Representation of Saturation in Stability Studies

Kundur writes (pg 110) that

"A rigorous treatment of synchronous machine performance including saturation effects is a futile exercise. Any practical method of accounting for saturation effects must be based on semi-heuristic reasoning and judiciously chosen approximations, with due consideration to simplicity of model structure, data availability, and accuracy of results."

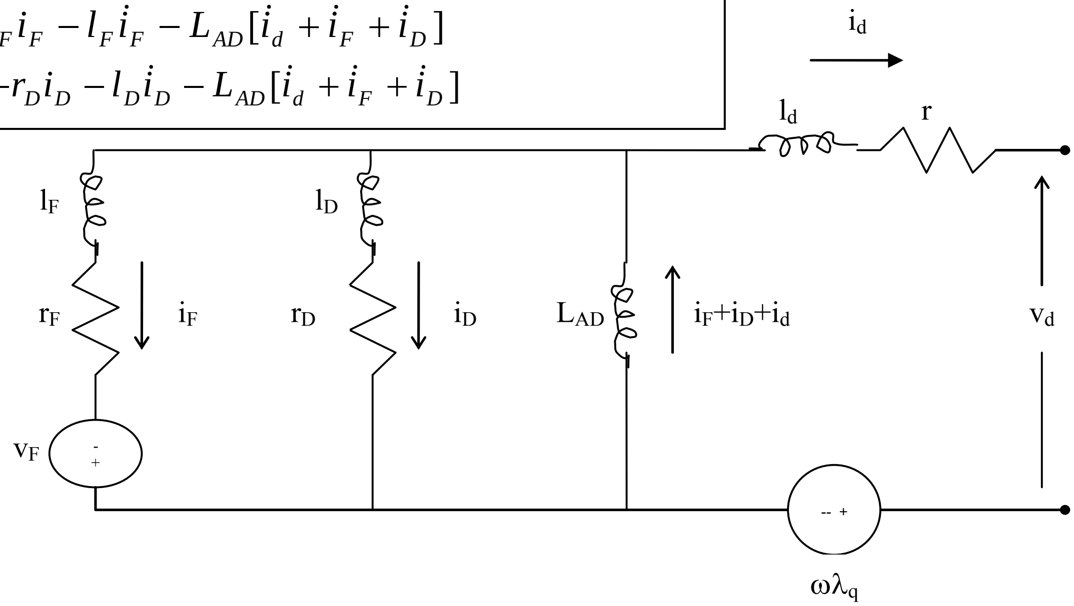
Some assumptions (see Kundur pg. 112-113) :

1. Leakage inductances are independent of saturation since they exist in the air for most of their path. Therefore we may confine our analysis of saturation to the mutual inductances, represented by L_{AD} and $L_{AQ.}$
2. The leakage fluxes do no contribute to the iron saturation. This is reasonable because these fluxes are small and their paths coincide with that of the main flux for only a small part of its path. So we may determine saturation of the inductances as a function of λ_{AD} and λ_{AQ}.
3. The saturation relationship between the resultant air-gap flux and the mmf under loaded conditions is the same as under no-load conditions. This allows the saturation characteristics to be represented by the open-circuit saturation curve, which is usually the only saturation data readily available.

An additional assumption that is sometimes made is that L_{AQ} does not saturate, simply because the quadrature axis flux is usually quite small in comparison to the direct axis flux due to the effect of the main field winding. This assumption is quite good for salient pole machines but not so good for round-rotor machines.

Recall, from our equivalent circuit (shown below), that
$\lambda_{AD}=(i_d+i_F+i_D)L_{AD}$.

$$v_d = -ri_d - l_d\dot{i}_d - L_{AD}[\dot{i}_d + \dot{i}_F + \dot{i}_D] - \omega\lambda_q$$
$$-v_F = -r_F i_F - l_F \dot{i}_F - L_{AD}[\dot{i}_d + \dot{i}_F + \dot{i}_D]$$
$$v_D = 0 = -r_D i_D - l_D \dot{i}_D - L_{AD}[\dot{i}_d + \dot{i}_F + \dot{i}_D]$$



