Linearized Analysis of the Synchronous Machine for PSS

Chapter 6 does two basic things:

1. Shows how to linearize the 7-state model (model #2, IEEE #2.1, called "full model without G-cct.") of a synchronous machine connected to an infinite bus using the current-state-space model (sections 6.2-6.3) and using the flux-linkage state-space model (section 6.4). This material is useful for understanding the modeling required for power system eigenvalue calculation programs such as the EPRI programs MASS and PEALS contained within SSSP. Some information on these tools follow:
   a. Kundur, Rogers, Wong, Wang, and Lauby, "A comprehensive computer program package for small signal stability analysis of power systems," IEEE Transactions on Power Systems, Vol. 5, No. 4, Nov., 1990.
   b. Wang, Howell, Kundur, Chung, and Xu, "A tool for small-signal security assessment of power systems," IEEE Transactions on Power Systems, …
   c. M. Crow, "Computational methods for electric power systems," chapter 7 on "Eigenvalue Problems," CRC Press, 2003.
2. Linearizes the one-axis model of a synchronous machine connected to an infinite bus (sections 6.5-6.7). This material is useful for the conceptual understanding of why power system stabilizers are needed.

My feeling is that (2) will be more useful to you, and so I will spend this class discussing it.

Some additional references for you on this issue are references [1,2] given at the end of chapter 6. These two references are:

[1] W. Heffron and R. Phillips, "Effect of modern apllidyne voltage regulators on under-excited operation of large turbine generators," AIEE Transactions, pt. III, vol. 71, pp. 692-696, 1952.
[2] F. de Mello and C. Concordia, "Concepts of synchronous machine stability as affected by excitation control," IEEE Transactions on Power Apparatus and Systems, PAS-88, pp 316-329, 1969.

Reference [1] came first and produced what is commonly referred to in the literature as the Heffron-Phillips model of the synchronous machine. Reference [2] extended the Heffron-Phillips model and is the most well known. Reference [2] is also viewed as the seminal work that motivated the need for power system stabilizers (PSS). This paper is on the web site for you to download, read, and place in your notebook. You will note that it contains material quite similar to what follows below.

Your text also provides background on this issue in several separate locations, found in the following sections:

- Section 3.5.1: Voltage regulator with one time lag
- Section 6.5: Simplified linear model
- Section 6.6: Block diagrams
- Section 6.7: State-space representation of simplified model
- Section 7.8: State-space description of the excitation system
- Section 8.4: Effect of excitation on dynamic stability
- Section 8.5: Root-locus analysis of a regulated machine connected to an infinite bus
- Section 8.7: Supplementary stabilizing signals

I will provide the minimal analysis necessary to see the basic issue.

The analysis uses the simplest model possible for which the excitation system may be represented – the one-axis model (model 7, IEEE #1.0), loaded through a connection to an infinite bus.

The one-axis model is a 3-state model, developed based on the following main assumptions (there are others as well – see page 222):

1. Only the field winding is represented (so no G-circuit and no amortisseur windings).
2. $d\lambda_d/dt = d\lambda_q/dt = 0$

The nonlinear equations for the one-axis model are given by eqs. (4.294) and (4.297), as follows:

$$\dot{E}'_q = \frac{1}{\tau'_{do}} E_{FD} - \frac{1}{\tau'_{do}} E_q \text{, where } E_q = E'_q + (x'_d - x_d) I_d$$

$$\dot{\omega} = \frac{1}{\tau_j} T_m - \frac{1}{\tau_j}\left[E'_q I_q + (x'_d - x_q) I_d I_q\right] - \frac{1}{\tau_j} D\omega$$

$$\dot{\delta} = \omega - 1$$

To identify basic concepts, Concordia and deMello assumed a single machine connected to an infinite bus through a transmission line having series impedance of $R_e + jX_e$, as illustrated in Fig. 1.

