, and Scenario analysis for various generation options. EGEAS is widely used by many utilities and regulators.

The Ventyx product STRATEGIST is composed of multiple application modules incorporating all aspects of utility planning and operations. This includes forecasted load modeling, marketing and conservation programs, production cost calculations including the dispatch of energy resources, benefit-cost (B/C) ratios calculation for different alternatives, capital project modeling, financial and rate impacts evaluation, and analysis of long-range rate strategy and the implications of utility plans on customer classes.

PLEXOS, from Plexos Solutions, is a versatile software system that finds optimal combinations of new generation units, unit retirements and transmission system upgrades on a least-cost basis over a long-term planning horizon. PLEXOS in itself does not incorporate the optimization engine but rather produces optimization code that can be read by an external solver such as CPLEX or MOSEK.

These tools are summarized in Table 3.

Table 3

| Features | Resource planning tools | | | |
|---|---|---|---|---|
| | PLEXOS | GEM | EGEAS | Strategist |
| Algorithm | Mixed integer linear programming | Mixed integer linear programming | Generalized benders decomposition and Dynamic Programming | Dynamic programming |
| Objective | Maximize portfolio profit or least cost | Least cost | Least cost | 10 different objective functions |
| Methods to represent system load | load duration curve | load duration curve | load duration curve | chronological load in twelve typical weeks per year |

## 3.4 National planning tools

These tools can be considered expansion planning tools but are somewhat distinct in that they generally account for (a) energy infrastructure beyond that of just the electric power system and (b) environmental impacts. Table 4 summarizes.

Table 4

| | | NEMS | MARKAL/TIMES | WASP-IV |
|---|---|---|---|---|
| Output | | Alternative energy assessment | Optimal investment plan | Optimal investment plan |
| Optimization model | Objective function | Single objective | Single objective | Single objective |
| | Stochastic events | √ | √ | √ |
| | Formulation | Modular | Generalized network | Generalized network, modular |
| Forecast horizon | | 20-25 years | Unconstrained | 30 years |
| Sustainability | Greenhouse gases | √ | √ | √ |
| | Other emissions | √ | √ | √ |
| Resiliency | | | | Loss of load |
| Energy represented | Primary energy sources | √ | √ | |
| | Electricity | √ | √ | √ |
| | Liquid fuels | √ | | |
| Transportation | Freight | √ ? | Only fuel demand | |
| | Passenger | ? | ? | |

# 4.0 Probability models

## 4.1 Load duration curves

A critical issue for planning is to identify the total load level for which to plan. One extremely useful tool for doing this is the so-called load duration curve, which is formed as follows. Consider that we have obtained, either through historical data or through forecasting, a plot of the load vs. time for a period T, as shown in Fig. 3 below.

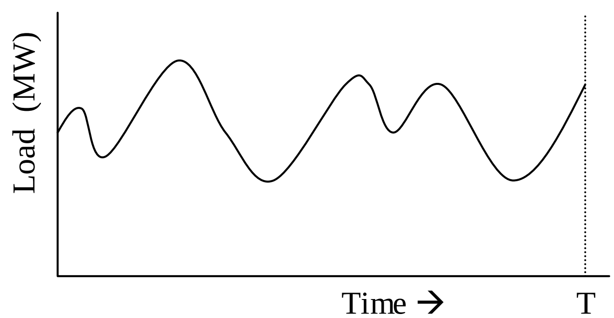


Fig. 3: Load curve (load vs. time)

Of course, the data characterizing Fig. 3 will be discrete, as illustrated in Fig. 4.

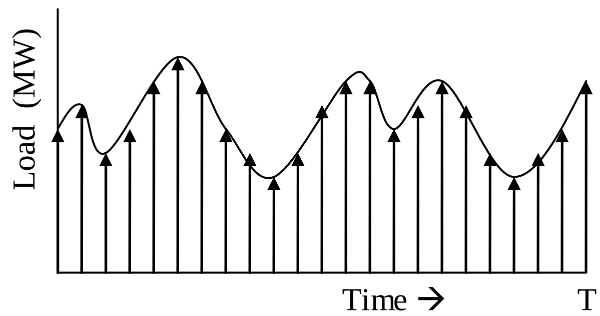


Fig. 4: Discretized Load Curve

We now divide the load range into intervals, as shown in Fig. 5.

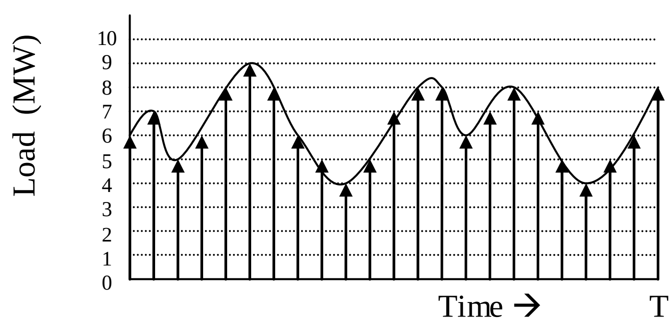


Fig. 5: Load range divided into intervals

This provides the ability to form a histogram by counting the number of time intervals contained in each load range. In this example, we assume that loads in Fig. 5 at the lower end of the range are “in” the range. The histogram for Fig. 5 is shown in Fig. 6.

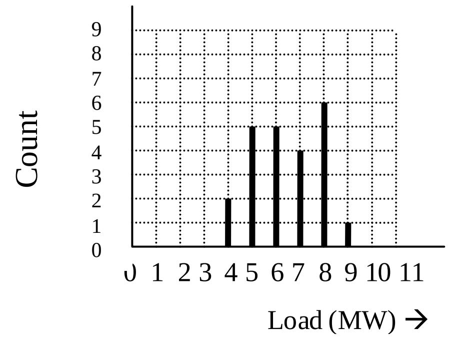


Fig. 6: Histogram

Figure 6 may be converted to a probability mass function, pmf, (which is the discrete version of the probability density function, pdf) by dividing each count by the total number of time intervals, which is 23. The resulting plot is shown in Fig. 7.

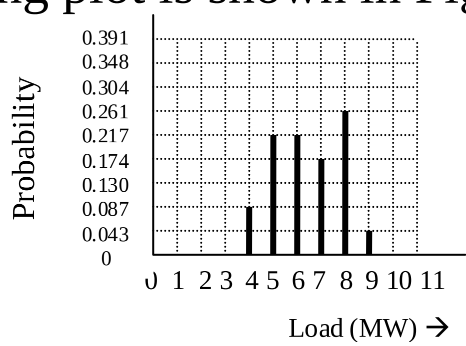


Fig. 7: Probability mass function

Like any pmf, the summation of all probability values should be 1, which we see by the following sum:

$$0.087+0.217+0.217+0.174+0.261+0.043=0.999$$

(It is not exactly 1.0 because there is some rounding error). The probability mass function provides us with the ability to compute the probability of the load being within a range according to:

$$\Pr(Load\ within\ Range) = \sum_{L\ in\ Range} \Pr(Load = L) \tag{2}$$

We may use the probability mass function to obtain the cumulative distribution function (CDF) as:

$$\Pr(Load \geq Value) = \sum_{L \geq Value} \Pr(Load = L) \tag{3}$$

From Fig. 7, we obtain:

$$\Pr(Load \geq 1) = \sum_{L \geq 1} \Pr(Load = L) = 1.0$$

$$\Pr(Load \geq 2) = \sum_{L \geq 2} \Pr(Load = L) = 1.0$$

$$\Pr(Load \geq 3) = \sum_{L \geq 3} \Pr(Load = L) = 1.0$$

$$\Pr(Load \geq 4) = \sum_{L \geq 2} \Pr(Load = L) = 1.0$$

$$\Pr(Load \geq 5) = \sum_{L \geq 5} \Pr(Load = L)$$
$$= 0.217 + 0.217 + 0.174 + 0.261 + 0.043 = 0.912$$

$$\Pr(Load \geq 6) = \sum_{L \geq 6} \Pr(Load = L)$$
$$= 0.217 + 0.174 + 0.261 + 0.043 = 0.695$$

$$\Pr(Load \geq 7) = \sum_{L \geq 6} \Pr(Load = L) = 0.174 + 0.261 + 0.043 = 0.478$$

$$\Pr(Load \geq 8) = \sum_{L \geq 8} \Pr(Load = L) = 0.261 + 0.043 = 0.304$$

$$\Pr(Load \geq 9) = \sum_{L \geq 9} \Pr(Load = L) = 0.043$$

$$\Pr(Load \geq 10) = \sum_{L \geq 10} \Pr(Load = L) = 0$$

Plotting these values vs. the load results in the CDF of Fig. 8.

