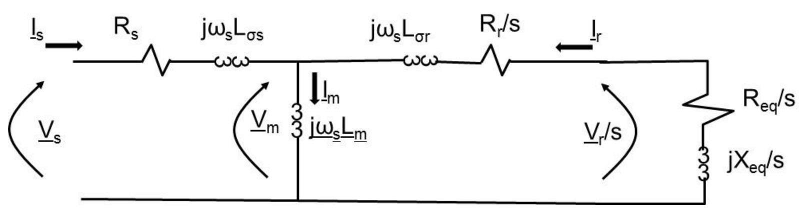
EE 559, Exam 2, Spring 2016, Dr. McCalley, 75 minutes allowed.
Closed Book, Closed Notes, Calculator Permitted, No Communication Devices

1. (36 pts) Consider a 1.5 MW, 690 v, 50 Hz, 1750 rpm DFIG wind energy system. The parameters of the generator are given in the table to the right. The generator operates with a maximum power point tracking (MPPT) system so that its mechanical torque T_{em} is proportional to the square of the rotor speed. The stator power factor is unity. For a rotor speed of 1750 rpm, determine:
   a. Slip
   b. T_{em} (kN-m)
   c. V_s (volts)
   d. I_s (amps)
   e. V_m (volts)
   f. I_m (amps)
   g. I_r (amps)
   h. V_r (volts)
   i. Z_{eq} (ohms)

Generator Type	DFIG, 1.5 MW, 690 V, 50 Hz	
Rated Mechanical Power	1.5 MW	1.0 pu
Rated Stator Line-to-line Voltage	690 V (rms)	
Rated Stator Phase Voltage	398.4 V (rms)	1.0 pu
Rated Rotor Phase Voltage	67.97 V (rms)	0.1706 pu
Rated Stator Current	1068.2 A (rms)	0.8511 pu
Rated Rotor Current	1125.6 A (rms)	0.8968 pu
Rated Stator Frequency	50 Hz	1.0 pu
Rated Rotor Speed	1750 rpm	1.0 pu
Nominal Rotor Speed Range	1200–1750 rpm	0.686–1.0 pu
Rated Slip	–0.1667	
Number of Pole Pairs	2	
Rated Mechanical Torque	8.185 kN·m	1.0 pu
Stator Winding Resistance, R_s	2.65 mΩ	0.0084 pu
Rotor Winding Resistance, R_r	2.63 mΩ	0.0083 pu
Stator Leakage Inductance, L_{ls}	0.1687 mH	0.167 pu
Rotor Leakage Inductance, L_{lr}	0.1337 mH	0.1323 pu
Magnetizing Inductance, L_m	5.4749 mH	5.419 pu
Base Current, $I_B = 1.5\text{ MW}/(\sqrt{3}\times 690\text{ V})$	1255.1 A (rms)	1.0 pu
Base Flux Linkage, Λ_B	1.2681 Wb (rms)	1.0 pu
Base Impedance, Z_B	0.3174 Ω	1.0 pu
Base Inductance, L_B	1.0103 mH	1.0 pu
Base Capacitance, C_B	10028.7 μF	1.0 pu

You are provided with the following relationship and circuit:

$$\underline{I}_s = \frac{V_s \pm \sqrt{V_s^2 - \frac{4R_s\omega_s}{3p}T_{em}}}{2R_s}$$



Solution:

a. Slip

$$\omega_s = 2\pi f = 2\pi(50) = 314.1592\text{rad/sec}$$

$$\Omega_m = \frac{1750 rev}{\min}\frac{2\pi rad}{rev}\frac{\min}{60\sec} = 183.26 rad/\sec$$

$$\omega_m = p\Omega_m = 2*183.26 = 366.52 rad/\sec$$

$$s = \frac{\omega_s - \omega_m}{\omega_s} = \frac{314.1592 - 366.52}{314.1592} = -0.1667$$

b. T_{em} (kN-m)

The "rated mechanical torque" in the table is for a speed of 1750rpm which is the speed given in this problem. So answer is 8.1851 kNm.

