Homework

(A) Given $T_{em} = 3pL_m \,\mathrm{Im}\{\underline{I}_s \underline{I}_r^*\}$

(1) Derive $T_{em} = 3p \,\mathrm{Im}\{\underline{\lambda}_s^* \underline{I}_s\}$

Solution: Use $\underline{I}_r = \dfrac{\underline{\lambda}_s - L_s \underline{I}_s}{L_m}$

$$\Rightarrow T_{em} = 3pL_m \,\mathrm{Im}\left\{\underline{I}_s \left(\frac{\underline{\lambda}_s - L_s \underline{I}_s}{L_m}\right)^*\right\}$$

$$= 3p\frac{L_m}{L_m} \,\mathrm{Im}\{\underline{I}_s \underline{\lambda}_s^* - \underbrace{L_s \underline{I}_s \underline{I}_s^*}_{\text{real}}\} = 3p \,\mathrm{Im}\{\underline{I}_s \underline{\lambda}_s^*\}$$

QED

(2) Derive $T_{em} = 3p \,\mathrm{Im}\{\underline{\lambda}_r \underline{I}_r^*\}$

Solution: Use $\underline{I}_s = \dfrac{\underline{\lambda}_r - L_r \underline{I}_r}{L_m}$

$$\Rightarrow T_{em} = 3pL_m \,\mathrm{Im}\left\{\left(\frac{\underline{\lambda}_r - L_r \underline{I}_r}{L_m}\right)\underline{I}_r^*\right\}$$

$$= 3p\frac{L_m}{L_m} \,\mathrm{Im}\{\underline{\lambda}_r \underline{I}_r^* - \underbrace{L_r \underline{I}_r \underline{I}_r^*}_{\text{real}}\} = 3p \,\mathrm{Im}\{\underline{\lambda}_r \underline{I}_r^*\}$$

QED

(3) Derive $T_{em} = 3\dfrac{L_m}{\sigma L_r L_s} p \,\mathrm{Im}\{\underline{\lambda}_r^* \underline{\lambda}_s\}$

Solution: Use $\underline{I}_s = \dfrac{\underline{\lambda}_s - L_m \underline{I}_r}{L_s}$ $\quad \underline{I}_r = \dfrac{\underline{\lambda}_r - L_m \underline{I}_s}{L_r}$

$$\Rightarrow T_{em} = 3pL_m \,\mathrm{Im}\left\{\left(\frac{\underline{\lambda}_s - L_m \underline{I}_r}{L_s}\right)\left(\frac{\underline{\lambda}_r - L_m \underline{I}_s}{L_r}\right)^*\right\} = 3p\frac{L_m}{L_s L_r} \,\mathrm{Im}\left\{(\underline{\lambda}_s - L_m \underline{I}_r)(\underline{\lambda}_r - L_m \underline{I}_s)^*\right\}$$

$$= 3p\frac{L_m}{L_s L_r} \,\mathrm{Im}\{\underline{\lambda}_s \underline{\lambda}_r^* - L_m \underline{\lambda}_s \underline{I}_s^* - L_m \underline{I}_r \underline{\lambda}_r^* + L_m^2 \underline{I}_r \underline{I}_s^*\}$$

$$= 3p\frac{L_m}{L_s L_r}\left[\mathrm{Im}\{\underline{\lambda}_s \underline{\lambda}_r^*\} - \mathrm{Im}\{L_m \underline{\lambda}_s \underline{I}_s^*\} - \mathrm{Im}\{L_m \underline{I}_r \underline{\lambda}_r^*\} + \mathrm{Im}\{L_m^2 \underline{I}_r \underline{I}_s^*\}\right]$$

Recall
$$\left.\begin{aligned} T_{em} &= 3p \,\mathrm{Im}\{\underline{I}_s \underline{\lambda}_s^*\} \Rightarrow \mathrm{Im}\{\underline{\lambda}_s \underline{I}_s^*\} = \frac{-T_{em}}{3p} \\ T_{em} &= 3p \,\mathrm{Im}\{\underline{\lambda}_r \underline{I}_r^*\} \Rightarrow \mathrm{Im}\{\underline{I}_r \underline{\lambda}_r^*\} = \frac{-T_{em}}{3p} \\ T_{em} &= 3pL_m \,\mathrm{Im}\{\underline{I}_s \underline{I}_r^*\} \Rightarrow \mathrm{Im}\{\underline{I}_r \underline{I}_s^*\} = \frac{-T_{em}}{3pL_m} \end{aligned}\right\}$$ Substitute into previous equation

$$\Rightarrow T_{em} = 3p\frac{L_m}{L_s L_r}\left[\mathrm{Im}\{\underline{\lambda}_s \underline{\lambda}_r^*\} + L_m\frac{T_{em}}{3p} + L_m\frac{T_{em}}{3p} - \frac{L_m T_{em}}{3p}\right]$$