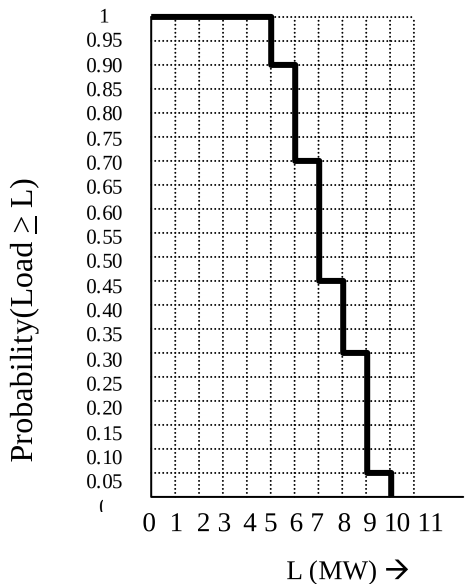


Fig. 8: Cumulative distribution function

The plot of Fig. 8 is often shown with the load on the vertical axis, as given in Fig. 9.

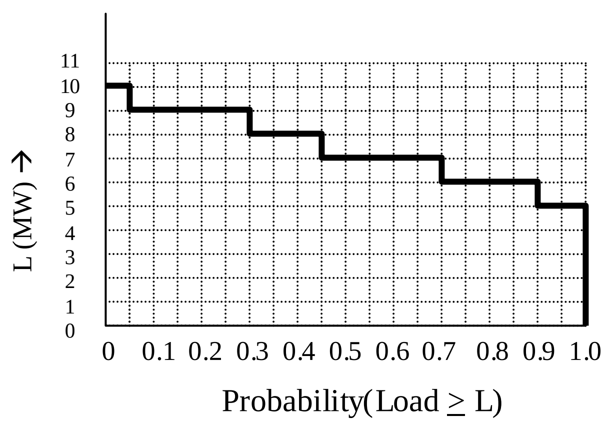


Fig. 9: CDF with axes switched

If the horizontal axis of Fig. 9 is scaled by the time duration of the interval over which the original load data was taken, T, we obtain the *load duration curve.* This curve provides the number of time intervals that the load equals, or exceeds, a given load level. For example, if the original load data had been taken over a year, then the load duration curve would show the number of hours out of that year for which the load could be expected to equal or exceed a given load level, as shown in Fig. 10.

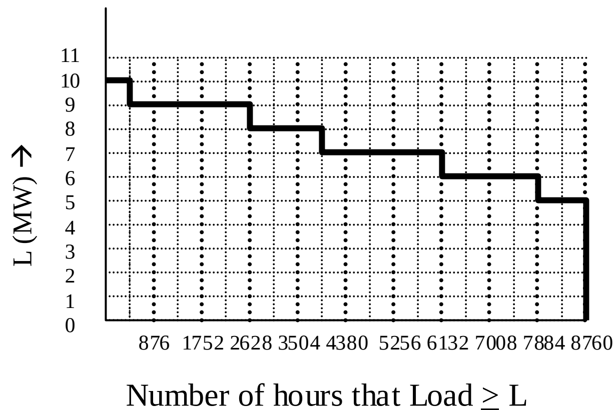


Fig. 10: Load duration curve

Load duration curves are useful in a number of ways.

- They provide guidance for judging different alternative plans. One plan may be satisfactory for loading levels of 90% of peak and less. One sees from Fig. 10 that such a plan would be unsatisfactory for 438 hours per year (or 5% of the time).
- They identify the base load. This is the value that the load *always* exceeds. In Fig. 10, this value is 5 MW.

- They provide convenient calculation of energy, since energy is just the area under the load duration curve. For example, Fig. 11 shows the area corresponding to the base load energy consumption, which is 5MWx8760hr=43800 MW-hrs.

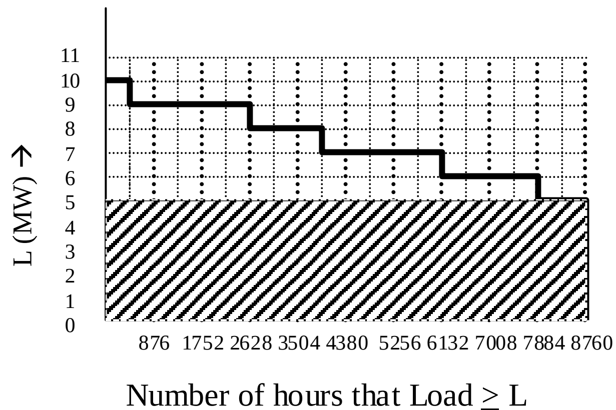


Fig. 11: Area corresponding to base load energy consumption

- They allow illustration of generation commitment policies and corresponding yearly unit energy production, as shown in Fig 12, where we see that the nuclear plant and fossil plant #1 are base loaded plants, supplying 26280 MWhrs and 17520 MWhrs, respectively. Fossil plant #2 and gas plant #1 are the mid-range plants, and gas plant #2 is a peaker.

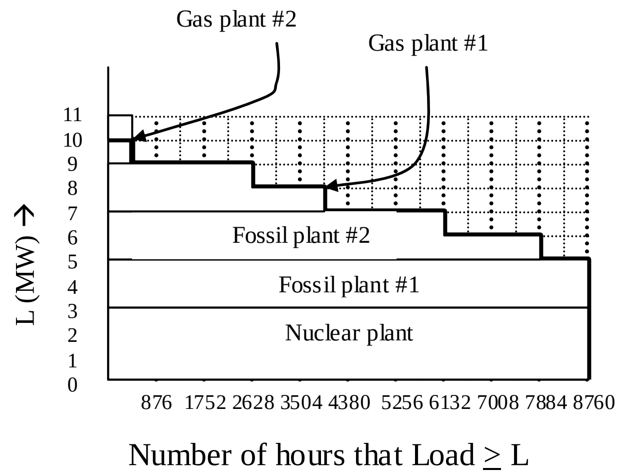


Fig. 12: Illustration of Unit commitment policy

Load duration curves are also used in reliability and production costing programs in computing different reliability indices, as we will see in Sections 5 and 6.

## 4.2 Generation probability models

We consider that generators obey a two-state model, i.e., they are either up or down, and we assume that the process by which each generator moves between states is Markov, i.e., the probability distribution of future states depends only on the current state and not on past states, i.e., the process is memoryless.

In this case, it is possible to show that unavailability (or *forced outage rate,* FOR) is the "steady-state" (or long-run) probability of a component not being available and is given by

$$U = p = \frac{\lambda}{\lambda + \mu} \tag{4}$$

and the availability is the long-run probability of a component being available and is given by

$$A = q = \frac{\mu}{\lambda + \mu} \tag{5}$$

where $\lambda$ is the "failure rate" and $\mu$ is the "repair rate." Substituting $\lambda=1/MTTF$ and $\mu=1/MTTR$, where *MTTF* is the mean time to failure, and *MTTR* is the mean time to repair, we get that

$$U = p = \frac{MTTR}{MTTF + MTTR} \tag{6}$$

$$A = q = \frac{MTTF}{MTTF + MTTR} \tag{7}$$

The probability mass function representing the outaged load corresponding to unit $j$ is then given as $f_{Dj}(d_j)$, expressed as

$$f_{Dj}(d_j) = q_j\delta(d_j) + p_j\delta(d_j - C_j) \tag{8}$$

and illustrated by Fig. 13.

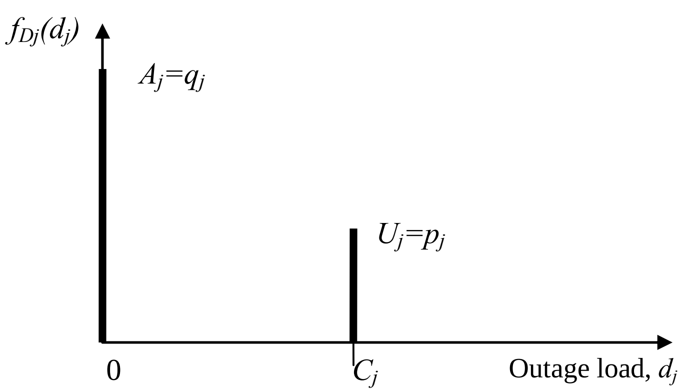


Fig. 13: Two state generator outage model

# 5.0 Reliability calculations

## 5.2 Preliminary Definitions

Let's characterize the load shape curve with $t=g(d)$, as illustrated in Fig. 14. It is important to note that the load shape curve characterizes the (forecasted) future time period and is therefore a probabilistic characterization of the demand.

