EE 559 Homework #2
Due Monday September 19, 2016

1. A 4-pole, 2.3 MW, 690 V, 50 Hz squirrel cage induction generator is used in a fixed-speed wind energy conversion system in Denmark (where the grid frequency is 50 Hz). Generator parameters are given below:

   R_1=0.001102 Ω X'_2=0.0204 Ω
   X_1=0.0204 Ω R'_2=0.001497 Ω
   R_C=∞ Ω
   X_m=0.6706 Ω

   At a given wind speed, the machine operates at the rated speed of 1512 rpm. Neglecting windage and friction losses, determine the following:
   a. The slip, and the rotor speed in mechanical rad/sec and in electrical rad/sec;
   b. The stator and rotor currents;
   c. The developed power and torque;
   d. The stator and rotor winding losses;
   e. The generator efficiency and power factor.

Solution:

a. The slip is given by:

$$s = \frac{n_s - n_r}{n_s} = \frac{1500 - 1512}{1500} = -0.008$$

The rotor speed in mechanical rad/sec is given by:

$$\Omega_m = n_m \frac{2\pi}{60} = 1512 \frac{2\pi}{60} = 158.336 \text{rad/sec}$$

With 4 poles, the number of pole pairs is p=2. Therefore, the rotor speed in electrical rad/sec is given by

$$\omega_m = \Omega_m \times p = 158.336 \text{rad/sec} \times 2 = 316.67 \text{rad/sec}$$

b. The 690 V is a line-to-line voltage. Therefore, we need to compute line-to-neutral voltage according to

$$V_1 = \frac{690}{\sqrt{3}} = 398.37 \text{volts}$$

We will assume the applied voltage is the reference, therefore the applied voltage phasor is given by

$$\overline{V}_1 = 398.37\angle 0° \text{volts}$$

The equivalent impedance is given by

$$Z_{gen} = R_1 + jX_1 + \left(R_C // jX_m\right) // \left(R'_2 + jX'_2 + R_{eq}\right)$$

However, R_C is an open circuit, therefore

$$Z_{gen} = R_1 + jX_1 + \left(jX_m\right) // \left(R'_2 + jX'_2 + R_{eq}\right)$$

We can compute R_{eq} as

$$R_{eq} = R'_2 \frac{1-s}{s} = 0.001497 \frac{1+.008}{-.008} = -0.188622$$

Therefore

$$Z_{gen} = 0.001102 + j0.0204 + \frac{(j0.6706)(0.001497 + j0.0204 - 0.188622)}{j0.6706 + 0.001497 + j0.0204 - 0.188622}$$

$$= 0.001102 + j0.0204 + \frac{(j0.6706)(-0.1871 + j0.0204)}{-0.1871 + j0.692497} = -0.1628 + j0.0844$$

$$= 0.1834\angle 152.6°$$

The stator current is then given by

$$\bar{I}_1 = \frac{\bar{V}_1}{Z_{gen}} = \frac{398.37\angle 0°}{0.1834\angle 152.6°} = 2172\angle -152.6°$$

The rotor current is then computed by current division:

$$\bar{I}'_2 = \bar{I}_1 \frac{jX_m}{jX_m + R'_2 + jX'_2 + R_{eq}}$$

$$= 2172\angle -152.6° \frac{j0.6706}{j0.6706 + 0.001497 + j0.0204 - 0.188622}$$

$$= (2172\angle -152.6°)(0.9042 - j0.2449)$$

$$= (2172\angle -152.6°)(0.9368)\angle -15.15°) = 2035\angle -167.65°$$

c. The developed power is given by:

$$P_D = 3|\bar{I}_2|^2 R_{eq} = 3(2035)^2(-0.188622) = -2.343\text{MW}$$

The torque is given by

$$T_D = P_D / \Omega_m = (-2.343E6)/158.336 = 14800\text{Nm}$$

d. The stator winding loss is given by:

$$P_{Loss,s} = 3|\bar{I}_1|^2 R_1 = 3(2172)^2(0.00102) = 15.60\text{kw}$$

The rotor winding loss is given by:

$$P_{Loss,r} = 3|\bar{I}'_2|^2 R'_2 = 3(2035)^2(0.001497) = 18.60\text{kw}$$

e. The generator efficiency can be computed as follows:

$$\eta = \frac{P_{out}}{P_{in}} = \frac{P_s}{|P_D|} = \frac{|P_D| - (P_{Loss,s} + P_{Loss,r})}{|P_D|} = \frac{2343 - (15.6 + 18.6)}{2343} = 0.9854\%$$

Observe that the induction generator is highly efficient!

The generator power factor depends on the power factor angle, which is the angle given by

$$\theta = \theta_v - \theta_i$$

i.e., it is the angle by which the applied stator voltage leads the stator current. Since from part (b), we have

$$\bar{V}_1 = 398.37\angle 0°\text{volts}$$

$$\bar{I}_1 = 2172\angle -152.6°$$

then the power factor angle is given by

$$\theta = \theta_v - \theta_i = 0 - (-152.6) = 152.6°$$

Then the power factor is given by

$pf = \cos\theta = \cos 152.6° = -0.888$ (minus sign indicates “power in” is negative).

This is a lagging power factor (consumes reactive power) because the current phasor is lagging the voltage phasor.