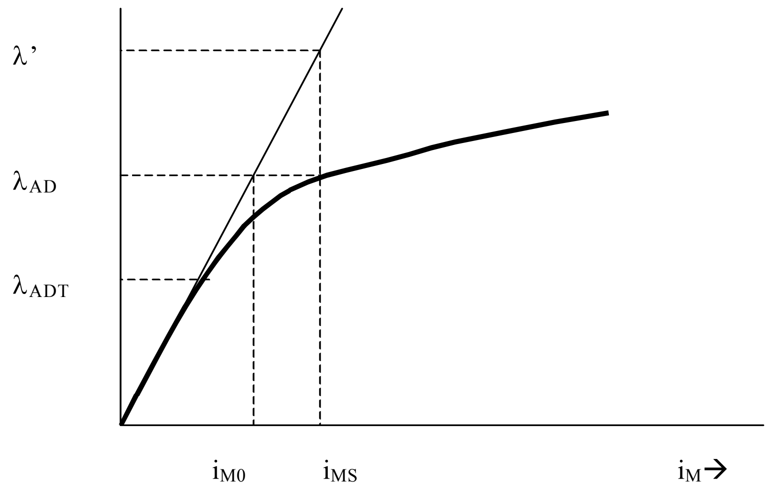
Direct-axis equivalent circuit:
The above is the same as Fig. 4.5 in your text

Define the following terms:

- Magnetization current: $i_M=(i_d+i_F+i_D)$ ➔ $\lambda_{AD}=L_{AD}\, i_M$
- Maximum per-unit flux linkage without saturation: λ_{ADT}
- i_{M0}: current that would produce λ_{AD} if no saturation effects
- i_{MS}: current that produces λ_{AD} with saturation effects
- λ': Flux linkage resulting from i_{MS} if no saturation effects

Define L_{AD0} as the inductance corresponding to the air-gap line. It is the inductance when i_M is small, i.e., it is the non-saturated inductance.

The magnetization curve appears as in the following figure:



From the figure, we can write that: $\frac{i_{M0}}{\lambda_{AD}} = \frac{i_{MS}}{\lambda'} \Rightarrow \lambda_{AD} = \frac{i_{M0}}{i_{MS}}\lambda'$

But from the air-gap line equation, $\lambda' = L_{AD0} i_{MS}$, and substitution of this relation into the previous one yields:

$$\lambda_{AD} = \frac{i_{M0}}{i_{MS}}\lambda' = \frac{i_{M0}}{i_{MS}} L_{AD0} i_{MS}$$

Define $K_S = i_{M0}/i_{MS}$, where $0 < K_S \leq 1$, then

$$\lambda_{AD} = \frac{i_{M0}}{i_{MS}}\lambda' = K_S L_{AD0} i_{MS}$$

So K_S is a factor that we will use to account for the difference between the magnetization curve and the air-gap line.

But how do we determine K_S?

Observe:

$$K_S = \frac{i_{M0}}{i_{MS}} = \frac{i_{M0}}{i_{M0} + \Delta i_M}$$

where $\Delta i_M = i_{MS} - i_{M0}$.

So evaluation of K_S requires evaluation of Δi_M, and our problem is now to get Δi_M.

Note from Fig. 1 below that Δi_M grows exponentially larger with $\lambda_{AD} - \lambda_{ADT}$.

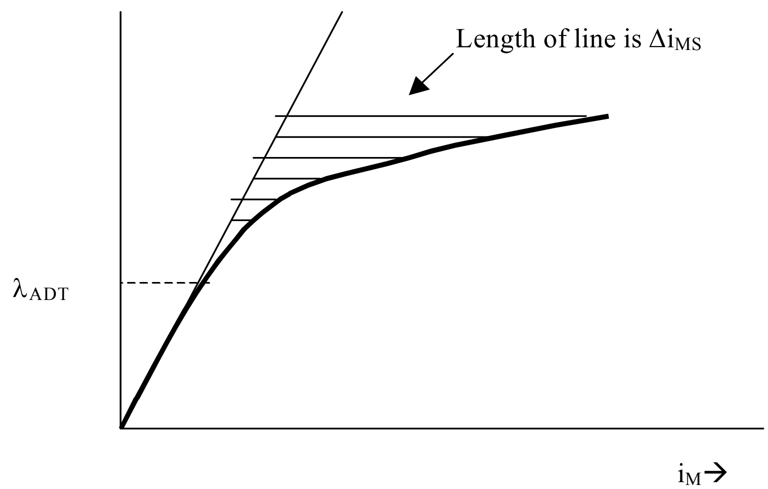


Fig. 1

So we reason that a good approximation to Δi_M is given by

$$\Delta i_M = A_S e^{B_S(\lambda_{AD} - \lambda_{ADT})}$$

So that

$$K_S = \frac{i_{M0}}{i_{M0} + A_S e^{B_S(\lambda_{AD} - \lambda_{ADT})}}$$

And it is clear from the above that K_S is a function of λ_{AD}, i.e.,

$$K_S = K_S(\lambda_{AD})$$

So that the mutual flux is given by

$$\lambda_{AD} = K_S(\lambda_{AD}) L_{AD0} i_{MS}$$

So how do we use it?