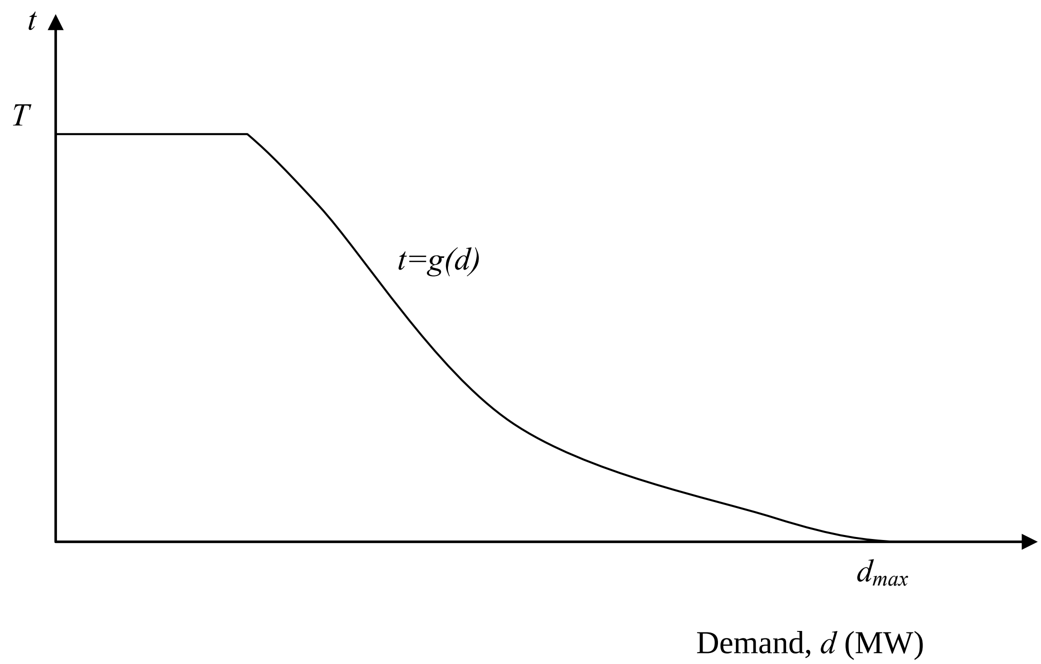


Fig. 14: Load shape $t=g(d)$

Here:

- $d$ is the system load
- $t$ is the number of time units in the interval $T$ for which the load is greater than $d$ and is most typically given in hours or days
- $t=g(d)$ expresses functional dependence of $t$ on $d$

- $T$ represents, most typically, a year

The cumulative distribution function (cdf) is given by

$$F_D(d) = P(D > d) = \frac{t}{T} = \frac{g(d)}{T} \qquad (9)$$

One may also compute the *total energy* $E_T$ consumed in the period $T$ as the area under the curve, i.e.,

$$E_T = \int_0^{d_{\max}} g(\lambda) d\lambda \qquad (10)$$

The *average demand* in the period $T$ is obtained from

$$d_{avg} = \frac{1}{T} E_T = \frac{1}{T} \int_0^{d_{\max}} g(\lambda) d\lambda = \int_0^{d_{\max}} F_D(\lambda) d\lambda \qquad (11)$$

Now let's assume that the planned system generation capacity, i.e, the installed capacity, is $C_T$, and that $C_T < d_{max}$. This is an undesirable situation, since we will not be able to serve some demands, even when there is no capacity outage! Nonetheless, it serves well to understand the relation of the load duration curve to several useful indices. The situation is illustrated in Fig. 15.

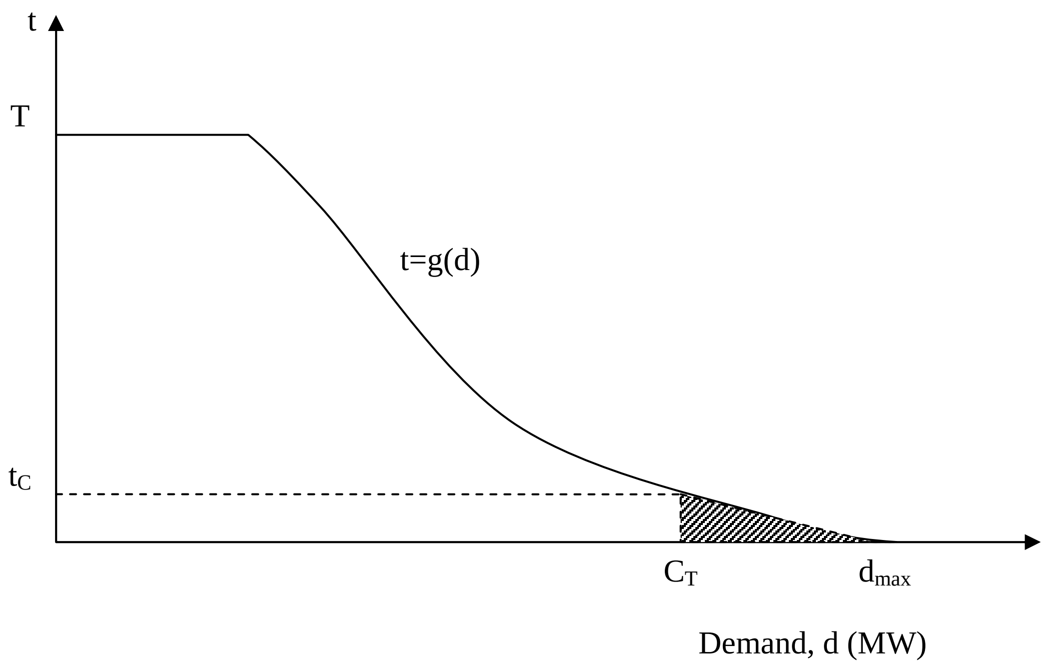


Fig. 15: Illustration of Unserved Demand

Then, under the assumption that the given capacity $C_T$ is perfectly reliable, we may express three useful reliability indices:

- <u>Loss of load expectation, *LOLE*</u>: the expected number of time units that load will exceed capacity

$$LOLE = t_{C_T} = g(C_T) \qquad (12)$$

- <u>Loss of load probability, *LOLP*</u>: the probability that the demand will exceed the capacity during $T$:

$$LOLP = P(D > C_T) = F_D(C_T) \qquad (13)$$

One may think that, if $d_{max}>C_T$, then $LOLP=1$. However, if $F_D(d)$ is a true probability distribution, then it describes the event $D>C_T$ with uncertainty associated with what the load is going to be, i.e., only with a probability. One can take an alternative view, that the load duration curve is certain, which would be the case if we were considering a previous year. In this case, LOLP should be thought of not as a probability but rather as the percentage of time during the interval $T$ for which the load exceeds capacity.

It is of interest to reconsider (9), repeated here for convenience:

$$F_D(d) = P(D > d) = \frac{t}{T} = \frac{g(d)}{T} \qquad (9)$$

Substituting $d=C_T$, we get:

$$F_D(C_T) = P(D > C_T) = \frac{t}{T} = \frac{g(C_T)}{T} \qquad (*)$$

By (12), $t=t_{Cr}=LOLE$; by (13), $P(D>C_T)=LOLP$, and so (*) becomes:

$$LOLP = \frac{LOLE}{T} \Rightarrow LOLE = LOLP \times T$$

which expresses that $LOLE$ is the expectation of the number of time units within $T$ that demand will exceed capacity.

- <u>Expected demand not served, *EDNS*</u>: If the average (or expected) demand is given by (11), then it follows that expected demand not served is:

$$EDNS = \int_{C_T}^{d_{\max}} F_D(\lambda) d\lambda \qquad (14)$$

which would be the same area as in Fig. 15 when the ordinate is normalized to provide $F_D(d)$ instead of $t$. Reference [2] provides a rigorous derivation for (14).

- Expected energy not served, EENS: This is the total amount of time multiplied by the expected demand not served, i.e.,

$$EENS = T \int_{C_T}^{d_{\max}} F_D(\lambda) d\lambda = \int_{C_T}^{d_{\max}} g(\lambda) d\lambda \qquad (15)$$

which is the area shown in Fig. 15.

## 5.2 Effective load

The notion of effective load is used to account for the unreliability of the generation, and it is essential for understanding the view taken in [2].

The basic idea is that the total system capacity is always $C_T$, and the effect of capacity outages are accounted for by changing the load model in an appropriate fashion, and then the different indices are computed as given in (13), (14), and (15).

A capacity outage of $C_i$ is therefore modeled as an increase in the demand, not as a decrease in capacity!

We have already defined $D$ as the random variable characterizing the demand. Now we define two more random variables:

- $D_j$ is the random increase in load for outage of unit $i$.
- $D_e$ is the random load accounting for outage of all units and represents the *effective* load.

Thus, the random variables $D$, $D_e$, and $D_j$ are related:

$$D_e = D + \sum_{j=1}^{N} D_j \qquad (16)$$

It is important to realize that, whereas $C_j$ represents the capacity of unit $j$ and is a deterministic value, $D_j$ represents the increase in load corresponding to outage of unit $j$ and is a random variable. The probability mass function (pmf) for $D_j$ is assumed to be as given in Fig. 16, i.e., a two-state model. We denote the pmf for $D_j$ as $f_{Dj}(d_j)$.