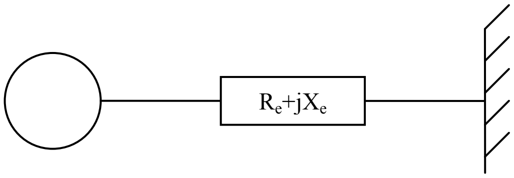

Fig. 1

The generator model connected to the infinite bus can be linearized and converted to the following state-space form, as given in eqs. (6.79) of Section 6.7:

$$\Delta\dot{E}'_q = -\left(\frac{1}{K_3\tau'_{do}}\right)\Delta E'_q - \left(\frac{K_4}{\tau'_{do}}\right)\Delta\delta + \left(\frac{1}{\tau'_{do}}\right)\Delta E_{FD}$$
$$\Delta\dot{\omega} = \left(\frac{-K_2}{\tau_j}\right)\Delta E'_q - \left(\frac{K_1}{\tau_j}\right)\Delta\delta + \left(\frac{1}{\tau_j}\right)\Delta T_m \qquad (6.79)$$
$$\Delta\dot{\delta} = (\Delta\omega)\omega_B$$

The LaPlace transform of the above equations results in the following relations:

$$\Delta E'_q = \frac{K_3}{1+K_3\tau'_{d0}\,s}\Delta E_{FD} - \frac{K_3K_4}{1+K_3\tau'_{d0}\,s}\Delta\delta$$
$$\Delta\omega = \frac{1}{s\tau_j}\left(\Delta T_m - \Delta T_e\right) \qquad (*)$$
$$\Delta\delta = \frac{1}{s}\Delta\omega$$

where the variables $\Delta E'_q$, $\Delta\omega$, and $\Delta\delta$ represent LaPlace transforms of their corresponding time-domain functions. Two additional relations may be obtained as well, according to the following:

$$\Delta T_e = K_1\Delta\delta + K_2\Delta E'_q + D\Delta\omega$$
$$\Delta V_t = K_5\Delta\delta + K_6\Delta E'_q \qquad (**)$$

Finally, we note that E_{FD}, the stator EMF produced by the field current and corresponding to the field voltage v_F, is a function of the voltage regulator. Under linearized conditions, the change in E_{FD} is proportional to the difference between changes in the reference voltage and changes in the terminal voltage, i.e.,

$$\Delta E_{FD} = G_e(s)(\Delta V_{ref} - \Delta V_t) \qquad (***)$$

where $G_e(s)$ is the transfer function of the voltage regulator.

In the above equations (*) and (**), the various constants K_1-K_6 are defined as follows:

$$K_1 = \left.\frac{\Delta T_e}{\Delta \delta}\right|_{E'_q = E'_{q0}} \qquad K_2 = \left.\frac{\Delta T_e}{\Delta E'_q}\right|_{\delta = \delta_0} \qquad K_4 = \left.\frac{-1}{K_3}\frac{\Delta E'_q}{\Delta \delta}\right|_{E_{FD} = constant}$$

$$K_5 = \left.\frac{\Delta V_t}{\Delta \delta}\right|_{E'_q = E'_{q0}} \qquad K_6 = \left.\frac{\Delta V_t}{\Delta E'_q}\right|_{\delta = \delta_0}$$

and K_3 is an impedance factor that accounts for the loading effect of the external impedance. Your text, on pages 223, 224, and 225, provides exact expressions for these constants for the case of the one-axis model we are analyzing. Two notes:

1. K_1 is the synchronizing power coefficient.
2. The book expresses K_4 as (see eq. 6.60)

$$K_4 = \left.\frac{1}{K_3}\frac{\Delta E'_q}{\Delta \delta}\right|_{E_{FD} = constant}$$

However, for K_4 and K_3 both positive constants, as is usually the case, the above expression suggests that E'_q would increase with an increase in angle (or loading). This is counter to the idea of armature reaction, where the internal flux decreases as a result of stator current. In fact, the book itself indicates as much via eq. (3.11) where it says that "K_4 is the demagnetizing effect of a change in the rotor angle (at steady-state)," which is given by the following relationship:

$$K_4 = -\frac{1}{K_3}\lim_{t\to\infty} \Delta E'(t)\Big|_{\substack{\Delta v_F = 0 \\ \Delta\delta = u(t)}} \qquad (3.11)$$

where we note the negative sign out front. Therefore, the book expression (eq. 6.60), without the negative sign, is incorrect.