Assume that we have values for λ_d, λ_F, λ_D, λ_q, λ_Q, and λ_G. Then the steps for including saturation are:

1. Use the auxiliary equations to obtain the unsaturated values of λ_{AD} and λ_{AQ}:

$$\lambda_{AD} = \frac{L_{MD}}{l_d}\lambda_d + \frac{L_{MD}}{l_F}\lambda_F + \frac{L_{MD}}{l_D}\lambda_D \qquad \lambda_{AQ} = \frac{L_{MQ}}{l_q}\lambda_q + \frac{L_{MQ}}{l_Q}\lambda_Q + \frac{L_{MQ}}{l_G}\lambda_G$$

where

$$\frac{1}{L_{MD}} = \left[\frac{1}{L_{AD}} + \frac{1}{l_d} + \frac{1}{l_F} + \frac{1}{l_D}\right] \qquad \frac{1}{L_{MQ}} = \left[\frac{1}{L_{AQ}} + \frac{1}{l_q} + \frac{1}{l_Q} + \frac{1}{l_G}\right]$$

2. For a salient pole machine, let $\lambda=\lambda_{AD}$.
   For a round-rotor machine, let $\lambda=\text{sqrt}\{\lambda_{AD}^2+\lambda_{AQ}^2\}$
3. Check if $\lambda>\lambda_{ADT}$. If not, use the unsaturated values. If so, proceed to step 4.
4. Get currents from 4.124', shown below:

$$\begin{bmatrix} i_d \\ i_F \\ i_D \\ i_q \\ i_Q \\ i_G \end{bmatrix} = \begin{bmatrix} \frac{1}{l_d} & 0 & 0 & -\frac{1}{l_d} & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{l_F} & 0 & -\frac{1}{l_d} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{l_D} & -\frac{1}{l_D} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{l_q} & 0 & 0 & -\frac{1}{l_q} \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{l_Q} & 0 & -\frac{1}{l_Q} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{l_G} & -\frac{1}{l_G} \end{bmatrix} \begin{bmatrix} \lambda_d \\ \lambda_F \\ \lambda_D \\ \lambda_{AD} \\ \lambda_q \\ \lambda_Q \\ \lambda_G \\ \lambda_{AQ} \end{bmatrix} \qquad (4.124')$$

5. Compute the magnetizing current as

$$i_{M0} = i_d + i_F + i_D$$

6. Compute K_S according to:

$$\Delta i_M = A_S e^{B_S(\lambda - \lambda_{ADT})}$$

$$i_{MS} = i_{M0} + \Delta i_M$$

$$K_S = \frac{i_{M0}}{i_{MS}}$$

7. Update λ_{AD} and λ_{AQ} according to
   a. Replace L_{AD} with $L_{AD} \leftarrow K_S L_{AD}$, and then compute:

$$\frac{1}{L_{MD}} = \left[\frac{1}{L_{AD}} + \frac{1}{l_d} + \frac{1}{l_F} + \frac{1}{l_D}\right] \Rightarrow \lambda_{AD} = \frac{L_{MD}}{l_d}\lambda_d + \frac{L_{MD}}{l_F}\lambda_F + \frac{L_{MD}}{l_D}\lambda_D$$

   b. If salient pole, then $\lambda_{AQ}=\lambda_{AQ}$ (i.e., no change), but if round-rotor, then replace L_{AQ} with $L_{AQ} \leftarrow K_S L_{AQ}$, and then compute:

$$\frac{1}{L_{MQ}} = \left[\frac{1}{L_{AQ}} + \frac{1}{l_q} + \frac{1}{l_Q} + \frac{1}{l_G}\right] \Rightarrow \lambda_{AQ} = \frac{L_{MQ}}{l_q}\lambda_q + \frac{L_{MQ}}{l_Q}\lambda_Q + \frac{L_{MQ}}{l_G}\lambda_G$$

