Solution for Homework2 (EE590 2008Fall)

1. (a)

$C(P) = 0.5P^2 \Rightarrow \frac{\partial C}{\partial P} = P \Rightarrow$ The supply function is: $P = y$

$U(m,P) = m + In(P) \Rightarrow \frac{\partial U}{\partial P} = \frac{1}{P} \Rightarrow$ The demand function is: $P = \frac{1}{y}$

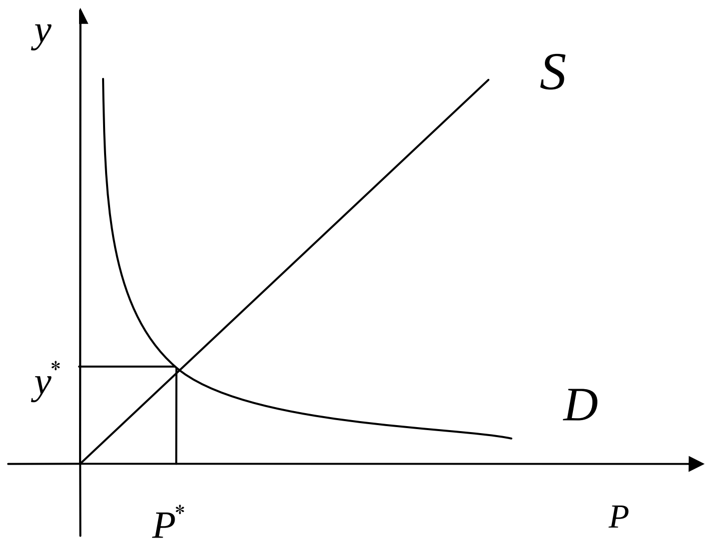

Fig 1: corresponding supply and demand functions for P

Competitive equilibrium: $y = \frac{1}{y} \Rightarrow y^* = 1, P^* = 1$

Check:

Max Social Surplus $= Max[U(m,P) - C(p)] = Max\left[m + In(P) - \frac{1}{2}P^2\right]$

Take the first derivative: $\frac{1}{P} - P = 0 \Rightarrow P^* = 1 \Rightarrow$ the same as the competitive equilibrium.

1. (b)

$C(P) = e^P \Rightarrow \frac{\partial C}{\partial P} = e^P \Rightarrow$ The supply function is: $P = Iny$

$U(m,P) = m + 2\sqrt{P} \Rightarrow \frac{\partial U}{\partial P} = \frac{1}{\sqrt{P}} \Rightarrow$ The demand function is: $P = \frac{1}{y^2}$

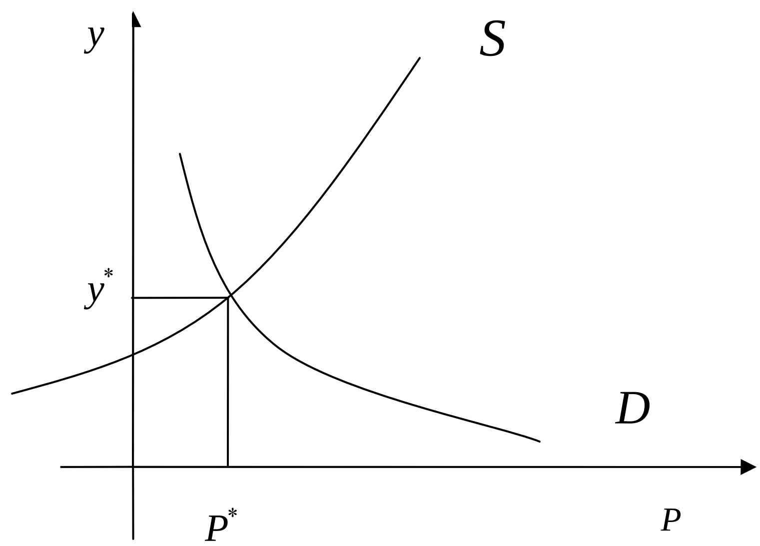

Fig 2: corresponding supply and demand functions for P

Competitive equilibrium: $Iny = \frac{1}{y^2} \Rightarrow y^* = 1.532, P^* = 0.426$

Check:

Max Social Surplus $= Max[U(m,P) - C(P)] = Max[m + 2\sqrt{P} - e^P]$

Take the first derivative: $\frac{1}{\sqrt{P}} - e^P = 0 \Rightarrow P^* = 0.426 \Rightarrow$ the same as the competitive equilibrium.