In eq. (***), $G_e(s)$ is the transfer function of the excitation system. We do not have time to study excitation systems, but suffice it (for now) to say that there are several different kinds (DC, AC Alternator, and static), each requiring somewhat different modeling. One kind that has become quite common is the "static" excitation system, represented by Fig. 2a, where K_A is the exciter gain and T_A is the excited time constant.

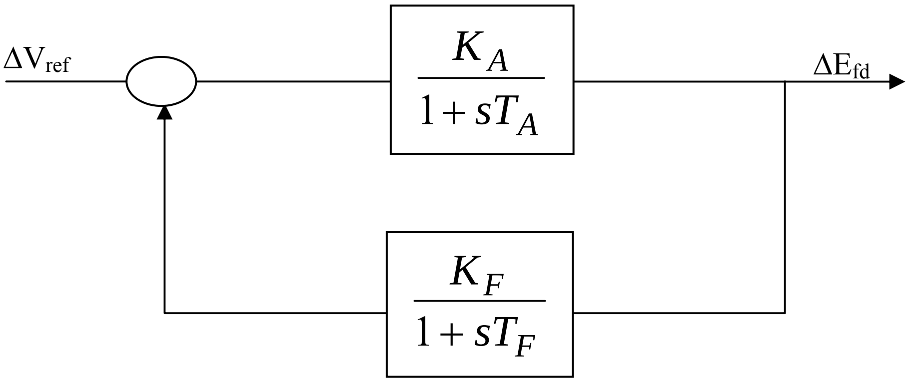

Fig. 2a

Fig. 2a is characterized by the following transfer function.

$$G_e(s) = \frac{(1+sT_F)K_A}{(1+sT_F)(1+sT_A)+K_F K_A} \qquad (****)$$

The static excitation is typically very fast (no rotating machine in the loop). Fast excitation response is beneficial for transient stability because generator terminal voltages see less voltage depression for less time during and after network faults. Such speed of excitation response can, however, cause problems for damping, as we shall see in what follows.

We may extract from the above equations (*), (**), and (***) a block diagram relation, as seen in Fig. 2. Note that in this block diagram, τ_j=M.