And then you can use the updated values of λ_{AD} and λ_{AQ} in the following to perform a numerical integration and get the next time step…

$$\dot{\lambda}_d = -\frac{r}{l_d}\lambda_d + \frac{r}{l_d}\lambda_{AD} - \omega\lambda_q - v_d \qquad (4.126)$$

$$\dot{\lambda}_F = -\frac{r_F}{l_F}\lambda_F + \frac{r_F}{l_F}\lambda_{AD} + v_F \qquad (4.128)$$

$$\dot{\lambda}_D = -\frac{r_D}{l_D}\lambda_D + \frac{r_D}{l_D}\lambda_{AD} \qquad (4.129)$$

$$\dot{\lambda}_q = -\frac{r}{l_q}\lambda_q + \frac{r}{l_q}\lambda_{AQ} + \omega\lambda_d - v_q \qquad (4.130)$$

$$\dot{\lambda}_Q = -\frac{r_Q}{l_Q}\lambda_Q + \frac{r_Q}{l_Q}\lambda_{AQ} \qquad (4.131)$$

$$\dot{\lambda}_G = -\frac{r_G}{l_G}\lambda_G + \frac{r_G}{l_G}\lambda_{AQ} \tag{4.131'}$$

$$\dot{\omega} = \frac{T_m}{\tau_j} + \left[\frac{\lambda_{AQ}}{l_q 3\tau_j}\lambda_d - \frac{\lambda_{AD}}{l_d 3\tau_j}\lambda_q\right] + \left[\frac{-D}{\tau_j}\right]\omega \tag{4.133}$$

$$\dot{\delta} = \omega - 1 \tag{4.102}$$

<u>Relation to input data to most commercial stability programs</u>:

The input requirements for characterizing generator saturation for most commercial-grade stability programs are in terms of a parameter called S, defined by

$$S = \frac{\Delta i_M}{i_{M0}}$$

where $\Delta i_M = i_{MS} - i_{M0}$ as before.

Recall that

$$K_S = \frac{i_{M0}}{i_{MS}} = \frac{i_{M0}}{i_{M0} + \Delta i_M}$$

The relation between S and K_S is derived from the below:

$$S = \frac{\Delta i_M}{i_{M0}} \Rightarrow S + 1 = \frac{\Delta i_M}{i_{M0}} + \frac{i_{M0}}{i_{M0}} = \frac{i_{M0} + \Delta i_M}{i_{M0}}$$

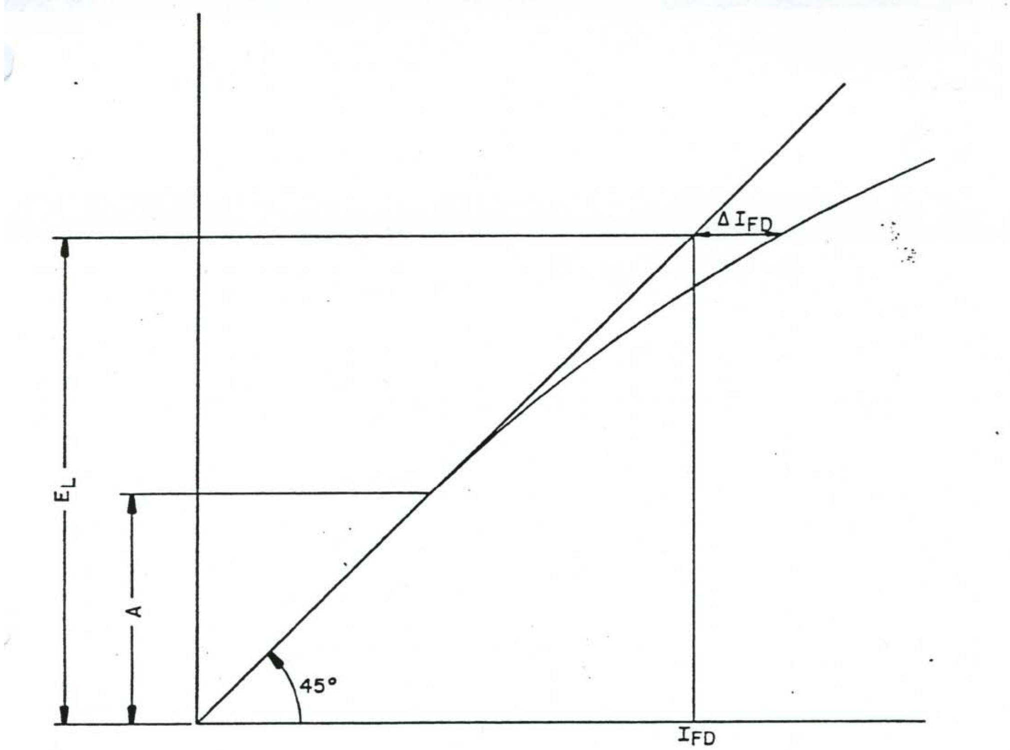
$$\Rightarrow \frac{1}{S+1} = \frac{i_{M0}}{i_{M0} + \Delta i_M} = K_S$$

