EE 590F, Power System Planning, Exam 1, 75 minutes, Closed book, Closed notes, Calculator Allowed, Off-campus must complete by 10/1 unless otherwise arranged.

1. **(40 pts) Consider an electricity market with 2 regions: North & South. Relevant data is below, with supply quantities given by q_N and q_S and demand quantities given by x_N and x_S, and all four of them given in per-unit (pu)-hour.**
   - **Cost function for Northern generators: $C_N(q_N)= q_N^2/2$**
   - **Cost function for Southern generators: $C_S(q_S)=4q_S + q_S^2/2$.**
   - **Utility function for Northern consumers: $U_N(m_N,x_N)=m_N+0.5(100-x_N)x_N$**
   - **Utility function for Southern consumers: $U_S(m_S, x_S)=m_S+0.5(50-x_S)x_S$**

   a. **(4 pts) Obtain expressions for the demand and supply functions of the economic agents in each region.**

   b. **Assume there is an infinite capacity transmission line that connects the two regions.**
      i. **(3 pts) Set up the appropriate constrained maximization problem and write down the first-order conditions.**
      ii. **(15 pts) Assume the solution is x_N=30.25, x_S=5.25, q_N=19.75, q_S=15.75, λ=19.75.**
         A. **What is the power flow across the line and what is the direction?**
         B. **What is the locational marginal price in the north?**
         C. **What is the locational marginal price in the south?**
         D. **Assume the ISO pays suppliers and collects from consumers. What does the ISO net?**
         E. **What is the total social surplus?**

   c. **Assume the transmission line connecting the regions has capacity of 10pu.**
      i. **(3 pts) Set up the appropriate constrained maximization problem and write down the first-order conditions.**
      ii. **(15 pts) Assume the solution is x_N=30.0, x_S=5.5, q_N=20.0, q_S=15.5, λ=20.0, μ=0.5. Answer the questions of part (b-ii, A-E) above.**

Solution:

4. (a)
In the North:
Supply Function: $q_N = p_N$
Demand Function: $x_N = 50 - p_N$

In the South:
Supply Function: $q_S = p_S$
Demand Function: $x_S = 25 - p_S$

(b-i) Objective function is:

$$U_N(m_N,x_N)+U_S(m_S,x_S)-C_N(q_N)-C_S(q_S)$$
$$= m_N + m_S + 0.5(100-x_N)x_N + 0.5(50-x_S)x_S - \frac{q_N^2}{2} - 4q_S - \frac{q_S^2}{2}$$

And so the constrained optimization problem is:

$$\underset{(x_S,x_N,q_N,q_S)}{Max}\; m_N + m_S + 0.5(100-x_N)x_N + 0.5(50-x_S)x_S - \frac{q_N^2}{2} - 4q_S - \frac{q_S^2}{2}$$
$$s.t.$$
$$q_N + q_S - x_N - x_S = 0$$

The Lagrangian function is:

$$L = 0.5(50-x_N)x_N + 0.5(25-x_S)x_S - \frac{q_N^2}{2} - 4q_S - \frac{q_S^2}{2} + \lambda(q_N+q_S-x_N-x_S)$$

Applying first-order conditions results in:

$$\begin{cases} \dfrac{\partial L}{\partial x_N} = 50 - x_N - \lambda = 0 \\ \dfrac{\partial L}{\partial x_S} = 25 - x_S - \lambda = 0 \\ \dfrac{\partial L}{\partial q_N} = -q_N + \lambda = 0 \\ \dfrac{\partial L}{\partial q_S} = -4 - q_S + \lambda = 0 \\ \dfrac{\partial L}{\partial \lambda} = q_N+q_S-x_N-x_S = 0 \end{cases} \Rightarrow \begin{cases} x_N^* = 30.25 \\ x_S^* = 5.25 \\ q_N^* = 19.75 \\ q_S^* = 15.75 \\ \lambda^* = 19.75 \end{cases}$$

(b-ii-A) From South to North; 15.75-5.25=10.5

(b-ii-B) The LMP in north is p_N=19.75.

(b-ii-C) The LMP in south is p_S=19.75.

(b-ii-D)

$$\begin{aligned} ISONet &= p_N(x_N) - p_N(q_N) + p_S(x_S) - p_S(q_S) \\ &= 19.75(30.25) - 19.75(19.75) + 19.75(5.25) - 19.75(15.75) \\ &= 207.375 - 207.375 = 0 \end{aligned}$$

(b-ii-E)

$$\begin{aligned} SocSurplus &= \int_0^{30.25} 50 - x_N - 19.75 dx_N + \int_0^{19.75} 19.75 - q_N dq_N + \int_0^{5.25} 25 - x_S - 19.75 dx_S + \int_0^{15.75} 19.75 - q_S dq_S \\ &= 457.5313 + 195.0313 + 13.7813 + 187.0313 \\ &= 853.3752 \end{aligned}$$

(c-i) Again, the objective function is

$$U_N(m_N, x_N) + U_S(m_S, x_S) - C_N(q_N) - C_S(q_S)$$
$$= m_N + m_S + 0.5(50 - x_N)x_N + 0.5(25 - x_S)x_S - \frac{q_N^2}{2} - 4q_S - \frac{q_S^2}{2}$$