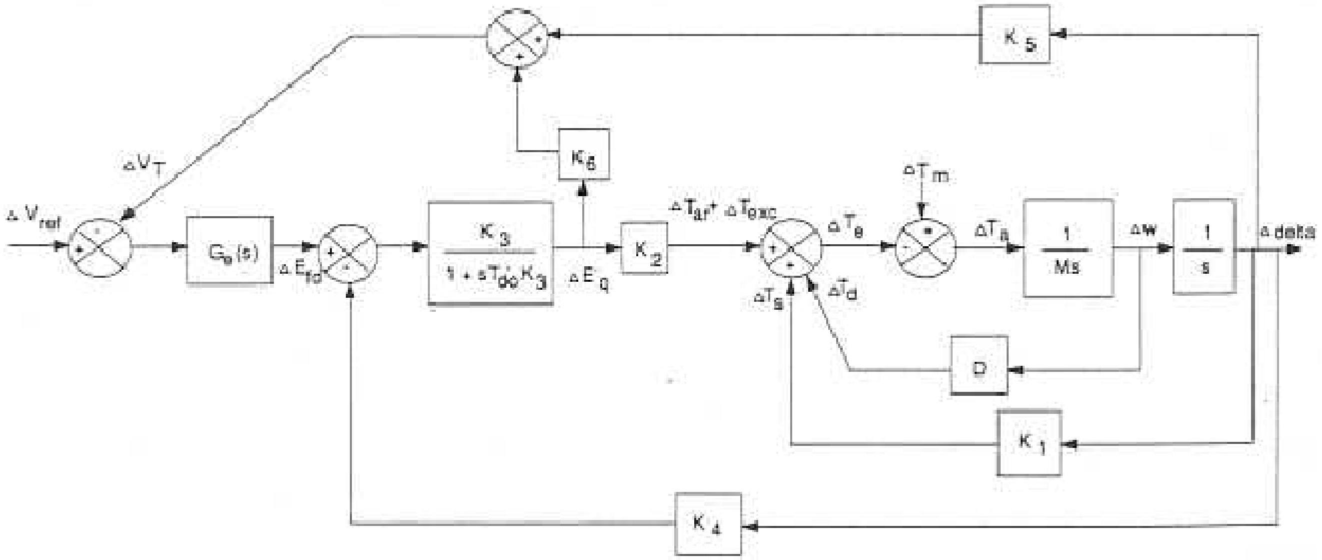

Fig. 2

We will use this block diagram to analyze the stability behavior of the machine. Although one can use a variety of methods to perform this analysis (Root locus, Routh's criterion, eigenanalysis), we will resort to a rather unconventional but quite intuitive analysis procedure that conforms to that originally done in the deMello-Concordia paper. This analysis is based on the following observations made of the block diagram.

1. ΔT_d, the damping torque, is in phase with speed deviation $\Delta\omega$.
2. ΔT_s, synchronizing torque, is in phase with angle deviation $\Delta\delta$. We call this *synchronizing torque* because the higher it is, the more "stable" the machine will be with respect to loss of synchronism. This is confirmed by noting that high K_1 means low loading, as indicated by the fact that K_1 is the slope of the tangent to the power-angle curve at the operating point.
3. Because $\Delta\delta=(1/s)\ \Delta\omega$, we see that angle deviation lags speed deviation by 90 degrees in phase.

This leads to a "stability criterion"….

For stability, the composite electrical torque must have positive damping torque, which means that it must have a component in phase with speed deviation.

So we can perform a qualitative analysis using the following ideas:

- Any electrical torque contribution in phase with angle deviation contributes positive synchronizing torque.
- Any electrical torque contribution in phase with speed deviation contributes positive damping torque.

<u>Inertial torques</u>:

Let's begin by just analyzing the "inertial" loops in the block diagram. These are the ones corresponding to D and K_1, as indicated by the two bold arrows in Fig. 3.