The specific data entry into most programs (including PSS/E) is

- S(1.0): value of S when open circuit terminal voltage is 1.0pu
- S(1.2): value of S when open circuit terminal voltage is 1.2pu

Note that S(1.2) should always be larger than S(1.0). In the Diablo Canyon data, S(1.0) is 0.0769 and S(1.2) is 0.41. The corresponding values of K_S are 0.9286 and 0.7092, respectively.

The figure below illustrates treatment of saturation from one commercial grade stability program.



THE SATURATION FACTOR S_D IS DEFINED AS $S_D = \Delta I_{FD} / I_{FD}$. INPUT DATA $S_{G1.0}$ AND $S_{G1.2}$ GIVES THE VALUE OF S_D FOR TWO POINTS ON THE SATURATION CURVE. THE WSCC STABILITY PROGRAM USES THE FOLLOWING FUNCTION TO DEFINE THE SATURATION FACTOR FOR OTHER POINTS ON THE SATURATION CURVE:

IF $E_L \le A$ $\quad S_D = 0$

IF $E_L > A$ $\quad S_D = B(E_L - A)^2 / E_L = \frac{\Delta I_{FD}}{I_{FD}}$

WHERE A AND B ARE CONSTANTS WHICH CAN BE CALCULATED BY SOLVING THE EQUATIONS

$S_{G1.0} = B(1.0 - A)^2 / 1.0$

$S_{G1.2} = B(1.2 - A)^2 / 1.2$

WSCC STABILITY PROGRAM GENERATOR MODELS	
WSCC TYPE F	FIGURE 3 APRIL 2, 1976
SATURATION FUNCTION	

Fig. 2

Final note on saturation. Section 4.12.3 develops a model where saturation is neglected. Such a model is useful for linearized analysis.

The approach is simple – just substitute the auxiliary equations, i.e., the expressions for λ_{AD} and λ_{AQ}, i.e.,

$$\lambda_{AD} = \frac{L_{MD}}{l_d}\lambda_d + \frac{L_{MD}}{l_F}\lambda_F + \frac{L_{MD}}{l_D}\lambda_D$$

$$\lambda_{AQ} = \frac{L_{MQ}}{l_q}\lambda_q + \frac{L_{MQ}}{l_Q}\lambda_Q + \frac{L_{MQ}}{l_G}\lambda_G$$

into the state equations:

$$\dot{\lambda}_d = -\frac{r}{l_d}\lambda_d + \frac{r}{l_d}\lambda_{AD} - \omega\lambda_q - v_d \qquad (4.126)$$

$$\dot{\lambda}_F = -\frac{r_F}{l_F}\lambda_F + \frac{r_F}{l_F}\lambda_{AD} + v_F \qquad (4.128)$$

$$\dot{\lambda}_D = -\frac{r_D}{l_D}\lambda_D + \frac{r_D}{l_D}\lambda_{AD} \qquad (4.129)$$

$$\dot{\lambda}_q = -\frac{r}{l_q}\lambda_q + \frac{r}{l_q}\lambda_{AQ} + \omega\lambda_d - v_q \qquad (4.130)$$

$$\dot{\lambda}_Q = -\frac{r_Q}{l_Q}\lambda_Q + \frac{r_Q}{l_Q}\lambda_{AQ} \qquad (4.131)$$

$$\dot{\lambda}_G = -\frac{r_G}{l_G}\lambda_G + \frac{r_G}{l_G}\lambda_{AQ} \qquad (4.131')$$

$$\dot{\omega} = \frac{T_m}{\tau_j} + \left[\frac{\lambda_{AQ}}{l_q 3\tau_j}\lambda_d - \frac{\lambda_{AD}}{l_d 3\tau_j}\lambda_q\right] + \left[\frac{-D}{\tau_j}\right]\omega \qquad (4.133)$$

$$\dot{\delta} = \omega - 1 \qquad (4.102)$$

Your book does this for all of the above, except, of course, for the equation corresponding to the G-winding. So we will do it for that one.