And so the optimization problem is:

$$\underset{(x_S, x_N, q_N, q_S)}{Max} \; m_N + m_S + 0.5(100 - x_N)x_N + 0.5(50 - x_S)x_S - \frac{q_N^2}{2} - 4q_S - \frac{q_S^2}{2}$$
$$s.t.$$
$$q_N + q_S - x_N - x_S = 0, \quad q_S - x_S \le 10$$

But since from part (b), with no transmission constraint, q_S-x_S=10.5, we know that the inequality constraint q_S-x_S≤10 will bind in this new optimization problem. Therefore, we may treat it as an equality, i.e., q_S-x_S=10. The Lagrangian is

$$L = 0.5(100 - x_N)x_N + 0.5(50 - x_S)x_S - \frac{q_N^2}{2} - 4q_S - \frac{q_S^2}{2} + \lambda(q_N + q_S - x_N - x_S) + \mu(10 - q_S + x_S)$$

Applying first-order conditions results in

$$\begin{cases} \frac{\partial L}{\partial x_N} = 50 - x_N - \lambda = 0 \\ \frac{\partial L}{\partial x_S} = 25 - x_S - \lambda + \mu = 0 \\ \frac{\partial L}{\partial q_N} = -q_N + \lambda = 0 \\ \frac{\partial L}{\partial q_S} = -4 - q_S + \lambda - \mu = 0 \\ \frac{\partial L}{\partial \lambda} = q_N + q_S - x_N - x_S = 0 \\ \frac{\partial L}{\partial \mu} = 10 - q_S + x_S = 0 \end{cases} \Rightarrow \begin{cases} x_N^* = 30 \\ x_S^* = 5.5 \\ q_N^* = 20 \\ q_S^* = 15.5 \\ \lambda^* = 20 \\ \mu^* = 0.5 \end{cases}$$

(c-ii-A) From South to North; 15.5-5.5=10.
(c-ii-B) The LMP in north is p_N=20.0
(c-ii-C) The LMP in south is p_S=20.0-0.5=19.5
(c-ii-D)

$$\begin{aligned} ISONet &= p_N(x_N) - p_N(q_N) + p_S(x_S) - p_S(q_S) \\ &= 20.0(30.0) - 20.0(20.0) + 19.5(5.5) - 19.5(15.5) \\ &= 200 - 195 = 5 \end{aligned}$$

(b-ii-E)

$$SocSurplus = \int_0^{30.} 50 - x_N - 20 dx_N + \int_0^{20} 20 - q_N dq_N + \int_0^{5.5} 25 - x_S - 19.5 dx_S + \int_0^{15.5} 19.5 - q_S dq_S$$

$$= 450 + 200 + 15.125 + 182.125$$

$$= 847.25$$

2. **(20 pts) A generation owner is trying to decide what kind of plant in which to invest. One is a pulverized coal power plant and one is a combined cycle plant, each characterized by data below. Assume a 40 year life for both plants with 0 salvage value.**

Data	**PC plant**	**NGCC plant**
Overnight cost	**$1600/kW**	**$850/kW**
Levelized fixed charge rate	**20%**	**20%**
Plant rating	**500 MW**	**800 MW**
Full-load heat rate	**9.6 MBTU/MWh**	**7.2 MBTU/MWh**
2010 fuel price	**$2.00/MBTU (coal)**	**$8.00/MBTU (gas)**
Fuel price inflation	**0%/year**	**0%/year**
Capacity factor	**0.9**	**0.6**
Discount rate	**9%**	**9%**

Assuming the above data accurately characterizes the future, compute:

a. **(4 pts) the lifetime operational cost for the two plants, assuming they operate at the given capacity factor 8760 hrs/year,**
b. **(4 pts) the annual fixed charges for the two plants**
c. **(4 pts) the total annual costs for the three plants**
d. **(4 pts) the present value of the investment plus operating cost,**

Also,

- **(2 pts) indicate which plant represents the least cost investment and**
- **(2 pts) identify reasons why the generation owner might invest in the higher-cost investment.**

One or more of the following relations may be helpful to you:

$$P = F \times \frac{1}{(1+i)^N} \text{ ; } P = A \times \frac{(1+i)^N - 1}{i(1+i)^N} \text{ ; } F = A \times \frac{(1+i)^N - 1}{i}$$

Solution:

a.

OpCost(t)=Cap*Capfactor*8760*Heatrate*Fuelprice(t)
For PC plant: OpCost(t)=500(0.9)(8760)(9.6)*2.00=75,686,400
For NGCC plant: OpCost(t)=800(0.6)(8760)(7.2)*8.00=242,196,480
Note that the above are annual operational costs.

b.

FixedCharges(t)=FixedChargeRate*Investment
For PC plant: FixedCharges(t)=0.2*1600*10^3*500=160,000,000

For NGCC plant: FixedCharges(t)=0.2*850*10^3*800=136,000,000
The above are annual fixed charges.

c.