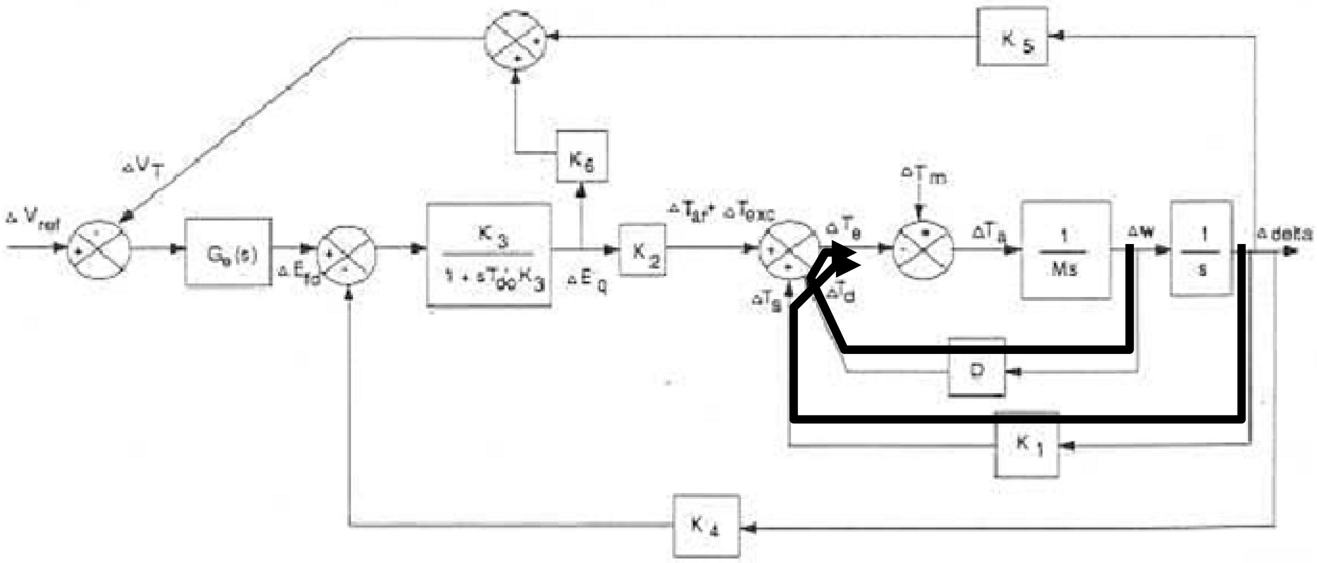

Fig. 3

- We see that the torque contribution through D, ΔT_D, is proportional to $\Delta\omega$ so it contributes positive damping, as expected.
- The torque contribution through K_1, ΔT_S, is proportional to $\Delta\delta$ so it contributes positive synchronizing torque, 90 degrees behind the damping torque. Figure 4 below illustrates.

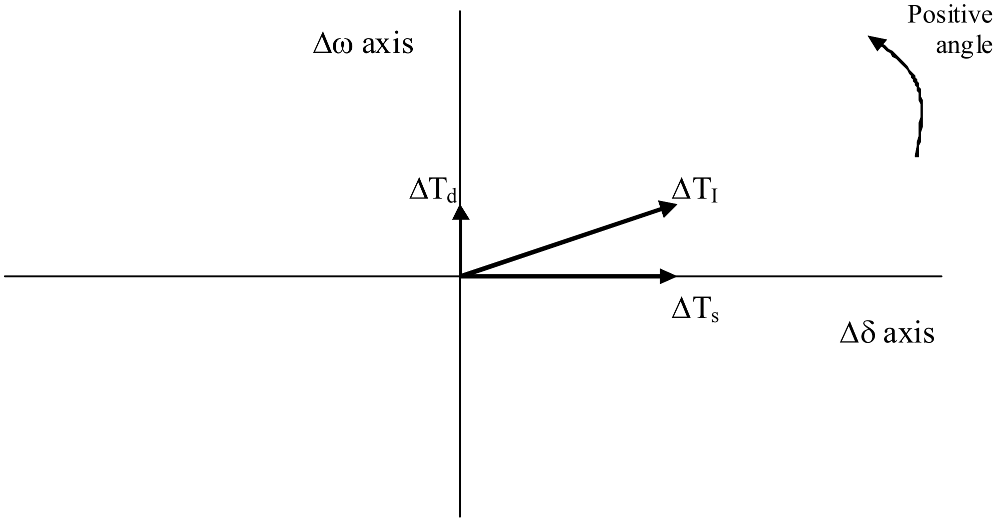

Fig. 4

So as long as D is positive and there are no other effects, we obtain positive damping contributions from the inertial torques.

Armature reaction torque:

But now let's consider the influence of armature reaction, when we get field weakening from the armature current. This effect is represented by the loop through K_4, K_3, and K_2, and is represented on the diagram by ΔT_{ar}, as indicated by the bold arrow in Fig. 5.