$$\dot{\lambda}_G = -\frac{r_G}{l_G}\lambda_G + \frac{r_G}{l_G}\left(\frac{L_{MQ}}{l_q}\lambda_q + \frac{L_{MQ}}{l_Q}\lambda_Q + \frac{L_{MQ}}{l_G}\lambda_G\right)$$

$$= -\frac{r_G}{l_G}\lambda_G + \frac{r_G L_{MQ}}{l_G l_q}\lambda_q + \frac{r_G L_{MQ}}{l_G l_Q}\lambda_Q + \frac{r_G L_{MQ}}{l_G^2}\lambda_G$$

$$= -\frac{r_G}{l_G}\left(1 - \frac{L_{MQ}}{l_G}\right)\lambda_G + \frac{r_G L_{MQ}}{l_G l_q}\lambda_q + \frac{r_G L_{MQ}}{l_G l_Q}\lambda_Q$$

The above state equation needs to be included to the state equations given in the book, eq. (4.138), which are provided below:

$$\begin{bmatrix} \dot{\lambda}_d \\ \dot{\lambda}_F \\ \dot{\lambda}_D \\ \dot{\lambda}_q \\ \dot{\lambda}_Q \\ \dot{\omega} \\ \dot{\delta} \end{bmatrix} = \begin{bmatrix} -\frac{r}{\ell_d}\left(1-\frac{L_{MD}}{\ell_d}\right) & \frac{r}{\ell_d}\frac{L_{MD}}{\ell_F} & \frac{r}{\ell_d}\frac{L_{MD}}{\ell_D} & -\omega & 0 & 0 & 0 \\ \frac{r_F}{\ell_F}\frac{L_{MD}}{\ell_d} & -\frac{r_F}{\ell_F}\left(1-\frac{L_{MD}}{\ell_F}\right) & \frac{r_F}{\ell_F}\frac{L_{MD}}{\ell_D} & 0 & 0 & 0 & 0 \\ \frac{r_D}{\ell_D}\frac{L_{MD}}{\ell_d} & \frac{r_D}{\ell_D}\frac{L_{MD}}{\ell_F} & -\frac{r_D}{\ell_D}\left(1-\frac{L_{MD}}{\ell_D}\right) & 0 & 0 & 0 & 0 \\ \omega & 0 & 0 & -\frac{r}{\ell_q}\left(1-\frac{L_{MQ}}{\ell_q}\right) & \frac{r}{\ell_q}\frac{L_{MQ}}{\ell_Q} & 0 & 0 \\ 0 & 0 & 0 & \frac{r_Q}{\ell_Q}\frac{L_{MQ}}{\ell_q} & -\frac{r_Q}{\ell_Q}\left(1-\frac{L_{MQ}}{\ell_Q}\right) & 0 & 0 \\ -\frac{L_{MD}}{3\tau_j \ell_d^2}\lambda_q & -\frac{L_{MD}}{3\tau_j \ell_d \ell_F}\lambda_q & -\frac{L_{MD}}{3\tau_j \ell_d \ell_D}\lambda_q & \frac{L_{MQ}}{3\tau_j \ell_q^2}\lambda_d & \frac{L_{MQ}}{3\tau_j \ell_q \ell_Q}\lambda_d & -\frac{D}{\tau_j} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \lambda_d \\ \lambda_F \\ \lambda_D \\ \lambda_q \\ \lambda_Q \\ \omega \\ \delta \end{bmatrix} + \begin{bmatrix} -v_d \\ v_F \\ 0 \\ -v_q \\ 0 \\ \frac{T_m}{\tau_j} \\ -1 \end{bmatrix} \quad (4.138)$$

Note the presence of v_d and v_q on the right-hand-side.

Recall that $\underline{v}_{0dq} = \underline{P}\underline{v}_{abc}$ and so v_d and v_q come from the phase voltages v_a, v_b, and v_c.

Since the phase voltages are affected by the load currents, so are v_d and v_q.

So, we need to represent the load in order to complete the model. This is the subject of section 4.13.