1. (c)

$C(P) = \frac{1}{2}P^4 \Rightarrow \frac{\partial C}{\partial P} = 2P^3 \Rightarrow$ The supply function is: $y = 2P^3$

$U(m,P) = m + 2P(10-P) \Rightarrow \frac{\partial U}{\partial P} = 20 - 4P \Rightarrow$ The demand function is: $y = 20 - 4P$

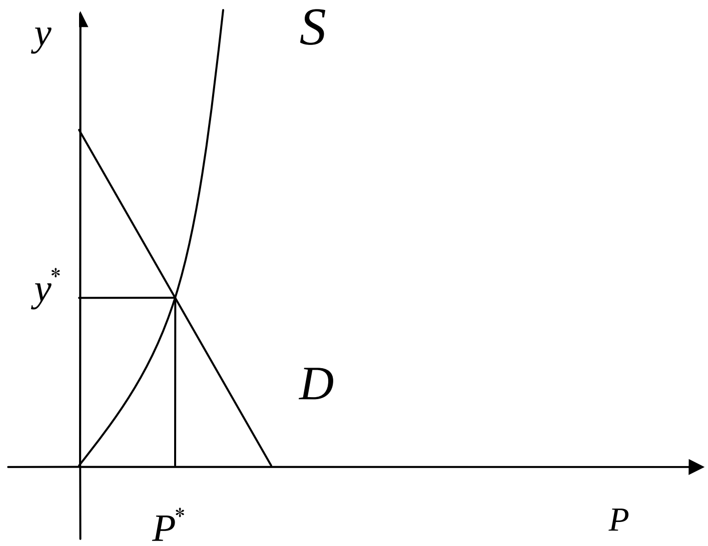

Fig 3: corresponding supply and demand functions for P

Competitive equilibrium: $2P^3 = 20 - 4P \Rightarrow P^* = 1.847, y^* = 12.61$

Check:

Max Social Surplus $= Max[U(m,P) - C(P)] = Max[m + 2P(10 - P) - 0.5P^4]$

Take the first derivative: $20 - 4P - 2P^3 = 0 \Rightarrow P^* = 1.847 \Rightarrow$ the same as the competitive equilibrium.

2. (a) $P^* = 180$ and $Q^* = 3$

2. (b)
1 and 2 would.
Either 3 or 4 would.
The others wouldn't

2. (c)
1: $20; 2: $10; All the others: $0

2. (d) 180*3=$540

2. (e)
The equilibrium price will be $175 and the equilibrium quality will remain the same.
No, they won't.
Yes, it will.

3. (a) 5, 4 and 3.
3. (b) 5: \$60; 4: \$80; 3: \$80
3. (c) 5 can sell his permit to 1 at \$190
3. (d) No.

4. (a)
In the North:
Supply Function: $q_N = p_N$
Demand Function: $x_N = 100 - p_N$

In the South:
Supply Function: $q_S = p_S$
Demand Function: $x_S = 50 - p_S$

4. (b)
In the North:

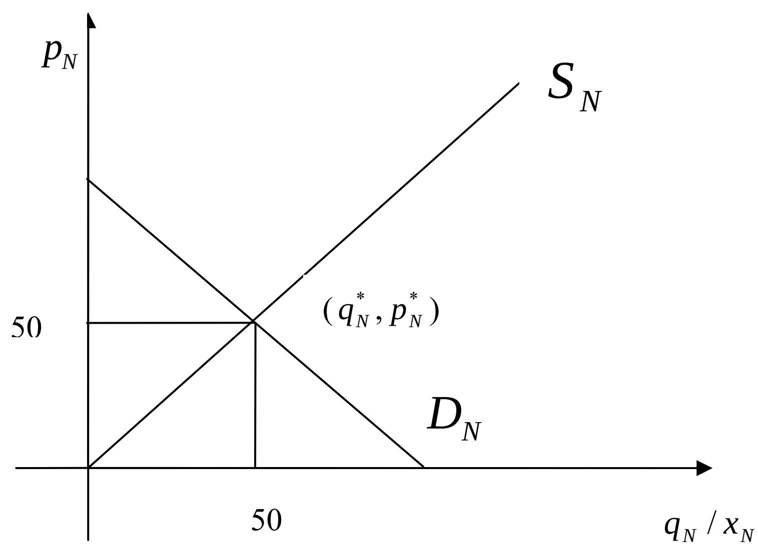

Fig 4: corresponding supply and demand functions in North

Competitive equilibrium: $p_N = 100 - p_N \Rightarrow p_N^* = 50, q_N^* = x_N^* = 50$

In the South:

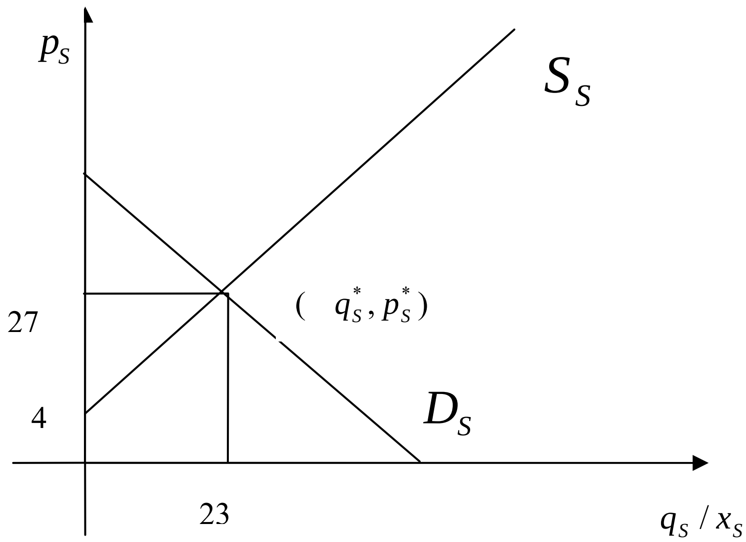

Fig 5: corresponding supply and demand functions in South

Competitive equilibrium: $p_S - 4 = 50 - p_S \Rightarrow p_S^* = 27, q_S^* = x_S^* = 23$

4. (c)

(i) Objective function is:

$$U_N(m_N, x_N) + U_S(m_S, x_S) - C_N(q_N) - C_S(q_S)$$

$$= m_N + m_S + \frac{(200 - x_N)x_N}{2} + \frac{(100 - x_S)x_S}{2} - \frac{q_N^2}{2} - 4q_S - \frac{q_S^2}{2}$$

And so the constrained optimization problem is:

$$\underset{(x_S, x_N, q_N, q_S)}{Max} \; m_N + m_S + \frac{(200 - x_N)x_N}{2} + \frac{(100 - x_S)x_S}{2} - \frac{q_N^2}{2} - 4q_S - \frac{q_S^2}{2}$$

$s.t.$

$$q_N + q_S - x_N - x_S = 0$$

The Lagrangian function is:

$$L = \frac{(200 - x_N)x_N}{2} + \frac{(100 - x_S)x_S}{2} - \frac{q_N^2}{2} - 4q_S - \frac{q_S^2}{2} + \lambda(q_N + q_S - x_N - x_S)$$

Applying first-order conditions results in:

$$\begin{cases} \dfrac{\partial L}{\partial x_N} = 100 - x_N - \lambda = 0 \\ \dfrac{\partial L}{\partial x_S} = 50 - x_S - \lambda = 0 \\ \dfrac{\partial L}{\partial q_N} = -q_N + \lambda = 0 \\ \dfrac{\partial L}{\partial q_S} = -4 - q_S + \lambda = 0 \\ \dfrac{\partial L}{\partial \lambda} = q_N + q_S - x_N - x_S = 0 \end{cases} \Rightarrow \begin{cases} x_N^* = 61.5 \\ x_S^* = 11.5 \\ q_N^* = 38.5 \\ q_S^* = 34.5 \\ \lambda^* = 38.5 \end{cases}$$