c. V_s (volts)

$$V_s = \frac{690}{\sqrt{3}} = 398.37$$

d. I_s (amps)

$$\underline{I}_s = \frac{V_s \pm \sqrt{V_s^2 - \frac{4R_s\omega_s}{3p}T_{em}}}{2R_s} = \frac{398.37 \pm \sqrt{398.37^2 - \frac{4(.00265)(314.1592)}{3(2)}(-8185.1)}}{2(.00265)}$$

$$= -1068.2 amps$$

e. V_m (volts)

$$\underline{V}_m = \underline{V}_s - \underline{I}_s\left(R_s + j\omega_s L_{\sigma s}\right) = 398.37 + 1068.2(.00265 + j(314.1592)(0.0001687))$$

$$= 401.2 + j56.61$$

f. I_m (amps)

$$\underline{I}_m = \frac{\underline{V}_m}{jX_m} = \frac{401.2 + j56.61}{j(.0054749)(314.1592)} = 32.91 - j233.2$$

g. I_r (amps)

$$\underline{I}_r = \underline{I}_m - \underline{I}_s = (32.91 - j233.2) - (-1068.2) = 1101.1 - j233.2$$

h. V_r (volts)

$$\underline{V}_r = s\underline{V}_m - \underline{I}_r\left(R_r + js\omega_s L_{\sigma r}\right) = -.1667(401.2 + j56.61) -$$

$$(1101.1 - j233.34)(0.00263 + j(-0.1667)(314.1592)(.0001337)$$

$$= -65.62 - j17.76$$

i. Z_{eq} (ohms)

$$Z_{eq} = \frac{\underline{V}_r}{-\underline{I}_r} = \frac{-65.62 - j17.76}{1101.1 - j233.2} = 0.0538 + j0.0275$$

2. (22 pts) We developed the below qd0 voltage equations for the double-fed induction generator.

$$\begin{bmatrix} v_{qs} \\ v_{ds} \\ v_{0s} \\ v_{qr} \\ v_{dr} \\ v_{0r} \end{bmatrix} = \begin{bmatrix} r_s & 0 & 0 & 0 & 0 & 0 \\ 0 & r_s & 0 & 0 & 0 & 0 \\ 0 & 0 & r_s & 0 & 0 & 0 \\ 0 & 0 & 0 & r_r & 0 & 0 \\ 0 & 0 & 0 & 0 & r_r & 0 \\ 0 & 0 & 0 & 0 & 0 & r_r \end{bmatrix} \begin{bmatrix} i_{qs} \\ i_{ds} \\ i_{0s} \\ i_{qr} \\ i_{dr} \\ i_{0r} \end{bmatrix} - \begin{bmatrix} 0 & -\omega & 0 & 0 & 0 & 0 \\ \omega & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -(\omega - \omega_m) & 0 \\ 0 & 0 & 0 & \omega - \omega_m & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \lambda_{qs} \\ \lambda_{ds} \\ \lambda_{0s} \\ \lambda_{qr} \\ \lambda_{dr} \\ \lambda_{0r} \end{bmatrix}$$

We also developed the following torque expression.

$$T_{em} = \frac{3}{2}\frac{L_m}{L_{\sigma r} + L_m} p\left(\lambda_{qs} i_{dr} - \lambda_{ds} i_{qr}\right)$$

Using the above relations together with the implications of stator-voltage-oriented-control (SVOC), show that T_{em} is a function of d-axis rotor current and d-axis stator voltage only. Clearly state how SVOC plays a role in this development. State any other assumptions that are necessary.

Solution:

Extracting the relations for vqs and vds, we have:

$$v_{qs} = r_s i_{qs} + \omega\lambda_{ds} \qquad v_{ds} = r_s i_{ds} - \omega\lambda_{qs}$$

Solving for λ_{ds} and λ_{qs}, respectively, we get

$$\lambda_{ds} = \frac{v_{qs} - r_s i_{qs}}{\omega} \qquad \lambda_{qs} = \frac{-(v_{ds} - r_s i_{ds})}{\omega}$$

Now substitute these relations for λ_{ds} and λ_{qs} into the torque relation:

$$T_{em} = \frac{3}{2}\frac{L_m}{L_{\sigma r} + L_m} p\left(\lambda_{qs} i_{dr} - \lambda_{ds} i_{qr}\right) = \frac{3}{2}\frac{L_m}{L_{\sigma r} + L_m} p\left(\frac{-(v_{ds} - r_s i_{ds})}{\omega} i_{dr} - \frac{v_{qs} - r_s i_{qs}}{\omega} i_{qr}\right)$$

$$T_{em} = \frac{3}{2\omega}\frac{L_m}{L_{\sigma r} + L_m} p\left(-(v_{ds} - r_s i_{ds}) i_{dr} - (v_{qs} - r_s i_{qs}) i_{qr}\right) = \frac{3}{2\omega}\frac{L_m}{L_{\sigma r} + L_m} p\left(-v_{ds} i_{dr} + r_s i_{ds} i_{dr} - v_{qs} i_{qr} + r_s i_{qs} i_{qr}\right)$$

