EE 459/559, HW#4

Consider the same 1.5 MW DFIG analyzed under unity power factor (data is repeated on the next slide). Once again, assume the generator operates with a maximum power point tracking (MPPT) system so that its mechanical torque T_{em} is proportional to the square of the rotor speed.

(1) Assume the stator power factor is 0.95 leading. For each of the following speeds: 1750, 1650, 1500, 1350, and 1200 rpm, compute:

- **Slip**
- **T_{em} (kN-m)**
- **$\underline{I}_s$ (use exact expression, i.e., with R_s)**
- **$\underline{V}_r$ (volts)**
- **$\underline{I}_r$ (amps)**
- **R_{eq} (ohms)**
- **X_{eq} (ohms)**

(2) Repeat (1) except assume the stator power factor is 0.95 lagging.

(3) Repeat (1) except use approximate expression to obtain $\underline{I}_s$.

Generator Type	DFIG, 1.5 MW, 690 V, 50 Hz	
Rated Mechanical Power	1.5 MW	1.0 pu
Rated Stator Line-to-line Voltage	690 V (rms)	
Rated Stator Phase Voltage	398.4 V (rms)	1.0 pu
Rated Rotor Phase Voltage	67.97 V (rms)	0.1706 pu
Rated Stator Current	1068.2 A (rms)	0.8511 pu
Rated Rotor Current	1125.6 A (rms)	0.8968 pu
Rated Stator Frequency	50 Hz	1.0 pu
Rated Rotor Speed	1750 rpm	1.0 pu
Nominal Rotor Speed Range	1200–1750 rpm	0.686–1.0 pu
Rated Slip	−0.1667	
Number of Pole Pairs	2	
Rated Mechanical Torque	8.185 kN·m	1.0 pu
Stator Winding Resistance, R_s	2.65 mΩ	0.0084 pu
Rotor Winding Resistance, R_r	2.63 mΩ	0.0083 pu
Stator Leakage Inductance, L_{ls}	0.1687 mH	0.167 pu
Rotor Leakage Inductance, L_{lr}	0.1337 mH	0.1323 pu
Magnetizing Inductance, L_m	5.4749 mH	5.419 pu
Base Current, $I_B = 1.5\ \text{MW}/(\sqrt{3} \times 690\ \text{V})$	1255.1 A (rms)	1.0 pu
Base Flux Linkage, Λ_B	1.2681 Wb (rms)	1.0 pu
Base Impedance, Z_B	0.3174 Ω	1.0 pu
Base Inductance, L_B	1.0103 mH	1.0 pu
Base Capacitance, C_B	10028.7 μF	1.0 pu

Solution:
Note that:

$$n_s = \frac{60\, f_s}{p}\ \text{rpm} = 60*50/2 = 1500\text{rpm}$$

So the synchronous speed is 1500rpm.

1. Assume 0.95 leading. This means reactive power is being supplied to the grid. We want to make the below calculations for n_m= 1750, 1650, 1500, 1350, and 1200 rpm.

The equations to use for each of the calculations are provided below:

- **Slip**

$$slip = s = \frac{n_s - n_m}{n_s};$$

- **T_{em} (kN-m)**

$$T_{em} = -8185.1\left(\frac{n_m}{1750}\right)^2;$$

- **$\underline{I}_s$ (use exact expression, i.e., with R_s)**

$$I_s = \frac{V_s \cos\phi \pm \sqrt{(V_s\cos\phi)^2 - \frac{4R_s\omega_s}{3p}T_{em}}}{2R_s}$$

Then, assuming that $\underline{V}_s$ is the reference (and has angle of 0 degrees), the phasor I_s is given by

$$\underline{I}_s = I_s\angle 180 - \cos^{-1}(0.95) = I_s\angle 161.8°$$

- **$\underline{I}_r$ (amps)**

$$\underline{V}_m = \underline{V}_s - \underline{I}_s\left(R_s + j\omega_s L_{\sigma s}\right)$$

$$\underline{I}_m = \frac{\underline{V}_m}{j\omega_s L_m}$$

$$\underline{I}_r = \underline{I}_m - \underline{I}_s = \frac{\underline{V}_s - \underline{I}_s\left(R_s + j\omega_s L_{\sigma s}\right)}{j\omega_s L_m} - \underline{I}_s$$

- **$\underline{V}_r$ (volts)**

$$\underline{V}_r / s = \underline{V}_m + \underline{I}_r\left(\frac{R_r}{s} + j\omega_s L_{\sigma r}\right)$$

$$= \underline{V}_s - \underline{I}_s(R_s + j\omega_s L_{\sigma s}) + \underline{I}_r\left(\frac{R_r}{s} + j\omega_s L_{\sigma r}\right)$$

- **Z_{eq} (ohms)**

$$Z_{eq} \equiv R_{eq} + js\omega_s L_{eq} = \frac{\underline{V}_r}{-\underline{I}_r} = \left(\frac{s\underline{V}_m + \underline{I}_r\left(R_r + js\omega_s L_{\sigma r}\right)}{\underline{I}_r}\right)$$

Observe here that the equation to obtain Z_{eq} has negative $-I_r$ on the denominator. This is as it should be, given our directionality of $\underline{I}_r$.