(ii) $\lambda^* = 38.5$

(iii) From South to North; 34.5-11.5=23

(iv) $q_N(p) + q_S(p) = x_N(p) + x_S(p) \Rightarrow p + p - 4 = 100 - p + 50 - p \Rightarrow p^* = 38.5$ (this is the equilibrium price), and $q_N^* = 38.5, q_S^* = 34.5, x_N^* = 61.5, x_S^* = 11.5$

(v)

In the North:

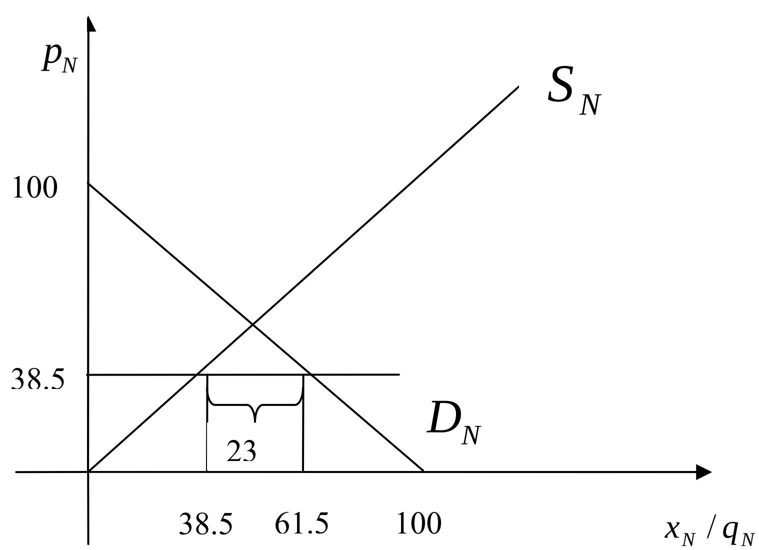

Fig 6: corresponding supply and demand functions in North

In the South:

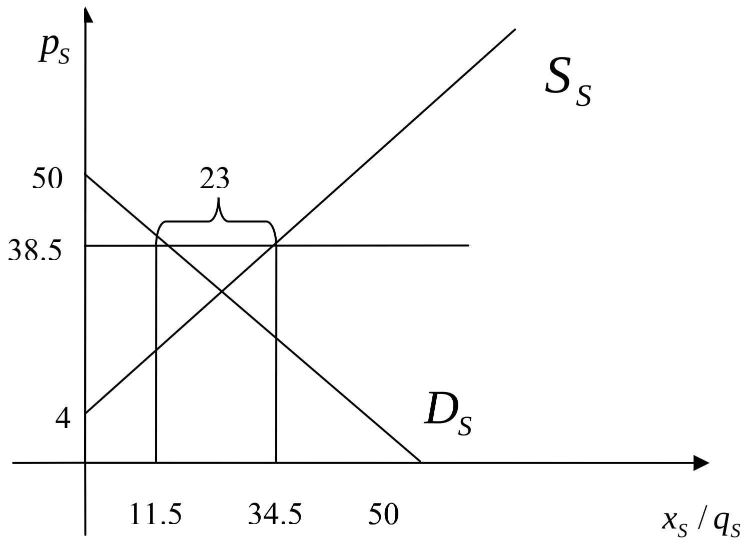

Fig 5: corresponding supply and demand functions in South

4. (d)

(i) Again, the objective function is

$$U_N(m_N, x_N) + U_S(m_S, x_S) - C_N(q_N) - C_S(q_S)$$

$$= m_N + m_S + \frac{(200 - x_N)x_N}{2} + \frac{(100 - x_S)x_S}{2} - \frac{q_N^2}{2} - 4q_S - \frac{q_S^2}{2}$$

And so the optimization problem is:

$$\underset{(x_S, x_N, q_N, q_S)}{Max} \; m_N + m_S + \frac{(200 - x_N)x_N}{2} + \frac{(100 - x_S)x_S}{2} - \frac{q_N^2}{2} - 4q_S - \frac{q_S^2}{2}$$