Now use SVOC which implies that $v_{qs}=0$, and we obtain

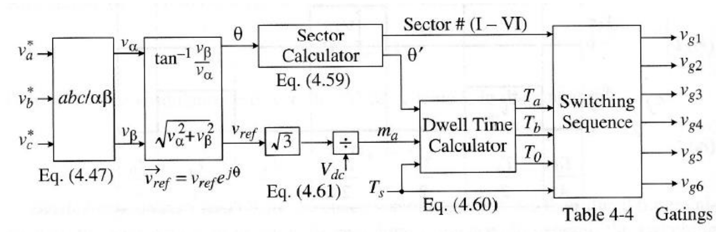
$$T_{em} = \frac{3}{2\omega}\frac{L_m}{L_{\sigma r} + L_m} p\left(-v_{ds} i_{dr} + r_s i_{ds} i_{dr} + r_s i_{qs} i_{qr}\right)$$

Assume that $r_s=0$, then

$$T_{em} = \frac{-3}{2\omega}\frac{L_m p v_{ds} i_{dr}}{L_{\sigma r} + L_m}$$

3. (24 pts) Consider a two-level voltage source inverter connected to a three-phase balanced RL load, and its DC voltage is 1220 volts. The inverter is modulated by space vector modulation and its simulation algorithm is given in the figure below. The switching frequency of the converter is 720Hz. At a given instance of time, the three-phase reference voltages v_a*, v_b*, and v_c*, are found to be 0 volts, 487.9 volts, and -487.9 volts, respectively. For this problem, you may need the matrix and the equations provided below.

$$\frac{2}{3}\begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \qquad T_a = \frac{\sqrt{3} v_{ref} T_S}{V_{DC}} \sin\left(\frac{\pi}{3} - \theta'\right) \qquad T_b = \frac{\sqrt{3} v_{ref} T_S}{V_{DC}} \sin\theta'$$



Determine:

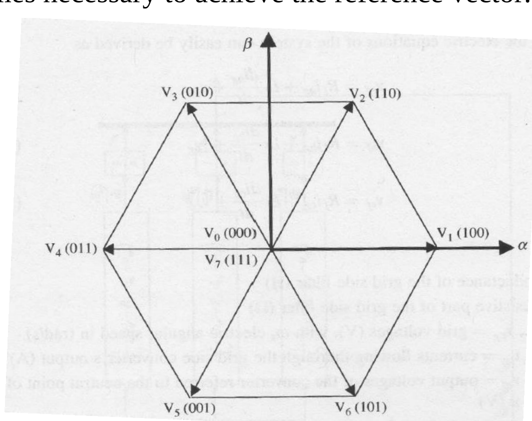
a. The α-β components of the three-phase reference voltages.

b. The specific switching sequence necessary to achieve the reference vector corresponding to your answer in (a). Here, you need to specify a sequence of particular states.

c. The 3 stationary vectors to be used in synthesizing the reference vector. Here, you need to identify appropriate vectors V_k in the space vector diagram shown below (no computation is required).

d. The dwell times necessary to achieve the reference vector.

Sector	Switching Sequence						
I	$\vec{V}_0$	$\vec{V}_1$	$\vec{V}_2$	$\vec{V}_0$	$\vec{V}_2$	$\vec{V}_1$	$\vec{V}_0$
	000	100	110	111	110	100	000
II	$\vec{V}_0$	$\vec{V}_3$	$\vec{V}_2$	$\vec{V}_0$	$\vec{V}_2$	$\vec{V}_3$	$\vec{V}_0$
	000	010	110	111	110	010	000
III	$\vec{V}_0$	$\vec{V}_3$	$\vec{V}_4$	$\vec{V}_0$	$\vec{V}_4$	$\vec{V}_3$	$\vec{V}_0$
	000	010	011	111	011	010	000
IV	$\vec{V}_0$	$\vec{V}_5$	$\vec{V}_4$	$\vec{V}_0$	$\vec{V}_4$	$\vec{V}_5$	$\vec{V}_0$
	000	001	011	111	011	001	000
V	$\vec{V}_0$	$\vec{V}_5$	$\vec{V}_6$	$\vec{V}_0$	$\vec{V}_6$	$\vec{V}_5$	$\vec{V}_0$
	000	001	101	111	101	001	000
VI	$\vec{V}_0$	$\vec{V}_1$	$\vec{V}_6$	$\vec{V}_0$	$\vec{V}_6$	$\vec{V}_1$	$\vec{V}_0$
	000	100	101	111	101	100	000