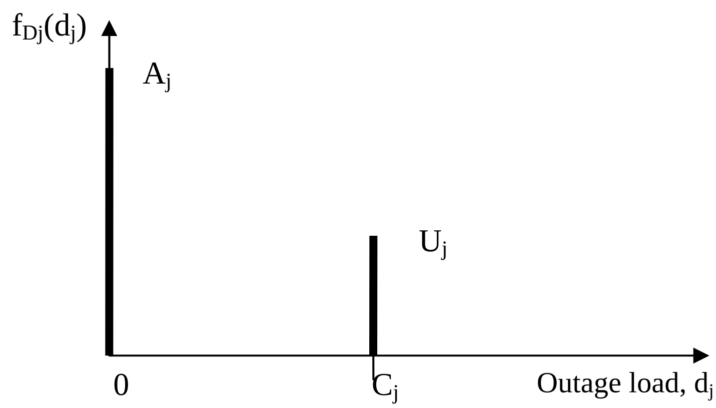


Fig. 16: Two state generator outage model

Recall from probability theory that the pdf of the sum of two independent random variables is the convolution of their individual pdfs, that is, for random variables *X* and *Y*, with *Z*=*X*+*Y*, then

$$f_Z(z) = \int_{\lambda=-\infty}^{\infty} f_X(z-\lambda) f_Y(\lambda) d\lambda \tag{17}$$

In addition, it is true that the cdf of two random variables can be found by convolving the cdf of one of them with the pdf (or pmf) of the other, that is, for random variables *X* and *Y*, with *Z*=*X*+*Y*, then

$$F_Z(z) = \int_{\lambda=-\infty}^{\infty} F_X(z-\lambda) f_Y(\lambda) d\lambda \tag{18}$$

Let's consider the case for only one unit, i.e., from (16),

$$D_e = D + D_j \tag{19}$$

Then, by (18), we have that:

$$F_{D_e}^{(1)}(d_e) = \int_{\lambda=-\infty}^{\infty} F_{D_e}^{(0)}(d_e-\lambda) f_{D_j}(\lambda) d\lambda \tag{20}$$

where the notation $F_D^{(j)}(\cdot)$ indicates the cdf after the $j^{th}$ unit is convolved in. Under this notation, then, (19) becomes

$$D_e^{(j)} = D_e^{(j-1)} + D_j \tag{21}$$

and the general case for (20) is:

$$F_{D_e}^{(j)}(d_e) = \int_{\lambda=-\infty}^{\infty} F_{D_e}^{(j-1)}(d_e - \lambda) f_{D_j}(\lambda) d\lambda \qquad (22)$$

which expresses the equivalent load after the $j^{th}$ unit is convolved in.

Since $f_{Dj}(d_j)$ is discrete (i.e., a pmf), we may rewrite (22) as

$$F_{D_e}^{(j)}(d_e) = \sum_{d_j=-\infty}^{\infty} F_{D_e}^{(j-1)}(d_e - d_j) f_{D_j}(d_j) \qquad (23)$$

From an intuitive perspective, (23) is providing the convolution of the load shape $F_D^{(j-1)}(\cdot)$ with the set of impulse functions comprising $f_{Dj}(d_j)$. When using a 2-state model for each generator, $f_{Dj}(d_j)$ is comprised of only 2 impulse functions, one at 0 and one at $C_j$. Recalling that the convolution of a function with an impulse function simply shifts and scales that function, (23) can be expressed for the 2-state generator model as:

$$F_{D_e}^{(j)}(d_e) = A_j F_{D_e}^{(j-1)}(d_e) + U_j F_{D_e}^{(j-1)}(d_e - C_j) \qquad (24)$$

So the cdf for the effective load following convolution with capacity outage pmf of the $j^{th}$ unit, is the sum of

- the original cdf, scaled by $A_j$ and
- the original cdf, scaled by $U_j$ and right-shifted by $C_j$.

Example 1: Fig. 17 illustrates the convolution process for a single unit $C_1$=4 MW supplying a system having peak demand $d_{max}$=4 MW, with demand cdf given as in plot (a) based on a total time interval of $T$=1 year.

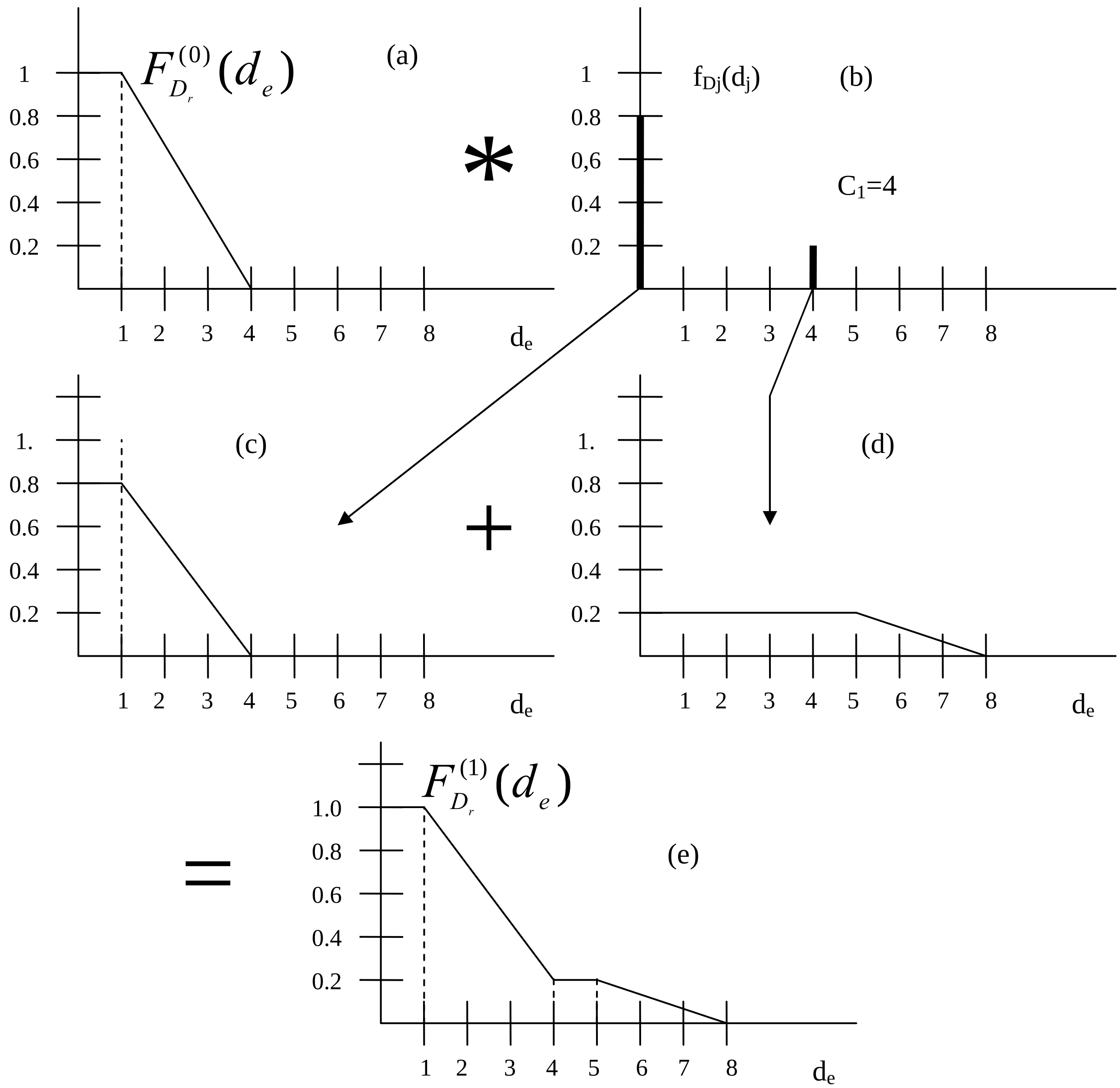


Fig. 17: Convolving in the first unit

Plots (c) and (d) represent the intermediate steps of the convolution where the original cdf $F_{D_e}^{(0)}(d_e)$ was scaled by $A_1$=0.8 and $U_1$=0.2, respectively, and right-shifted by 0 and $C_1$=4, respectively. Note the effect of convolution is to spread the original cdf.

Plot (d) may raise some question since it appears that the constant part of the original cdf has been extended too far to the left. The reason for this apparent discrepancy is that all of the original cdf, in plot (a), was not shown. The complete cdf is illustrated in Fig. 18 below, which shows clearly that $F_{D_e}^{(0)}(d_e)=1$ for $d_e<0$, reflecting the fact that $P(D_e>d_e)=1$ for $d_e<0$.