- **R_{eq} (ohms)**
  ➔R_{eq} is the real part of Z_{eq}
- **X_{eq} (ohms)**
  ➔X_{eq} is the imaginary part of Z_{eq}

Applying these expressions (see matlab code at end of this document), we obtain the following for the 0.95 leading condition:

n_m	1200	1350	1500	1650	1750
s (slip)	0.2	0.1	0	-0.1	-0.1667
T_{em} (ntn-m)	-3848.7	-4871.0	-6013.5	-7276.4	-8185.1
$\underline{I}_s$ (amps)	530.5∟161.8°	670.8∟161.8°	827.2∟161.8°	999.7∟161.8°	1123.6∟161.8°
$\underline{I}_r$ (amps)	657.4∟-37.8°	795.1∟-34.3°	951.1∟-31.6°	1124.9∟-29.6°	1250.4∟-28.4°
$\underline{V}_r$ (volts)	86.9∟5.65°	45.0∟6.25°	2.5∟-31.6°	42.81∟-165.8°	73.6∟-165.4°
R_{eq} (ohms)	-0.0959	-0.0430	-0.0026	0.0275	0.0431
X_{eq} (ohms)	-0.0909	-0.0368	0	0.0263	0.0402

(2) Repeat (1) except assume the stator power factor is 0.95 lagging.

- **Slip**

$$slip = s = \frac{n_s - n_m}{n_s};$$

- **T_{em} (kN-m)**

$$T_{em} = -8185.1\left(\frac{n_m}{1750}\right)^2;$$

- **$\underline{I}_s$ (use exact expression, i.e., with R_s)**

$$I_s = \frac{V_s \cos\phi \pm \sqrt{(V_s \cos\phi)^2 - \frac{4R_s\omega_s}{3p}T_{em}}}{2R_s}$$

Then, assuming that $\underline{V}_s$ is the reference (and has angle of 0 degrees), the phasor I_s is given by

$$\underline{I}_s = I_s\angle -180 + \cos^{-1}(0.95) = I_s\angle -161.8°$$

- **$\underline{I}_r$ (amps)**

$$\underline{V}_m = \underline{V}_s - \underline{I}_s(R_s + j\omega_s L_{\sigma s})$$

$$\underline{I}_m = \frac{\underline{V}_m}{j\omega_s L_m}$$

$$\underline{I}_r = \underline{I}_m - \underline{I}_s = \frac{\underline{V}_s - \underline{I}_s(R_s + j\omega_s L_{\sigma s})}{j\omega_s L_m} - \underline{I}_s$$

- **$\underline{V}_r$ (volts)**

$$\underline{V}_r / s = \underline{V}_m + \underline{I}_r\left(\frac{R_r}{s} + j\omega_s L_{\sigma r}\right)$$

$$= \underline{V}_s - \underline{I}_s(R_s + j\omega_s L_{\sigma s}) + \underline{I}_r\left(\frac{R_r}{s} + j\omega_s L_{\sigma r}\right)$$

- **Z_{eq} (ohms)**

$$Z_{eq} \equiv R_{eq} + js\omega_s L_{eq} = \frac{\underline{V}_r}{-\underline{I}_r} = \left(\frac{s\underline{V}_m + \underline{I}_r(R_r + js\omega_s L_{\sigma r})}{\underline{I}_r}\right)$$

- **R_{eq} (ohms)**

  ➔R_{eq} is the real part of Z_{eq}
- **X_{eq} (ohms)**

  ➔X_{eq} is the imaginary part of Z_{eq}

Applying these expressions (see matlab code at end of this document), we obtain the following for the 0.95 lagging condition:

n_m	1200	1350	1500	1650	1750
s (slip)	0.2	0.1	0	-0.1	-0.1667
T_{em} (ntn-m)	-3848.7	-4871.0	-6013.5	-7276.4	-8185.1
$\underline{I}_s$ (amps)	530.5∟-161.8°	670.8∟-161.8°	827.2∟-161.8°	999.7∟-161.8°	1123.6∟-161.8°
$\underline{I}_r$ (amps)	523.4∟-6.8°	657.4∟-1.4°	811.1∟2.4°	983.5∟5.2°	1108.3∟6.7°
$\underline{V}_r$ (volts)	80.6∟6.9°	41.2∟8.6°	2.1∟2.4°	36.6∟-165.8°	62.3∟-164.2°
R_{eq} (ohms)	-0.1498	-0.0616	-0.0026	0.0368	0.0555
X_{eq} (ohms)	-0.0363	-0.0109	0	0.0059	0.0089

(3) Repeat (1) except use approximate expression to obtain $\underline{I}_s$.