$$\Rightarrow T_{em} = 3p\frac{L_m}{L_s L_r}\,\mathrm{Im}\left(\underline{\lambda}_s\,\underline{\lambda}_r^*\right) + \frac{L_m^2}{L_s L_r}T_{em} + \cancel{\frac{L_m^2}{L_s L_r}T_{em}} - \cancel{\frac{L_m^2}{L_s L_r}T_{em}}$$

$$\Rightarrow T_{em}\left(1 - \frac{L_m^2}{L_s L_r}\right) = 3p\frac{L_m}{L_s L_r}\,\mathrm{Im}\left(\underline{\lambda}_s\,\underline{\lambda}_r^*\right)$$

$$\Rightarrow T_{em} = \frac{3pL_m}{\left(1 - \frac{L_m^2}{L_s L_r}\right)L_s L_r}\,\mathrm{Im}\left(\underline{\lambda}_s\,\underline{\lambda}_r^*\right)$$

Define $\sigma = 1 - \dfrac{L_m^2}{L_s L_r}$

$$\Rightarrow T_{em} = \frac{3pL_m}{\sigma L_s L_r}\,\mathrm{Im}\left(\underline{\lambda}_s\,\underline{\lambda}_r^*\right)$$

B) REACTIVE POWER
(very similar to work done on slides # 17-18 for P_s and P_r)

1. STATOR, Q_s

$$Q_s = 3\,\mathrm{Im}\{\underline{V}_s \underline{I}_s^*\} \quad (1)$$

But from slide #35, the voltage equation is

$$\underline{V}_s = \underline{I}_s R_s + j\omega_s\left[L_{\sigma s}\underline{I}_s + (\underline{I}_s + \underline{I}_r)L_m\right] \quad (2)$$

Substitution of (2) into (1) results in

$$Q_s = 3\,\mathrm{Im}\left\{\left(\underline{I}_s R_s + j\omega_s\left[L_{\sigma s}\underline{I}_s + (\underline{I}_s+\underline{I}_r)L_m\right]\right)\underline{I}_s^*\right\}$$

$$= 3\,\mathrm{Im}\left\{I_s^2 R_s + j\omega_s\left[L_{\sigma s} I_s^2 + I_s^2 L_m + \underline{I}_r \underline{I}_s^* L_m\right]\right\}$$

$$= 3\omega_s I_s^2 (L_{\sigma s} + L_m) + 3\omega_s L_m \mathrm{Im}\{j\underline{I}_r \underline{I}_s^*\}$$

We know $\mathrm{Im}\{j\underline{a}\} = \mathrm{Re}\{\underline{a}\}$, therefore

$$\boxed{Q_s = 3\omega_s I_s^2 (L_{\sigma s} + L_m) + 3\omega_s L_m \mathrm{Re}\{\underline{I}_r \underline{I}_s^*\}}$$

2. Rotor, Q_r

$$Q_r = 3\,\mathrm{Im}\{\underline{V}_r \underline{I}_r^*\} \quad (3)$$

But from slide #35, the voltage equation is

$$\underline{V}_r = \underline{I}_r R_r + js\omega_s\left[L_{\sigma r}\underline{I}_r + (\underline{I}_s + \underline{I}_r)L_m\right] \quad (4)$$

Substitution of (4) into (3) results in:

$$Q_r = 3\,\mathrm{Im}\left\{\left(\underline{I}_r R_r + js\omega_s\left(L_{\sigma r}\underline{I}_r + (\underline{I}_s+\underline{I}_r)L_m\right)\right)\underline{I}_r^*\right\}$$

$$= 3\,\mathrm{Im}\left\{I_r^2 R_r + j\omega_s s\left[L_{\sigma r} I_r^2 + \underline{I}_s \underline{I}_r^* L_m + I_r^2 L_m\right]\right\}$$

$$= 3\omega_s s\left(L_{\sigma r} I_r^2 + L_m I_r^2\right) + 3\omega_s s L_m \mathrm{Im}\{j\underline{I}_s \underline{I}_r^*\}$$

We know $\mathrm{Im}\{j\underline{a}\} = \mathrm{Re}\{\underline{a}\}$, therefore

$$\boxed{Q_r = 3\omega_s s (L_{\sigma r} + L_m) I_r^2 + 3\omega_s s L_m \mathrm{Re}\{\underline{I}_s \underline{I}_r^*\}}$$