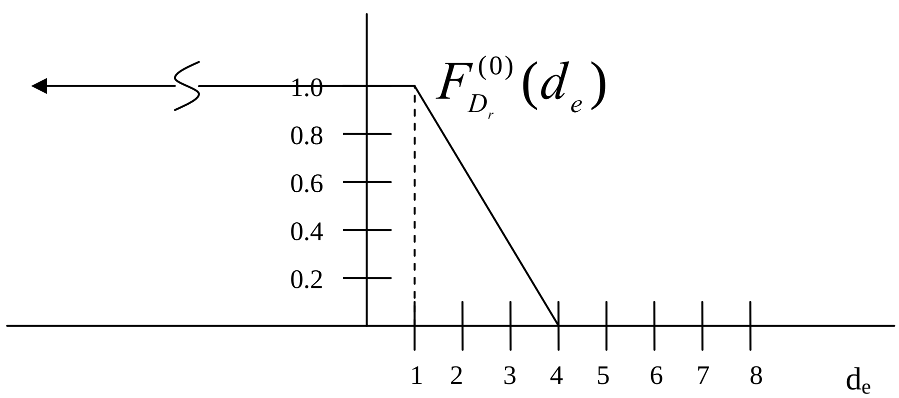


Fig. 18: Complete cdf including values for $d_e<0$

Let's consider that the "first" unit we just convolved in is actually the only unit. If that unit were perfectly reliable, then, because $C_1=4$ and $d_{max}=4$, our system would never have loss of load. This would be the situation if we applied the ideas of Fig. 15 to Fig. 17, plot (a).

However, Fig. 17, plot (e) tells a different story. Fig. 19 applies the ideas of Fig. 15 to Fig. 17, plot (e) to show how the cdf on the *equivalent load* indicates that, for a total capacity of $C_T=4$, we do in fact have some chance of losing load.

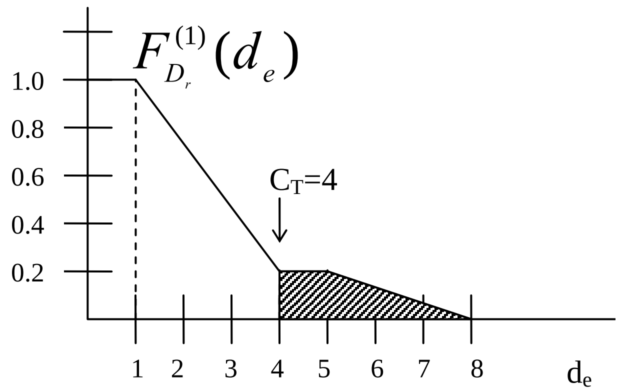


Fig. 19: Illustration of loss of load region

The desired indices are obtained from (12),(13), (14):

$$LOLE = t_{C_T} = g_e(C_T) = T \times F_{D_r}(C_T = 4) = 1 \times 0.2 = 0.2\,years$$

A LOLE of 0.2 years is 73 days, a very poor reliability level that reflects the fact we have only a single unit with a high FOR=0.2.

The LOLP is given by:

$$LOLP = P(D_e > C_T) = F_{De}(C_T) = 0.2$$

and the EDNS is given by:

$$EDNS = \int_{C_T}^{d_{e,\max}} F_{De}(\lambda)\,d\lambda$$

which is just the shaded area in Fig. U19.23, most easily computed using the basic geometry of the figure, according to:

$$0.2(1)+\frac{1}{2}(3)(0.2)=0.5MW$$

The EENS is given by

$$EENS = T\int_{C_T}^{d_{e,\max}} F_{De}(\lambda)d\lambda = \int_{C_T}^{d_{e,\max}} g_e(\lambda)d\lambda$$

or $T\times EDNS$=1(0.5)=0.5MW-years,

or 8760(0.5)=4380MWhrs.

Example 2: This example is from [3].
A set of generation data is provided in Table 5.

Table 5

| Unit No. | Unit name | Rated capacity (MW(e)) | Forced outage (%) | Type of fuel | Variable cost ($/MW·h) |
|---|---|---|---|---|---|
| 1 | NUC1 | 200 | 20 | Nuclear | 6.5 |
| 2 | NUC2 | 200 | 20 | Nuclear | 6.5 |
| 3 | COAL1 | 200 | 10 | Coal | 27.0 |
| 4 | COAL2 | 200 | 10 | Coal | 27.0 |
| 5 | OIL1 | 100 | 10 | Oil | 58.1 |
| 6 | OIL2 | 100 | 10 | Oil | 58.1 |
| 7 | OIL3 | 100 | 10 | Oil | 58.1 |
| 8 | OIL4 | 100 | 10 | Oil | 58.1 |
| 9 | CT1 | 100 | 5 | Distillate oil | 113.2 |
| System capacity | | 1300 | | | |

The 4$^{th}$ column provides the force outage rate, which we have denoted by *U*. The two-state generator outage model for each unit is obtained from this value, together with the rated capacity, as illustrated in Fig. 20, for unit 1. Notice that the units are ordered from least cost to highest cost.

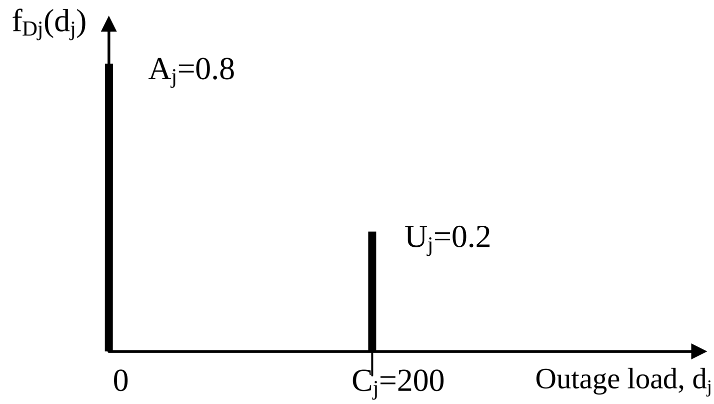


Fig. 20: Two-state outage model for Unit 1

Load duration data is provided in Table 6 and plotted in Fig. 21.

Table 6

| Load MW(e) | Fraction of time load exceeds given load |
|---|---|
| 0 | 1.00 |
| 100 | 1.00 |
| 200 | 1.00 |
| 300 | 1.00 |
| 400 (minimum load) | 1.00 |
| 500 | 0.80 |
| 600 | 0.40 |
| 700 | 0.20 |
| 800 | 0.10 |
| 900 | 0.05 |
| 1000 (peak load) | 0.00 |

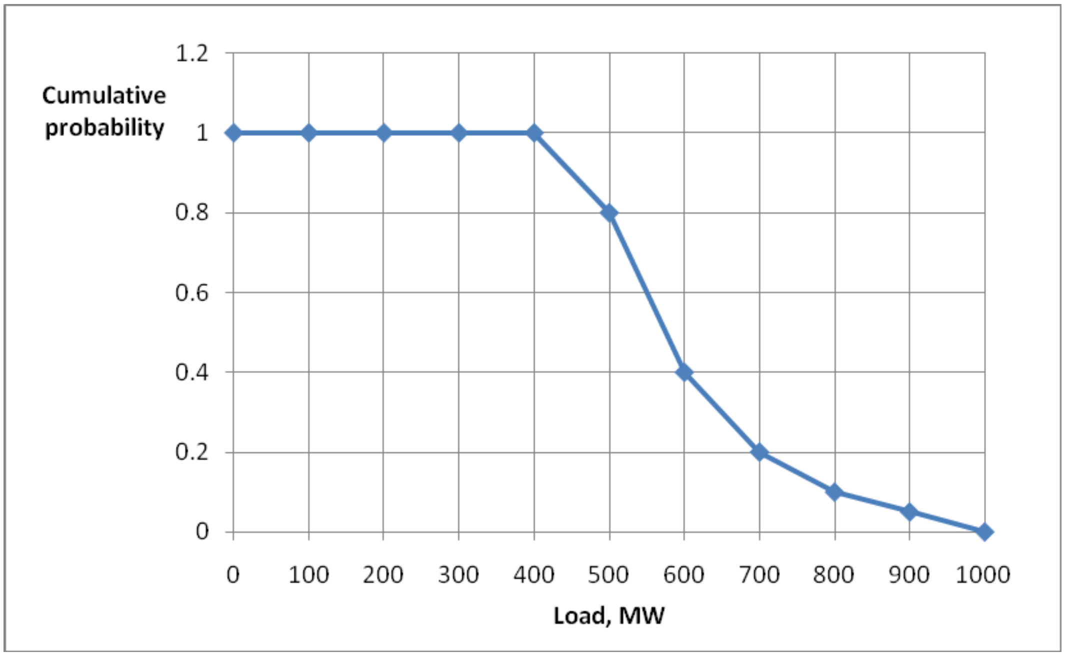

Fig. 21

We now deploy (24), repeated here for convenience,

$$F_{D_e}^{(j)}(d_e) = A_j F_{D_e}^{(j-1)}(d_e) + U_j F_{D_e}^{(j-1)}(d_e - C_j) \qquad (24)$$

to convolve in the unit outage models with the load duration curve of Fig. 21. The procedure is carried out in an Excel spread sheet, and the result is provided in Fig. 22. In Fig. 22, we have shown

- Original load duration curve, F0;
- Load duration curve with unit 1 convolved in, F1.
- Load duration curve with all units convolved in, F9

We could, of course, show the load duration curves for any number of units convolved in, but this would be a cluttered plot.