$s.t.$

$$q_N + q_S - x_N - x_S = 0, \quad q_S - x_S \le 16$$

But since from part (c) that, with no transmission constraint, q_S-x_S=23, we know that the inequality constraint q_S-$x_S \le 16$ will bind in this new optimization problem. Therefore, we may treat it as an equality, i.e., q_S-x_S=16. The Lagrangian is

$$L = \frac{(200 - x_N)x_N}{2} + \frac{(100 - x_S)x_S}{2} - \frac{q_N^2}{2} - 4q_S - \frac{q_S^2}{2} + \lambda(q_N + q_S - x_N - x_S) + \mu(16 - q_S + x_S)$$

Applying first-order conditions results in

$$\begin{cases} \dfrac{\partial L}{\partial x_N} = 100 - x_N - \lambda = 0 \\ \dfrac{\partial L}{\partial x_S} = 50 - x_S - \lambda + \mu = 0 \\ \dfrac{\partial L}{\partial q_N} = -q_N + \lambda = 0 \\ \dfrac{\partial L}{\partial q_S} = -4 - q_S + \lambda - \mu = 0 \\ \dfrac{\partial L}{\partial \lambda} = q_N + q_S - x_N - x_S = 0 \\ \dfrac{\partial L}{\partial \mu} = 16 - q_S + x_S = 0 \end{cases} \Rightarrow \begin{cases} x_N^* = 58 \\ x_S^* = 15 \\ q_N^* = 42 \\ q_S^* = 31 \\ \lambda^* = 42 \\ \mu^* = 7 \end{cases}$$

(ii) $\lambda^* = 42, \mu^* = 7$

(iii) From South to North; 31-15=16

(iv) Let p be the price of power in the North and t be the price of transmission service. Then the price of power in the south will be *p-t*. Let's consider the situation from the southern supplier and consumer's point of view.

The southern supplier will be able to supply according to a cost function that accounts for transmission price, as follows:

$$C_S(q_S) = 4q_S + \frac{q_S^2}{2} + t(q_S - x_S)$$

The southern supplier will maximize their profit when $p = C'_S(q_S)$, that is,

$$\Rightarrow p = 4 + q_S + t \Rightarrow q_S = p - 4 - t$$

The southern consumer have utility dependent on how much money they have, m_S, and how much power they have, x_S, according to $U_S(m_S,x_S)=m_S+0.5(100-x_S)x_S$. Let's assume that m is the amount of money the southern consumer has before purchasing an amount of power x_S at a price *p-t*, then $m_S=m-(p-t)x_S$. Notice that the southern consumer purchases power at a price *t* less than what the northern consumer purchases. Therefore, the utility of the southern consumer is

$$U_S\left[m - (p-t)x_S, x_S\right] = m_S - (p-t)x_S + 0.5(100 - x_S)x_S$$

Note that this is equivalent to what we did in the notes where we essentially added a constraint $m_S=m-(p-t)x_S$ to our maximization problem.

The southern consumer will maximize their utility when $U'_S(m_S,x_S)=0$, or

$$\text{Utility Max} \Rightarrow -p + t + 50 - x_S = 0 \Rightarrow x_S = 50 + t - p$$

When the market clears, we will have that the total supply will equal total demand, and there will be 16 units flowing from south to north. Writing these expressions, and substituting the above expressions for q_S and x_S, respectively, we have:

$$\begin{cases} q_S + q_n = x_S + x_N \Rightarrow p - 4 - t + p = 50 + t - p + 100 - p \\ q_S - x_S = 16 \Rightarrow p - 4 - t - 50 - t + p = 16 \end{cases} \Rightarrow \begin{cases} p^* = 42 \\ t^* = 7 \end{cases}$$ and

$q_N^* = 42, q_S^* = 31, x_N^* = 58, x_S^* = 15$, $p_S^* = 35$

(v) In the North:

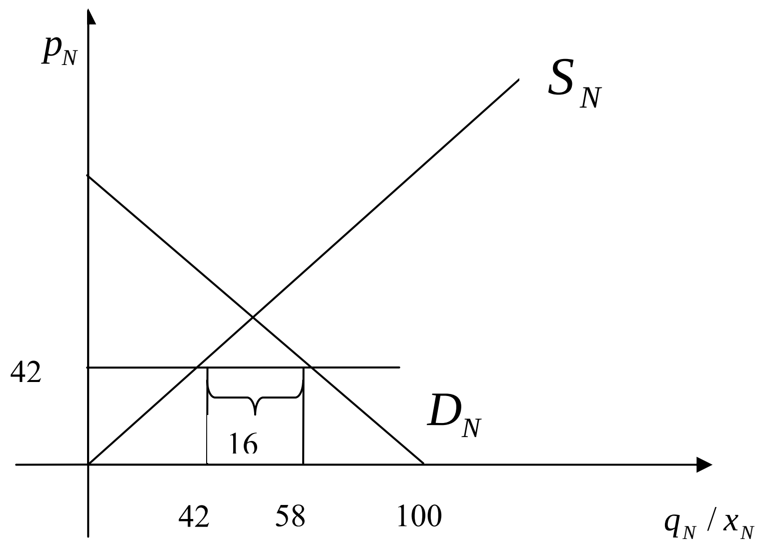

Fig 7: corresponding supply and demand functions in North

In the South:

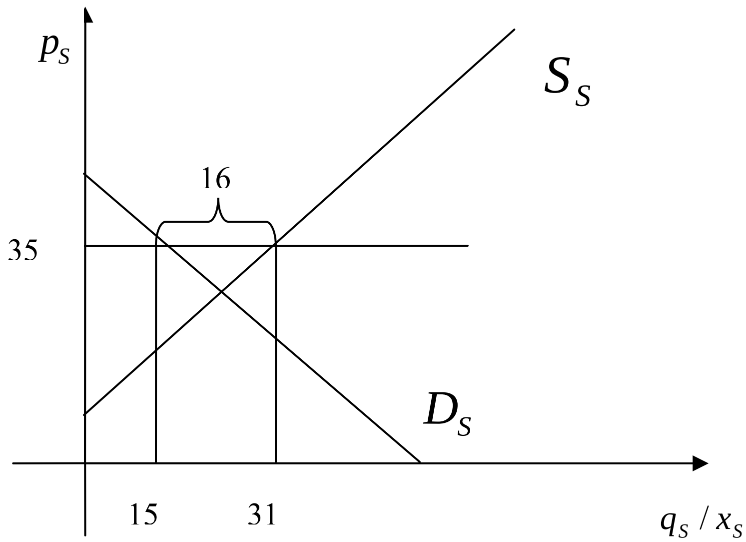

Fig 8: corresponding supply and demand functions in South

5.
The base value is 100MW

$$C(P_{G1}) = 1.8P_{G1}$$
$$C(P_{G4}) = 20P_{G4}$$

5. (a) Without 2-3 circuit

Objective function:

$$Z(x) = C^T x = (1.8,\ 20,\ 0,0,0,0,0,0,0,0)\begin{bmatrix} P_{G1} \\ P_{G4} \\ P_{S12} \\ P_{S13} \\ P_{S24} \\ P_{S34} \\ \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \end{bmatrix}$$

Equality constraints:

$$D=\begin{bmatrix} 10 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 10 \end{bmatrix} \quad A=\begin{bmatrix} 1 & -1 & 0 & 0 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \end{bmatrix} \quad D*A=\begin{bmatrix} 10 & -10 & 0 & 0 \\ 5 & 0 & -5 & 0 \\ 0 & 5 & 0 & -5 \\ 0 & 0 & 10 & -10 \end{bmatrix}$$

$$B' = \begin{bmatrix} 15 & -10 & -5 & 0 \\ -10 & 15 & 0 & -5 \\ -5 & 0 & 15 & -10 \\ 0 & -5 & -10 & 15 \end{bmatrix}$$

$$A_{eq}x = \begin{bmatrix} 0 & 0 & -1 & 0 & 0 & 0 & 10 & -10 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 5 & 0 & -5 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 5 & 0 & -5 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 10 & -10 \\ -1 & 0 & 0 & 0 & 0 & 0 & 15 & -10 & -5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -10 & 15 & 0 & -5 \\ 0 & 0 & 0 & 0 & 0 & 0 & -5 & 0 & 15 & -10 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & -5 & -10 & 15 \end{bmatrix}$$

beq=[0 0 0 0 0 0 0 -1]';