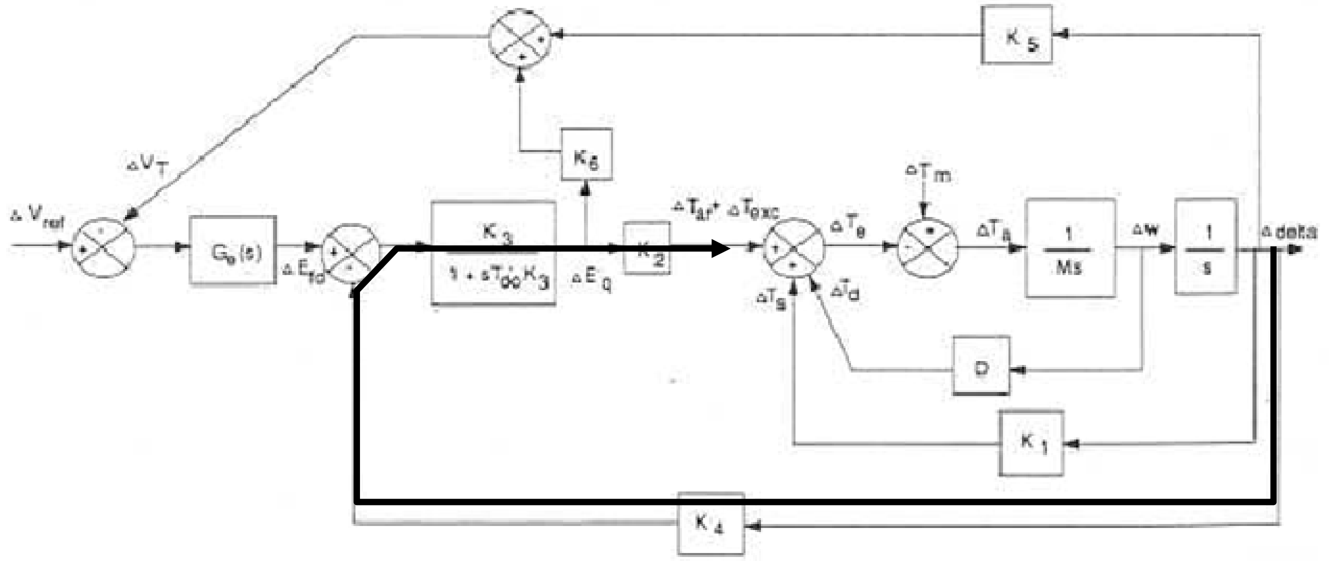

Fig. 5

The transfer function for ΔT_{ar} is given by:

$$\frac{\Delta T_{ar}}{\Delta\delta} = \frac{-K_2 K_3 K_4}{1 + sK_3\tau'_{d0}} = \frac{K_2 K_3 K_4 \angle -180^\circ}{1 + sK_3\tau'_{d0}}$$

From this last transfer function, we can identify the phase of the electrical torque contribution relative to $\Delta\delta$, which is:

$$\phi_{ar} = -180 - \tan^{-1} K_3\tau'_{d0}\,\omega_{osc}$$

where ω_{osc} is the frequency corresponding to the weakly damped electromechanical modes of oscillation (from 0.2 Hz up to about 2.0 Hz).

What does this do to the resulting torque? Since it is negative, we draw the vector with an angle measured opposite the positive angle. We clearly get -180°, but we also get an additional negative angle from the $\tan^{-1}$ term. Since $t'_{d0}\omega_{osc}$ is positive, this additional angle must between 0 and 90°. The effect is shown in Fig. 6 below.