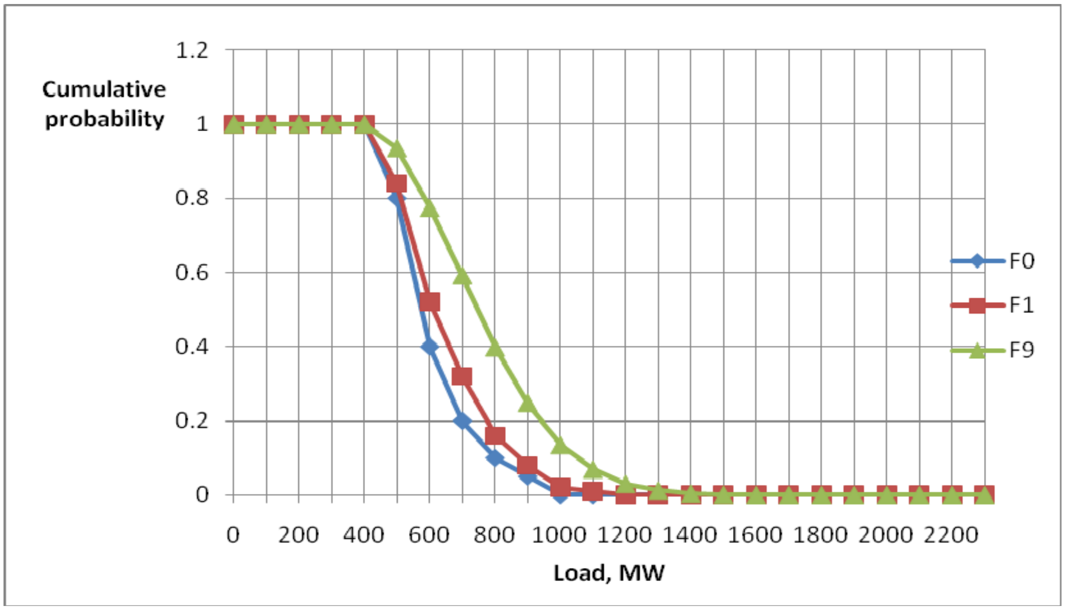


Fig. 22

We also show, in Table 7, the results of the calculations performed to obtain the series of load duration curves (LDC) F0-F9. Notice the following:

- Each LDC is a column FO-F9
- The first column, in MW, is the load.
    - It begins at -200 to facilitate the convolution for the largest unit, which is a 200 MW unit.
    - Although it extends to 2300 MW, the largest actual load is 1000 MW; the extension is to obtain the equivalent load corresponding to a 1000 MW load with 1300 MW of failable generation.
- The entries in the table show the % time the load exceeds the given value.
- LOLP is, for a particular column, the % time load exceeds the total capacity corresponding to that column, and is underlined.

For example, one observes that *LOLP=1* if we only have units 1 (F1, $C_T=200$) or only units 1 and 2 (F2, $C_T=400$). This is because the capacity would never be enough to satisfy the load, at any time. And *LOLP=0.6544* if we have only units 1, 2, and 3 (F3, $C_T=600$). This is because we would be able to supply the load for some of the time with this capacity. And *LOLP=0.012299* if we have all units (F9, $C_T=1300$), which is non-0 (in spite of the fact that $C_T>1000$) because units can fail.

Table 7

| | F0 | F1 | F2 | F3 | F4 | F5 | F6 | F7 | F8 | F9 |
|---|---|---|---|---|---|---|---|---|---|---|
| | | 0.2 | 0.2 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.05 |
| | | 0.8 | 0.8 | 0.9 | 0.9 | 0.9 | 0.9 | 0.9 | 0.9 | 0.95 |
| | | 200 | 200 | 200 | 200 | 100 | 100 | 100 | 100 | 100 |
| Load (MW) | Fraction of time load exceeds given load | | | | | | | | | |
| -200 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| -100 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 100 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 200 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 300 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 400 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 500 | 0.8 | 0.84 | 0.872 | 0.8848 | 0.89632 | 0.906688 | 0.916019 | 0.924417 | 0.931976 | 0.935377 |
| 600 | 0.4 | 0.52 | 0.616 | 0.6544 | 0.68896 | 0.709696 | 0.729395 | 0.748058 | 0.765694 | 0.774008 |
| 700 | 0.2 | 0.32 | 0.424 | 0.4688 | 0.5104 | 0.528256 | 0.5464 | 0.5647 | 0.583035 | 0.592168 |
| 800 | 0.1 | 0.16 | 0.232 | 0.2704 | 0.3088 | 0.32896 | 0.34889 | 0.368641 | 0.388247 | 0.397986 |
| 900 | 0.05 | 0.08 | 0.128 | 0.1576 | 0.18872 | 0.200728 | 0.213551 | 0.227085 | 0.241241 | 0.248591 |
| 1000 | 0 | 0.02 | 0.048 | 0.0664 | 0.0868 | 0.096992 | 0.107366 | 0.117984 | 0.128894 | 0.134512 |
| 1100 | 0 | 0.01 | 0.024 | 0.0344 | 0.04672 | 0.050728 | 0.055354 | 0.060556 | 0.066298 | 0.069428 |
| 1200 | 0 | 0 | 0.004 | 0.0084 | 0.0142 | 0.017452 | 0.02078 | 0.024237 | 0.027869 | 0.02979 |
| 1300 | 0 | 0 | 0.002 | 0.0042 | 0.00722 | 0.007918 | 0.008871 | 0.010062 | 0.01148 | 0.012299 |
| 1400 | 0 | 0 | 0 | 0.0004 | 0.0012 | 0.001802 | 0.002414 | 0.003059 | 0.00376 | 0.004146 |
| 1500 | 0 | 0 | 0 | 0.0002 | 0.0006 | 0.00066 | 0.000774 | 0.000938 | 0.00115 | 0.001281 |
| 1600 | 0 | 0 | 0 | 0 | 0.00004 | 0.000096 | 0.000152 | 0.000215 | 0.000287 | 0.00033 |
| 1700 | 0 | 0 | 0 | 0 | 0.00002 | 0.000022 | 2.94E-05 | 4.17E-05 | 5.9E-05 | 7.04E-05 |
| 1800 | 0 | 0 | 0 | 0 | 0 | 0.000002 | 0.000004 | 6.54E-06 | 1.01E-05 | 1.25E-05 |
| 1900 | 0 | 0 | 0 | 0 | 0 | 0 | 2E-07 | 5.8E-07 | 1.18E-06 | 1.62E-06 |
| 2000 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2E-08 | 7.6E-08 | 1.31E-07 |
| 2100 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2E-09 | 5.7E-09 |
| 2200 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1E-10 |
| 2300 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

## 6.0 Production cost models

The most basic production cost model obtains production costs of thermal units over a period of time, say 1 year, by building upon the equivalent load duration curve described in Section 5.

To perform this, we will assume that generator variable cost, in \$/MWhr, for unit $j$ operating at $P_j$ over a time interval $t$, is expressed by

$$C_j(E_j)=b_jE_j$$

where $E_j=P_jt$ is the energy produced by the unit during the hour and $b_j$ is the unit's average variable costs of producing the energy.

The production cost model begins by assuming the existence of a loading (or merit) order, which is how the units are expected to be called upon to meet the demand facing the system. We assume for simplicity that each unit consists of a single "block" of capacity equal to the maximum capacity. It is possible, and more accurate, to divide each unit into multiple capacity blocks, but there is no conceptual difference to the approach when doing so.

Table 5, listed previously in Example 2, provides the variable cost for each unit in the appropriate loading order. This table is repeated here for convenience.

Table 5

| Unit No. | Unit name | Rated capacity (MW(e)) | Forced outage (%) | Type of fuel | Variable cost ($/MW·h) |
|---|---|---|---|---|---|
| 1 | NUC1 | 200 | 20 | Nuclear | 6.5 |
| 2 | NUC2 | 200 | 20 | Nuclear | 6.5 |
| 3 | COAL1 | 200 | 10 | Coal | 27.0 |
| 4 | COAL2 | 200 | 10 | Coal | 27.0 |
| 5 | OIL1 | 100 | 10 | Oil | 58.1 |
| 6 | OIL2 | 100 | 10 | Oil | 58.1 |
| 7 | OIL3 | 100 | 10 | Oil | 58.1 |
| 8 | OIL4 | 100 | 10 | Oil | 58.1 |
| 9 | CT1 | 100 | 5 | Distillate oil | 113.2 |
| System capacity | | 1300 | | | |

The criterion for determining loading order is clearly economic. Sometimes it is necessary to alter the economic loading order to account for must-run units or spinning reserve requirements. We will not consider these issues in the discussion that follows.

To motivate the approach, we introduce the concept of a unit's *observed load* as the load "seen" by a unit just before it is committed in the loading order. Thus, it will be the case that all higher-priority units will have been committed.

If all higher-priority units would have been perfectly reliable ($A_j=1$), then the observed load seen by the next unit would have been just the total load less the sum of the capacities of the committed units.

However, all higher-priority units are not perfectly reliable, i.e., they may fail according to the forced outage rate $U_j$. This means we must account for their stochastic behavior over time. This can be done in a straight-forward fashion by using the equivalent load duration curve developed for the last unit committed.