Solution:

a. The α-β components of the three-phase reference voltages.

$$\begin{bmatrix} v_\alpha \\ v_\beta \end{bmatrix} = \frac{2}{3}\begin{bmatrix} 1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \end{bmatrix}\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = \frac{2}{3}\begin{bmatrix} 1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \end{bmatrix}\begin{bmatrix} 0 \\ 487.9 \\ -487.9 \end{bmatrix} = \begin{bmatrix} 0 \\ 563.38 \end{bmatrix}$$

b. The angle of the reference voltage vector & its corresponding sector number is given by

$$\theta = \tan^{-1}\frac{v_\beta}{v_\alpha} = \tan^{-1}\frac{563.38}{0} = 90°$$

It will be in sector II, therefore the switching sequence is V_0, V_3, V_2, V_0, V_2, V_1, V_0

c. The three stationary vectors to be used in synthesizing the reference vector are V_2, V_3, and V_0.

d. This is in sector II, therefore:

$$\theta' = \theta - (k-1)\frac{\pi}{3} \qquad \text{for } 0 \le \theta' < \frac{\pi}{3}$$

$$\theta' = \theta - (2-1)\frac{\pi}{3} = \theta - \frac{\pi}{3}$$

So the reference vector, with θ=90deg, will have θ'=45deg.

$$f_{sw} = 720Hz \Rightarrow T_s = 1/720 = 0.001389$$

$$T_a = \frac{\sqrt{3}v_{ref}T_S}{V_{DC}}\sin\left(\frac{\pi}{3} - \theta'\right) =$$

$$\frac{\sqrt{3}(563.38)(0.001389)}{1220}\sin(60-45)$$

$$= 0.00028754\,\text{sec}$$

$$T_b = \frac{\sqrt{3}v_{ref}T_S}{V_{DC}}\sin\theta' = \frac{\sqrt{3}*563.38*0.0014}{1220}\sin(45) = 0.0007854$$

$$T_0 = T_S - T_a - T_b = 0.001389 - .00028754 - .0007854 = 0.00031606$$

4. (18 pts) A 60 Hz power system has 1000 MW of conventional generation. Assume:
   - The inertia constant for all conventional generation is H=6 seconds (on the base of the generator);
   - A trip of a 100 MVA unit occurs with the unit producing 100 MW.
   - The frequency nadir following generation trips occurs 1.5 second after the trip.

   a. Estimate the nadir assuming all generation is conventional.

   b. Estimate the nadir assuming 200 MW of conventional units are displaced by 500 MW of double-fed induction generators.

   c. Give 3 "nested" reasons why it is important to avoid low frequency deviations.

<u>**Solution**</u>:

a. Total capacity of conventional generation following the trip is 1000MW-100MW=900MW. Using a 100 MVA base, the loss of the 100 MW results in
ΔP_L=100/100=1 pu

$$\sum_{i=1}^{n} H_i = \frac{900*6}{100} = 54$$

$$m_f = \frac{d\overline{\Delta f}}{dt} = \frac{-\Delta P_L f_{\mathrm{Re}}}{2\sum_{i=1}^{n} H_i} \equiv \frac{-1(60)}{2*54} = -0.5556 Hz/\sec$$

With t1=1.5 seconds, then Δf=-0.5556*1.5=-0.8334 Hz, so that the nadir is 60-0.8334=59.17Hz.

b. Total capacity of conventional generation following the trip is 800MW-100MW=700MW. Using a 100 MVA base, the loss of the 100 MW results in

ΔP_L=100/100=1 pu

$$\sum_{i=1}^{n} H_i = \frac{700*6}{100} = 42$$

$$m_f = \frac{d\overline{\Delta f}}{dt} = \frac{-\Delta P_L f_{\mathrm{Re}}}{2\sum_{i=1}^{n} H_i} \equiv \frac{-1(60)}{2*42} = -0.7143 Hz/\sec$$

With t1=1.5 seconds, then Δf=-0.7143*1.5=-1.0714 Hz, so that the nadir is 60-0.7143=58.93Hz.

c. Turbine failure or loss of life; turbine trip via under- or over-frequency relays; under-frequency load shedding.