TotalCost(t)=OpCost(t)+FixedCharges(t)
For PC plant : TotalCost(t)=75,686,400+160,000,000=235,686,400
For NGCC plant: TotalCost(t)= 242,196,480+136,000,000=378,196,480

d.

For PC plant:

$$P = Investment + A \times \frac{(1+i)^N - 1}{i(1+i)^N}$$

$$= 1600*1000*500 + 235686400\,\frac{(1+0.09)^{40} - 1}{0.09(1+0.09)^{40}} = 3{,}335{,}400{,}000$$

For NGCC plant:

$$P = Investment + A \times \frac{(1+i)^N - 1}{i(1+i)^N}$$

$$= 850*1000*800 + 378196480\,\frac{(1+0.09)^{40} - 1}{0.09(1+0.09)^{40}} = 4{,}748{,}400{,}000$$

So the PC plant is the minimum cost investment. However, it still might be of interest to invest in the NGCC plant because it is a higher capacity plant, offering possible peaking benefits that the 500 MW PC plant cannot, and because it is expected to supply more energy from it, as indicated by the product of capacity factor and capacity, which gives the average plant loading throughout the year:

PC plant: 0.9*500=450MW
NGCC plant: 0.6*800=480 MW.

3. **(10 pts)**
(a) (8 pts) Consider that the function *f* is concave in *x* for all values of *θ*. An unconstrained maximization problem is given by $\max_x f(x,\theta)$. We know a solution *x*(θ)* satisfies the first order condition

$$\frac{\partial f(x,\theta)}{\partial x} = 0. \qquad \textbf{(3-a)}$$

Let's denote the optimal value of the objective function by

$$V(\theta) = f(x^*(\theta),\theta) \qquad \textbf{(3-b)}$$

Prove that $\frac{dV(\theta)}{d\theta} = \frac{\partial f(x^*(\theta),\theta)}{\partial \theta}$, which is the "simple version" of the envelope theorem ("simple" because it is for the unconstrained maximization problem). In doing so, begin with (3-b) and use the "fact" below to express $\frac{dV(\theta)}{d\theta}$. Then simplify.

Fact: If $y = f(z_1(t), z_2(t))$, then $\frac{dy}{dt} = \frac{\partial f(z_1(t), z_2(t))}{\partial z_1}\frac{\partial z_1}{\partial t} + \frac{\partial f(z_1(t), z_2(t))}{\partial z_2}\frac{\partial z_2}{\partial t}$

(b) (2 pts) We used one form of the envelope theorem in class. Identify, in words, the application for which we used this theorem (no need to express anything analytically here).

Solution:

(a) $\frac{dV(\theta)}{dt} = \frac{\partial f(x^*(\theta), \theta)}{\partial x}\frac{\partial x}{\partial \theta} + \frac{\partial f(x^*(\theta), \theta)}{\partial \theta}$

But by (3-a), the first factor in the first term is zero, therefore the first term is zero. So

$\frac{dV(\theta)}{dt} = \frac{\partial f(x^*(\theta), \theta)}{\partial \theta}$

The proof is complete.

(b) We used this theorem is obtaining the expression for LMP, when we differentiated the Lagrangian with respect to the demand.

4. **(9 pts) Investing in a power plant is a long-term commitment to the fuel(s) the power plant is capable of using and the form(s) of energy the power plant is capable of supplying. Given the uncertainty of the future, many investors are looking for power plants that can utilize more than one kind of fuel and/or provide more than one form of energy. Of the generation technologies discussed in class, identify the most attractive from this point of view, and indicate the fuels that can be utilized and the energy forms that can be provided.**

   Solution:

   Either of the following answers will be accepted:

   - IGCC or most gasification processes: Can use coal and/or biomass. Produces syngas and hydrogen, and if Fisher-Tropsch is employed, synfuel as well.
   - IPCC or most pyrolysis processes: Can use coal and/or biomass. Produces syngas, char, and bio-oil.

5. **True-false (3 pts each):**

   ____**a. The electric power sector is the #2 greenhouse gas emitter, next only to the "direct fuel use" sector (which includes CO2 emitted transportation, industrial process heat, space heating, and cooking fueled by petroleum, natural gas, or coal).**

   ____**b. The top five SO_2 allowance supplier and acquirer power plants in 2002 are representative of the majority of transfers in that they are reallocations, intra-utility, or intraholding company movements of allowances.**

   ____**c. The US Environmental Protection Agency's defined "criteria pollutants" covers almost all harmful pollutants that US industry emits.**

   ____**d. The objective of the Midwest ISO's long-term planning analysis is to determine the transmission plan which will provide the most options to create value under whatever future supply and demand resource state exists**

   ____**e. Steam-turbine power plants utilize the Brayton thermodynamic cycle, whereas combustion turbines utilize the Rankine thermodynamic cycle.**

   ____**f. Among renewable resources, today, more electric energy is produced from wind energy in the United States than any other fuel.**

a. T
b. T
c. F
d. T
e. F
f. F
g. T

____g. Based on existing estimates, if all available US 5-50 mile off-shore wind capacity were developed, and all US 10000-30000 foot enhanced geothermal systems were developed, this capacity would meet or exceed the current US peak load of 1000GW.