NOTE: In the above expressions, we could substitute

$$L_{\sigma s} + L_m = L_s \quad (\text{for B-1})$$

$$L_{\sigma r} + L_m = L_r \quad (\text{for B-2})$$

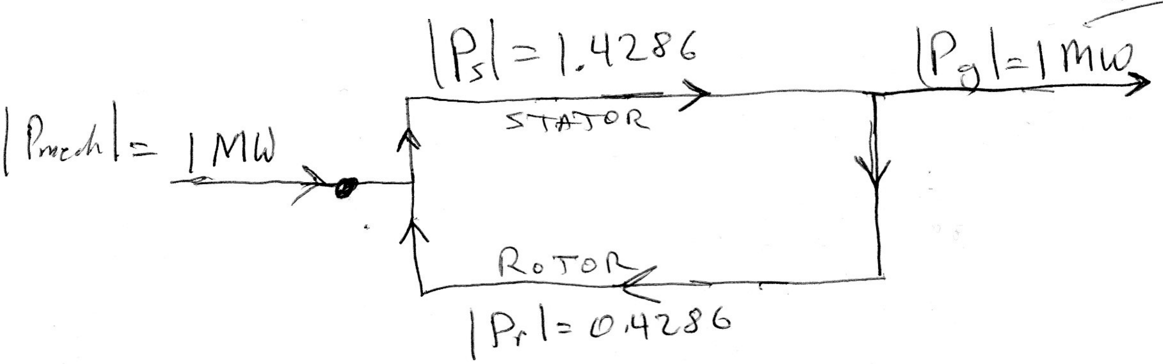
1. $P_{mech} = -1\,MW$, $s = +0.3$ (subsynchronous)

$$P_{airgap} = P_s = \frac{P_{mech}}{1-s} = \frac{-1}{1-0.3} = \frac{-1}{.7} = -1.4286$$

$$\Rightarrow P_{airgap} = -1.4286\,MW$$

$$P_{slip} = P_r = -sP_s = -0.3(-1.43) = 0.4286\,MW$$

$$\Rightarrow P_{SLIP} = 0.4286\,MW$$



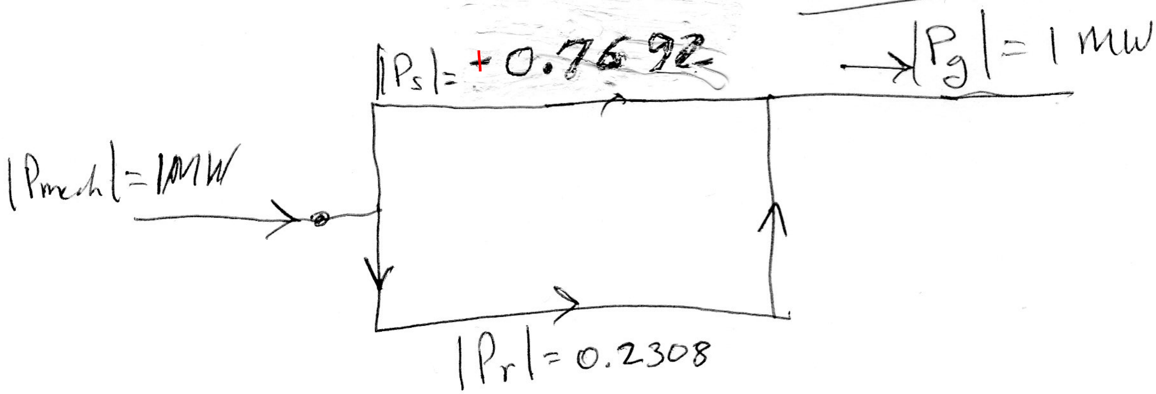
2. $P_{mech} = -1\,MW$, $s = -0.3$ (supersynchronous)

$$P_{airgap} = P_s = \frac{P_{mech}}{1-s} = \frac{-1}{1+.3} = -0.7692$$

$$\Rightarrow P_{airgap} = -0.7692\,MW$$

$$P_{SLIP} = P_r = -sP_s = -(-0.3)(-0.7692) = -0.2308\,MW$$

$$\Rightarrow P_{slip} = -0.2308\,MW$$



$L_{base} = \frac{\lambda_{base}}{I_{base}}$ $I_{base} = 1760$ Ampers

Part D.

$V_{base} = \frac{690}{\sqrt{3}} = 398.37$ volts

$\Rightarrow Z_{base} = \frac{398.37}{1760} = 0.2263\ \Omega$; $\Omega_s = \frac{1500\ rev \times 2\pi\ rad \times min}{min \times rev \times 60\ sec}$

$= 157.08$ rad/sec

$\lambda_{base} = \frac{V_{base}}{\omega_{base}} = \frac{398.37}{314.16}$ $\Rightarrow \omega_s = p\Omega_s = 2(157.08)$

$= 314.16$ rad/sec

$= 1.268$ weber

$\Rightarrow L_{base} = \frac{\lambda_{base}}{I_{base}} = \frac{1.268}{1760A} = .0007205$

$r_s = \frac{.0026\ \Omega}{.2263} = 0.0115$ pu

$l_{os} = \frac{0.000087}{0.0007205} = 0.1207$ pu

$l_m = \frac{}{0.0007205} = 3.47$ pu

$r_r = \frac{0.0029}{0.2263} = 0.0128$ pu

$l_{or} = \frac{.000087}{.0007205} = 0.1207$ pu