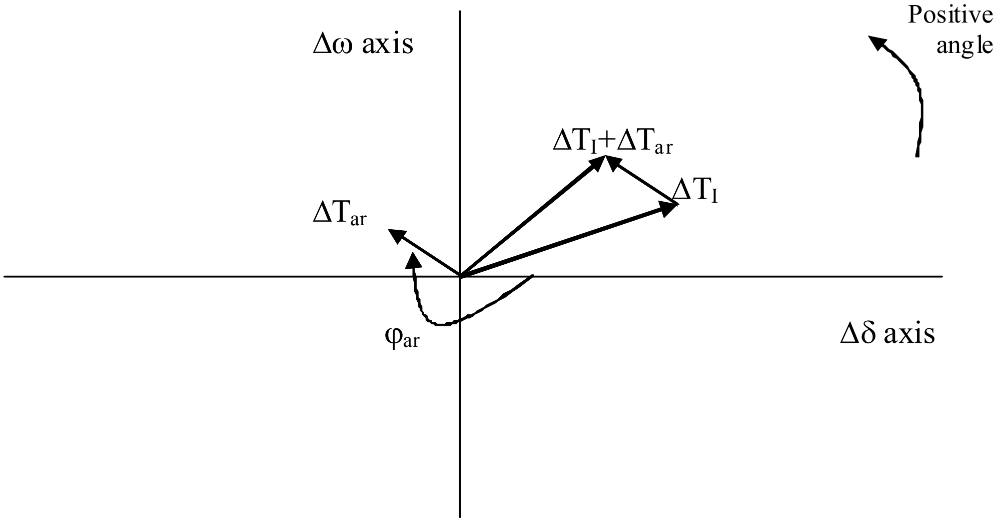

Fig. 6

Note that the effect of armature reaction on composite torque is to increase damping torque (in phase with Δω) and to decrease synchronizing torque (in phase with Δδ).

Excitation system torque:

This is the electrical torque that results from the K_5 and K_6 loops, as shown in Fig. 7 below.

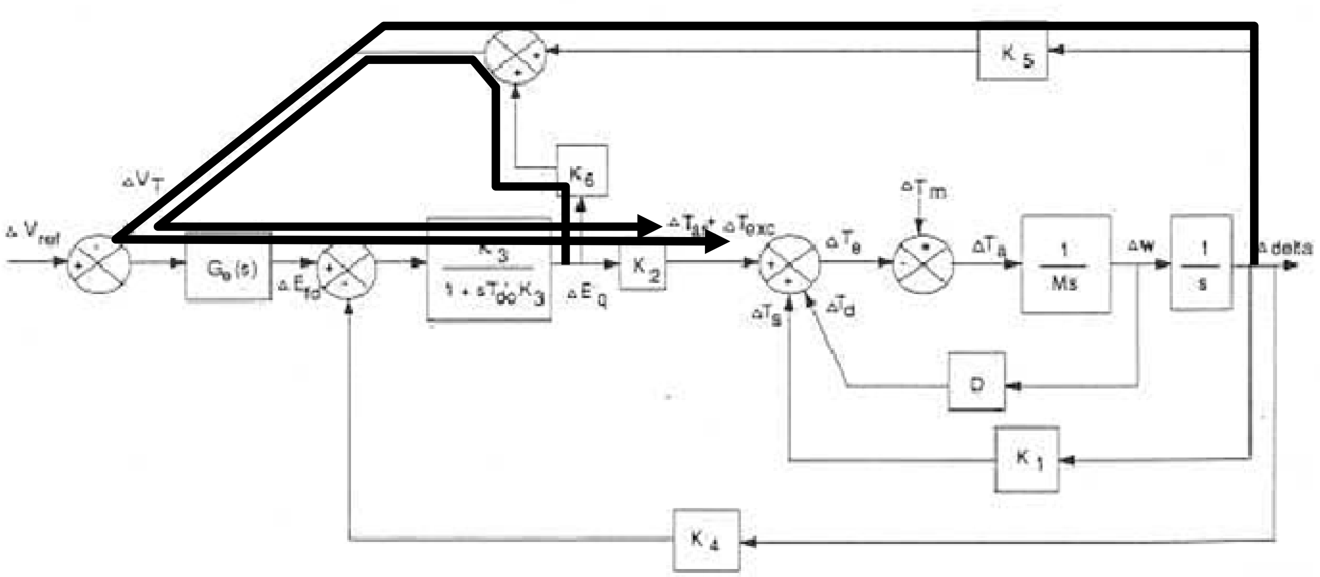

Fig. 7

This torque may be expressed based on the block diagram as:

$$\Delta T_{exc} = \frac{K_2 K_3}{1 + s\tau'_{d0} K_3} G_e(s)\left(\Delta V_{ref} - \Delta V_t\right)$$