- **Slip**

$$slip = s = \frac{n_s - n_m}{n_s};$$

- **T_{em} (kN-m)**

$$T_{em} = -8185.1\left(\frac{n_m}{1750}\right)^2;$$

- **$\underline{I}_s$ (use exact expression, i.e., with R_s)**

$$\Rightarrow I_s = \frac{\omega_s T_{em}}{3pV_s \cos\phi}$$

Then, assuming that $\underline{V}_s$ is the reference (and has angle of 0 degrees), the phasor I_s is given by

$$\underline{I}_s = I_s\angle 180 - \cos^{-1}(0.95) = I_s\angle 161.8°$$

- **$\underline{I}_r$ (amps)**

$$\underline{V}_m = \underline{V}_s - \underline{I}_s(R_s + j\omega_s L_{\sigma s})$$

$$\underline{I}_m = \frac{\underline{V}_m}{j\omega_s L_m}$$

$$\underline{I}_r = \underline{I}_m - \underline{I}_s = \frac{\underline{V}_s - \underline{I}_s(R_s + j\omega_s L_{\sigma s})}{j\omega_s L_m} - \underline{I}_s$$

- **$\underline{V}_r$ (volts)**

$$\underline{V}_r / s = \underline{V}_m + \underline{I}_r\left(\frac{R_r}{s} + j\omega_s L_{\sigma r}\right)$$

$$= \underline{V}_s - \underline{I}_s(R_s + j\omega_s L_{\sigma s}) + \underline{I}_r\left(\frac{R_r}{s} + j\omega_s L_{\sigma r}\right)$$

- **Z_{eq} (ohms)**

$$Z_{eq} \equiv R_{eq} + js\omega_s L_{eq} = \frac{\underline{V}_r}{-\underline{I}_r} = \left(\frac{s\underline{V}_m + \underline{I}_r(R_r + js\omega_s L_{\sigma r})}{\underline{I}_r}\right)$$

- **R_{eq} (ohms)**
  ➔R_{eq} is the real part of Z_{eq}
- **X_{eq} (ohms)**
  ➔X_{eq} is the imaginary part of Z_{eq}

Applying these expressions (see matlab code at end of this document), we obtain the following for the 0.95 leading condition. Comparing to the solutions obtained in (1), we observe that the approximate evaluation of stator current magnitude Is seems to incur little error.

n_m	1200	1350	1500	1650	1750
s (slip)	0.2	0.1	0	-0.1	-0.1667
T_{em} (ntn-m)	-3848.7	-4871.0	-6013.5	-7276.4	-8185.1
$\underline{I}_s$ (amps)	532.5∟161.8°	673.9∟161.8°	832.0∟161.8°	1006.7∟161.8°	1132.5∟161.8°
$\underline{I}_r$ (amps)	659.3∟-37.8°	798.3∟-34.3°	956.0∟-31.6°	1132.0∟-29.5°	1259.4∟-28.3°
$\underline{V}_r$ (volts)	86.9∟5.67°	45.0∟6.28°	2.5∟-31.6°	42.84∟-165.7°	73.7∟-165.3°
R_{eq} (ohms)	-0.0957	-0.0428	-0.0026	0.0273	0.0428
X_{eq} (ohms)	-0.0906	-0.0367	0	0.0262	0.0399

MATLAB CODE FOR CALCULATIONS

Note that:

- one must change “phi” to set the right power factor and to indicate whether it is leading or lagging;
- one must change “n” to set the speed;
- one must remove the “%” from the code to use the exact evaluation of Is (and then add the “%” to the code for the approximate relation).

```
Vsll=690;
Rs=0.00265;
Rr=0.00263;
Lls=0.0001687;
Llr=0.0001337;
Lm=0.0054749;
phi=161.8*pi/180;
pp=2;
Vs=Vsll/sqrt(3);
n=1200;
omega_m=pp*n*2*pi/60;
omega_s=2*pi*50;
Tem=-8185.1*(n/1750)^2;
s=(omega_s-omega_m)/omega_s
%EXACT EVALUATION OF Is
%Isroot=sqrt((Vs*cos(phi))^2-4*Rs*Tem*omega_s/(3*pp));
%Isplus=(Vs*cos(phi)+Isroot)/(2*Rs)
%Isminus=(Vs*cos(phi)-Isroot)/(2*Rs);
%APPROXIMATE EVALUATION OF Is
Isplus=omega_s*Tem/(3*pp*Vs*cos(phi))
Is=abs(Isplus)*(cos(phi)+i*sin(phi));
Vm=Vs-Is*(Rs+i*omega_s*Lls);
Im=Vm/(i*omega_s*Lm);
Ir=Im-Is;
Irmag=abs(Ir)
Irangle=atan2(imag(Ir), real(Ir))*180/pi
Vr=s*Vm+Ir*(Rr+i*s*omega_s*Llr);
Vrmag=abs(Vr)
Vrangle=atan2(imag(Vr), real(Vr))*180/pi
```

```
Zeq=Vr/(-1*Ir)
```