Inequality constraints:

$$\begin{bmatrix} 0 \\ 0 \\ -0.55 \\ -0.55 \\ -1 \\ -1 \\ -\pi \\ -\pi \\ -\pi \\ -\pi \\ \end{bmatrix} \le \begin{bmatrix} P_{G1} \\ P_{G4} \\ P_{S12} \\ P_{S13} \\ P_{S24} \\ P_{S34} \\ \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \\ \end{bmatrix} \le \begin{bmatrix} 1 \\ 1 \\ 0.55 \\ 0.55 \\ 1 \\ 1 \\ \pi \\ \pi \\ \pi \\ \pi \\ \end{bmatrix}$$

Matlab Code:

```
%Build objective function vector.
c=[1.8 20 0 0 0 0 0 0 0 0]';

%Build A matrix for inequality constraints Ax<b.
A=[];
%Build b, the right-hand-side of inequality constraints.
b=[];

%Build Aeq matrix for equality constraints.
Aeq=[0  0  -1 0  0  0  10  -10  0   0;
     0  0  0  -1 0  0  5   0    -5  0;
     0  0  0  0  -1 0  0   5    0   -5;
     0  0  0  0  0  -1 0   0    10  -10;
     -1 0  0  0  0  0  15  -10  -5  0;
```

```
     0  0  0  0  0  0  -10 15   0   -5;
     0  0  0  0  0  0  -5  0    15  -10;
     0 -1  0  0  0  0  0   -5   -10 15;];

%Build right-hand side of equality constraint. It will be vector of
zeros
beq=[0 0 0 0 0 0 0 -1]';

%Build upper and lower bounds on decision variables.
LB=[0   0 -0.55 -0.55 -1 -1 -pi -pi -pi -pi]';
UB=[1   1  0.55  0.55  1  1  pi  pi  pi  pi]';
[X,FVAL,EXITFLAG,OUTPUT,LAMBDA]=linprog(c,A,b,Aeq,beq,LB,UB);
% Compute settlements assuming pay-at-bid/offer:
dollars=c.*X;
```

LMP on four buses:
6.924144029015576
6.924144035339617
6.924144035339773
6.924144035729284

5. (b) With 2-3 circuit

Objective function:

$$Z(x) = C^T x = (1.8, 20, 0,0,0,0,0,0,0,0,0) \begin{bmatrix} P_{G1} \\ P_{G4} \\ P_{S12} \\ P_{S13} \\ P_{S24} \\ P_{S34} \\ P_{S23} \\ \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \end{bmatrix}$$

Equality constraints:

$$D=\begin{bmatrix}10 & 0 & 0 & 0 & 0\\ 0 & 5 & 0 & 0 & 0\\ 0 & 0 & 5 & 0 & 0\\ 0 & 0 & 0 & 10 & 0\\ 0 & 0 & 0 & 0 & 5\end{bmatrix}\quad A=\begin{bmatrix}1 & -1 & 0 & 0\\ 1 & 0 & -1 & 0\\ 0 & 1 & 0 & -1\\ 0 & 0 & 1 & -1\\ 0 & -1 & 1 & 0\end{bmatrix}\quad D*A=\begin{bmatrix}10 & -10 & 0 & 0\\ 5 & 0 & -5 & 0\\ 0 & 5 & 0 & -5\\ 0 & 0 & 10 & -10\\ 0 & -5 & 5 & 0\end{bmatrix}$$

$$B^{'}=\begin{bmatrix}15 & -10 & -5 & 0\\ -10 & 20 & -5 & -5\\ -5 & -5 & 20 & -10\\ 0 & -5 & -10 & 15\end{bmatrix}$$

$$A_{eq}x=\begin{bmatrix}0 & 0 & -1 & 0 & 0 & 0 & 0 & 10 & -10 & 0 & 0\\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 5 & 0 & -5 & 0\\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 5 & 0 & -5\\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 10 & -10\\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & -5 & 5 & 0\\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 15 & -10 & -5 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -10 & 20 & -5 & -5\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -5 & -5 & 20 & -10\\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & -5 & -10 & 15\end{bmatrix}$$

beq=[0 0 0 0 0 0 0 0 -1]';