In the notation of (24) unit $j$ sees a load characterized by $F_{D_e}^{(j-1)}(d_e)$. Thus, the energy provided by unit $j$ is proportional to the area under $F_{D_e}^{(j-1)}(d_e)$ from $x_{j-1}$ to $x_j$, where

- $x_{j-1}$ is the summed capacity over all previously committed units and
- $x_j$ is the summed capacity over all previously committed units and unit $j$.

But unit $j$ is only going to be available $A_j$% of the time. In addition, since $F_{D_e}^{(j-1)}(d_e)$ is a probability function, we must multiply it by $T$, resulting in the following expression for energy provided by unit $j$ [4]:

$$E_j = TA_j \int_{x_{j-1}}^{x_j} F_{D_e}^{(j-1)}(\lambda) d\lambda \qquad (25)$$

where

$$x_j = \sum_{i=1}^{j} C_i \, , \quad x_{j-1} = \sum_{i=1}^{j-1} C_i \qquad (26)$$

Referring back to Example 2, we describe the computations for the first three entries. This description is adapted from [3].

For unit 1, the original load duration curve F0 is used, as forced outages of any units in the system do not affect unit l's observed load. The energy requested by the system from unit 1, excluding unit l's forced outage time, is the area under $F_{D_e}^{(0)}(d_e)$ over the range of 0 to 200 MW (unit 1 's position in the loading order) times the number of hours in the period (8760) times $A_1$. The area under $F_{D_e}^{(0)}(d_e)$ from 0 to 200, illustrated in Fig. 23 below, is 200.

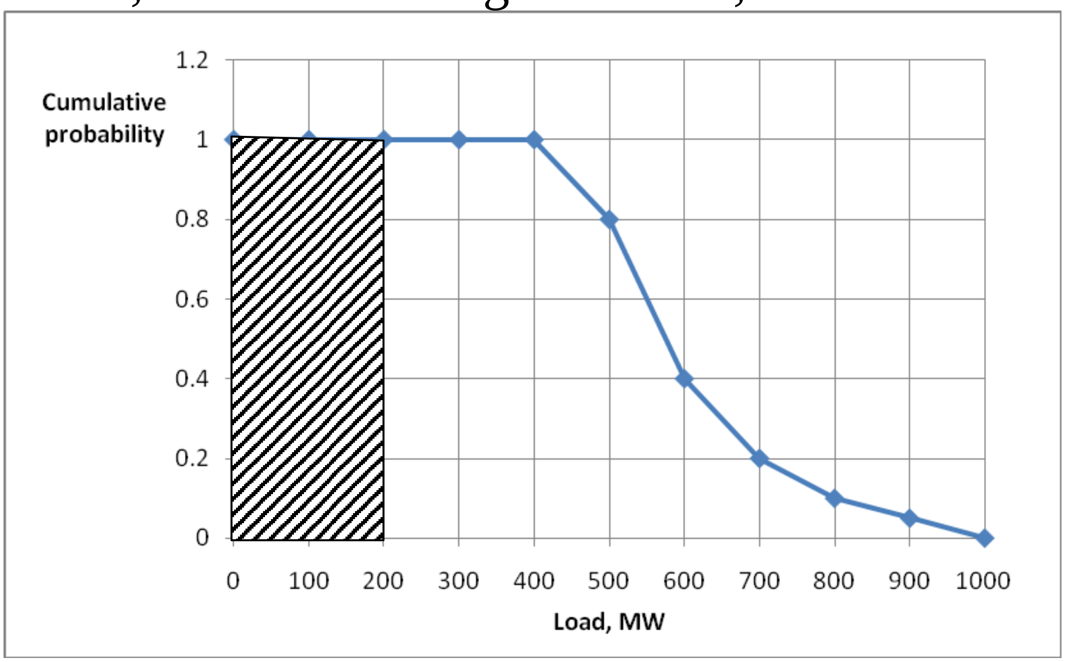


Fig. 23

Therefore,

$$E_1 = 8760 \times 0.8 \times 200 = 1{,}401{,}600\,\text{MWhrs}$$

For unit 2, the load duration curve F1 is used, as forced outage of unit 1 will affect unit 2's observed load. The energy requested by the system from unit 2, excluding unit 2's forced outage time, is the area under $F_{D_e}^{(1)}(d_e)$ over the range of 200 to 400 MW (unit 2 's position in the loading order) times the number of hours in the period (8760) times $A_2$. The area under $F_{D_e}^{(1)}(d_e)$ from 200 to 400, illustrated in Fig. 24 below, is 200.

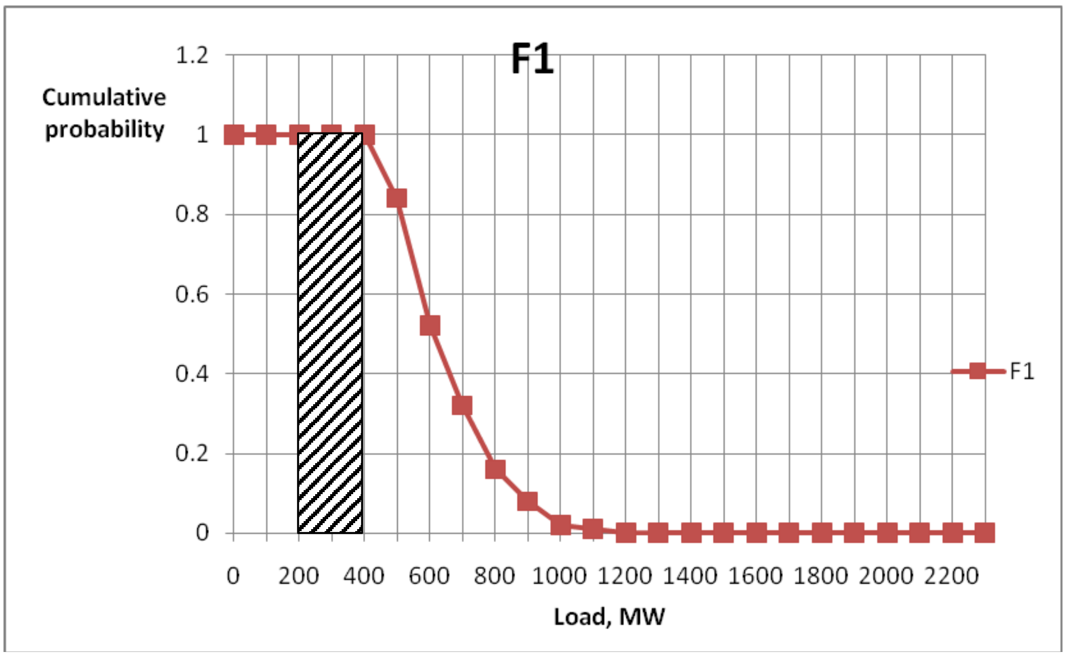


Fig. 24

Therefore,

$$E_2 = 8760 \times 0.8 \times 200 = 1{,}401{,}600 \, \text{MWhrs}$$

For unit 3, the load duration curve F2 is used, as forced outage of units 1 and 2 will affect unit 3's observed load. The energy requested by the system from unit 3, excluding unit 3's forced outage time, is the area under $F_{D_e}^{(2)}(d_e)$ over the range of 400 to 500 MW (unit 3 's position in the loading order) times the number of hours in the period (8760) times $A_3$. The area under $F_{D_e}^{(2)}(d_e)$ from 400 to 600, illustrated in Fig. 25, is calculated below Fig 25. The coordinates on Fig. 25 are obtained from Table 7, repeated on the next page for convenience.

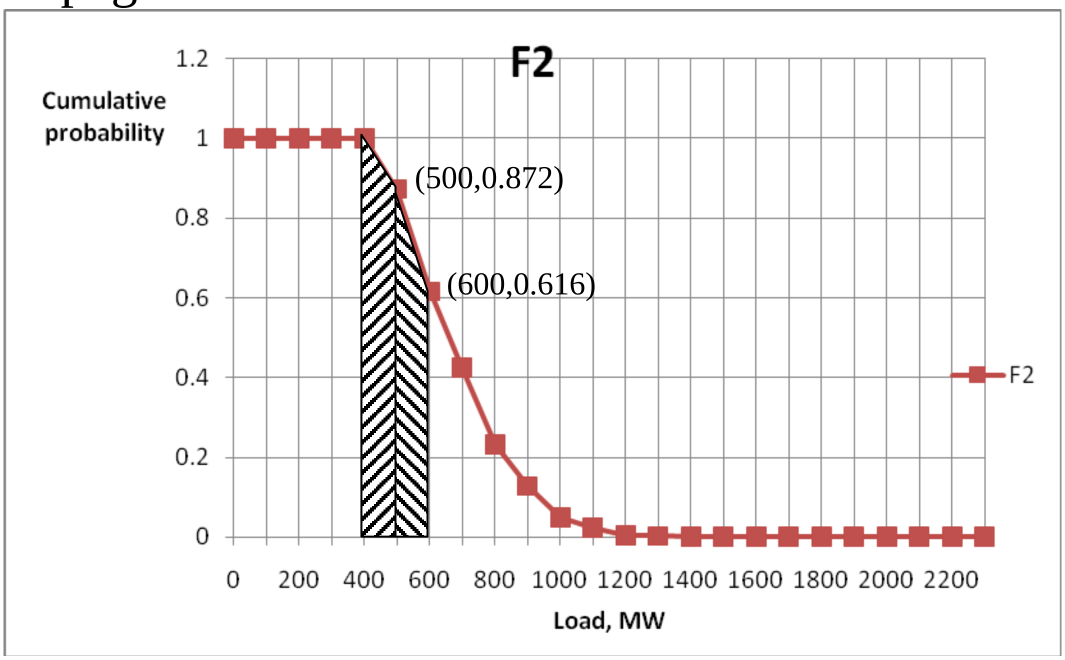


Fig. 25

The area, indicated in Fig. 25, is obtained as two applications of a trapezoidal area (1/2)(h)(a+b), as