Inequality constraints:

$$\begin{bmatrix}0\\ 0\\ -0.55\\ -0.55\\ -1\\ -1\\ -1\\ -\pi\\ \pi\\ \pi\\ \pi\end{bmatrix}\le\begin{bmatrix}P_{G1}\\ P_{G4}\\ P_{S12}\\ P_{S13}\\ P_{S24}\\ P_{S34}\\ P_{S23}\\ \theta_1\\ \theta_2\\ \theta_3\\ \theta_4\end{bmatrix}\le\begin{bmatrix}1\\ 1\\ 0.55\\ 0.55\\ 1\\ 1\\ 1\\ \pi\\ \pi\\ \pi\\ \pi\end{bmatrix}$$

Matlab Code:

```
%Build objective function vector.
c=[1.8 20 0 0 0 0 0 0 0 0 0]';

%Build A matrix for inequality constraints Ax<b.
A=[];
%Build b, the right-hand-side of inequality constraints.
b=[];

%Build Aeq matrix for equality constraints.
Aeq=[0  0  -1 0  0  0  0   10  -10  0   0;
     0  0  0  -1 0  0  0   5   0    -5  0;
     0  0  0  0  -1 0  0   0   5    0   -5;
     0  0  0  0  0  -1 0   0   0    10  -10;
     0  0  0  0  0  0  -1  0   -5   5   0;
     -1 0  0  0  0  0  0   15  -10  -5  0;
     0  0  0  0  0  0  0   -10 20   -5  -5;
     0  0  0  0  0  0  0   -5  -5   20  -10;
     0 -1  0  0  0  0  0   0   -5   -10 15;];

%Build right-hand side of equality constraint. It will be vector of
zeros
beq=[0 0 0 0 0 0 0 0 -1]';

%Build upper and lower bounds on decision variables.
LB=[0   0 -0.55 -0.55 -1 -1 -1 -pi -pi -pi -pi]';
UB=[1   1  0.55  0.55  1  1  1 pi  pi  pi  pi]';
[X,FVAL,EXITFLAG,OUTPUT,LAMBDA]=linprog(c,A,b,Aeq,beq,LB,UB);
% Compute settlements assuming pay-at-bid/offer:
dollars=c.*X;
```

LMP on four buses:
1.799999999999992
26.06666666666656
16.966666666666665
19.999999999999993

5. (c)

Conclusion of the paper:
" Without the Wheatstone bridge (and ignoring losses and reactive power), the inexpensive generator at bus 1 will produce 100 MW and serve the entire load. Due to the symmetry in the network, 50 MW will flow along each of the two paths from bus 1 to bus 4.
The network will have identical locational marginal prices (LMP) of $12.11/MWperiod at each of the four buses
The total system cost of serving the load is $1,620 per period.

Once the Wheatstone bridge is added to the network, the pattern of flows shifts, causing congestion on lines *S12* and *S34* in the network.1
The generator at bus 1 produces only 91.67 MW of real power, while the remainder must be made up by the expensive generator at bus 4.
The presence of congestion alters the LMPs at all four buses in the system; in particular, the LMP at the load bus increases to $51.67/MW-period.
The total system cost of serving the load rises to $1,945 per period. ”

The LMP is not the same as what the paper shows, while the power flow is nearly the same.

6. (a) CO_2
6. (b) In 2003, governors from Connecticut, Delaware, Maine, Massachusetts, New Hampshire, New Jersey, New York, Rhode Island, and Vermont began discussions to develop a regional cap-and-trade program addressing carbon dioxide emissions from power plants.
6. (c) The Regional Greenhouse Gas Initiative (RGGI) is a cooperative effort by ten Northeast and Mid-Atlantic states to limit greenhouse gas emissions. RGGI is the first mandatory, market-based CO_2 emissions reduction program in the United States.
6. (d) emission offset and allowance auction
6. (f) Present indication is that it is determined by each state and so varies.