$$\underbrace{\frac{1}{2}(100)(1+.872)}_{LeftPortion}+\underbrace{\frac{1}{2}(100)(.872+.616)}_{RightPortion}$$

$$=93.6+74.4=168$$

Therefore,

$$E_3=8760\times 0.9\times 168=1{,}324{,}512\,\text{MWhrs}$$

Table 7

| | F0 | F1 | F2 | F3 | F4 | F5 | F6 | F7 | F8 | F9 |
|---|---|---|---|---|---|---|---|---|---|---|
| | | 0.2 | 0.2 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.05 |
| | | 0.8 | 0.8 | 0.9 | 0.9 | 0.9 | 0.9 | 0.9 | 0.9 | 0.95 |
| | | 200 | 200 | 200 | 200 | 100 | 100 | 100 | 100 | 100 |
| Load (MW) | Fraction of time load exceeds given load | | | | | | | | | |
| -200 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| -100 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 100 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 200 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 300 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 400 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 500 | 0.8 | 0.84 | 0.872 | 0.8848 | 0.89632 | 0.906688 | 0.916019 | 0.924417 | 0.931976 | 0.935377 |
| 600 | 0.4 | 0.52 | 0.616 | 0.6544 | 0.68896 | 0.709696 | 0.729395 | 0.748058 | 0.765694 | 0.774008 |
| 700 | 0.2 | 0.32 | 0.424 | 0.4688 | 0.5104 | 0.528256 | 0.5464 | 0.5647 | 0.583035 | 0.592168 |
| 800 | 0.1 | 0.16 | 0.232 | 0.2704 | 0.3088 | 0.32896 | 0.34889 | 0.368641 | 0.388247 | 0.397986 |
| 900 | 0.05 | 0.08 | 0.128 | 0.1576 | 0.18872 | 0.200728 | 0.213551 | 0.227085 | 0.241241 | 0.248591 |
| 1000 | 0 | 0.02 | 0.048 | 0.0664 | 0.0868 | 0.096992 | 0.107366 | 0.117984 | 0.128894 | 0.134512 |
| 1100 | 0 | 0.01 | 0.024 | 0.0344 | 0.04672 | 0.050728 | 0.055354 | 0.060556 | 0.066298 | 0.069428 |
| 1200 | 0 | 0 | 0.004 | 0.0084 | 0.0142 | 0.017452 | 0.02078 | 0.024237 | 0.027869 | 0.02979 |
| 1300 | 0 | 0 | 0.002 | 0.0042 | 0.00722 | 0.007918 | 0.008871 | 0.010062 | 0.01148 | 0.012299 |
| 1400 | 0 | 0 | 0 | 0.0004 | 0.0012 | 0.001802 | 0.002414 | 0.003059 | 0.00376 | 0.004146 |
| 1500 | 0 | 0 | 0 | 0.0002 | 0.0006 | 0.00066 | 0.000774 | 0.000938 | 0.00115 | 0.001281 |
| 1600 | 0 | 0 | 0 | 0 | 0.00004 | 0.000096 | 0.000152 | 0.000215 | 0.000287 | 0.00033 |
| 1700 | 0 | 0 | 0 | 0 | 0.00002 | 0.000022 | 2.94E-05 | 4.17E-05 | 5.9E-05 | 7.04E-05 |
| 1800 | 0 | 0 | 0 | 0 | 0 | 0.000002 | 0.000004 | 6.54E-06 | 1.01E-05 | 1.25E-05 |
| 1900 | 0 | 0 | 0 | 0 | 0 | 0 | 2E-07 | 5.8E-07 | 1.18E-06 | 1.62E-06 |
| 2000 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2E-08 | 7.6E-08 | 1.31E-07 |
| 2100 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2E-09 | 5.7E-09 |
| 2200 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1E-10 |
| 2300 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

Continuing in this way, we obtain the energy produced by all units. This information, together with the average variable costs from Table 5, are provided in Table 8 below.

Table 8

| Unit | MW-hrs | Avg. Variable Costs, $/MWhr | Costs, $ |
|---|---|---|---|
| 1 | 1,401,600 | 6.5 | 9,110,400 |
| 2 | 1,401,600 | 6.5 | 9,110,400 |
| 3 | 1,324,500 | 27.0 | 35,761,500 |
| 4 | 734,200 | 27.0 | 19,823,400 |
| 5 | 196,100 | 58.1 | 11,393,410 |
| 6 | 117,400 | 58.1 | 6,820,940 |
| 7 | 64,100 | 58.1 | 3,724,210 |
| 8 | 33,400 | 58.1 | 1,940,540 |
| 9 | 16,400 | 113.2 | 1,856,480 |
| Total | 5,289,300 | | 99,541,280 |

It is interesting to note that the total energy supplied, 5,289,300 MWhrs, differs from what one obtained when the original load duration curve is integrated. This integration can be done by applying our trapezoidal approach to curve F0 in Table 7. Doing so results in 5,299,800 MWhrs. The difference is 5,299,800-5,289,300=10,500 MWhrs. This is the expected energy not served (EENS), sometimes called the expected unserved energy (EUE). It results from the fact that even though there is 1300 MW of capacity to serve load with maximum of 1000 MW, there is always some (small) chance that several units will simultaneously fail. This small chance corresponds to the area under the F9 load duration curve to the right of 1300, as illustrated in Fig. 26, and as indicated in Table 7.

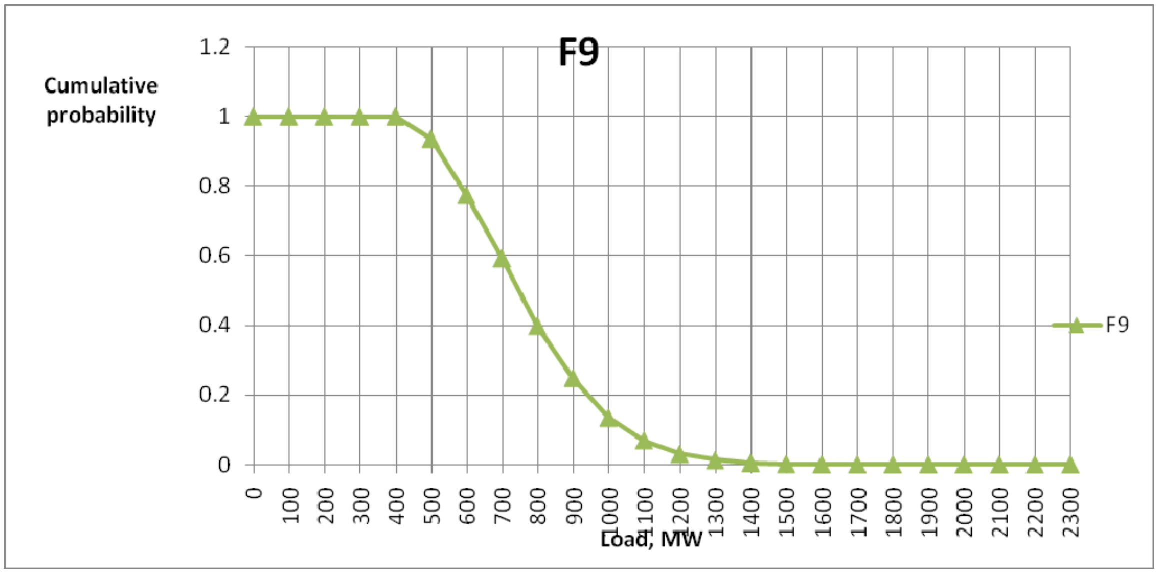


Fig. 26

[1] H. Stoll, “Least-cost electric utility planning,” 1989, John Wiley.
[2] R. Sullivan, “Power System Planning,” McGraw Hill, 1977.
[3] International Atomic Energy Agency, “Expansion planning for electrical generation systems,” Technical Reports Series No 241, 1984.
[4] X. Wang and J. McDonald, “Modern Power System Planning,” McGraw-Hill, 1994.