Ignoring ΔV_{ref} (it represents manual changes in the voltage setting), and using (from eq. (**)):

$$\Delta V_t = K_5 \Delta\delta + K_6 \Delta E'_q$$

we obtain

$$\Delta T_{exc} = \frac{K_2 K_3}{1 + s\tau'_{d0} K_3} G_e(s)\left(-K_5 \Delta\delta - K_6 \Delta E'_q\right)$$

We want to express each torque as a function of $\Delta\delta$ or $\Delta\omega$. But the last expression has a $\Delta E'_q$. We can address this by noticing the relation (from the block diagram) that $\Delta T_{exc}=K_2\Delta E'_q \Rightarrow \Delta E'_q=\Delta T_{exc}/K_2$, and so we can write that

$$\Delta T_{exc} = \frac{K_2 K_3}{1+s\tau'_{d0}K_3} G_e(s)\left(-K_5\Delta\delta - \frac{K_6}{K_2}\Delta T_{exc}\right)$$

Solving for ΔT_{exc} yields

$$\Delta T_{exc} = \frac{-K_2K_3K_5}{1+s\tau'_{d0}K_3 + K_3K_6G_e(s)} G_e(s)(\Delta\delta)$$

On substituting equation (****) for the static excitation transfer function $G_e(s)$, we obtain, after some algebra, the following:

$$\Delta T_{exc} = \frac{-K_2K_3K_5K_A(1+sT_F)}{[(1+sT_F)(1+sT_A)+K_FK_A](1+s\tau'_{d0}K_3)+K_3K_6K_A(1+sT_F)}(\Delta\delta)$$

The above relation appears quite challenging to analyze, but we can simplify the task greatly by observing that the denominator is third order. Thus, it will be possible to write the above relation as:

$$\Delta T_{exc} = \frac{-K_2K_3K_5K_A(1+sT_F)}{(s+p_1)(s+p_2)(s+p_3)}(\Delta\delta)$$

where p_i are the poles. We may have 3 real poles or 1 real with 2 complex. In general, we express each pole as $p_i=\sigma_i+j\omega_i$. Thus, the transfer function becomes:

$$\Delta T_{exc} = \frac{-K_2K_3K_5K_A(1+sT_F)}{(s+\sigma_1+j\omega_1)(s+\sigma_2+j\omega_2)(s+\sigma_3+j\omega_3)}(\Delta\delta)$$

We want to evaluate the transfer function at $s=j\omega_{osc}$, where ω_{osc} is the frequency of oscillation of concern (we assume this frequency to be an interarea oscillation). Therefore,

$$\Delta T_{exc} = \frac{-K_2 K_3 K_5 K_A (1 + j\omega_{osc} T_F)}{\left(j\omega_{osc} + \sigma_1 + j\omega_1\right)\left(j\omega_{osc} + \sigma_2 + j\omega_2\right)\left(j\omega_{osc} + \sigma_3 + j\omega_3\right)}(\Delta\delta)$$

On combining imaginary terms in the denominator, we get:

$$\Delta T_{exc} = \frac{-K_2 K_3 K_5 K_A (1 + j\omega_{osc} T_F)}{(\sigma_1 + j(\omega_1 + \omega_{osc}))(\sigma_2 + j(\omega_2 + \omega_{osc}))(\sigma_3 + j(\omega_3 + \omega_{osc}))}(\Delta\delta)$$

We are interested in the phase of ΔT_{exc} relative to $\Delta\delta$.

Fact: When the generator is heavily loaded, it is possible for K_5 to be negative. See Fig. 6.1and section 8.4.3 in your text. This makes the numerator of the previous transfer function positive.

A simulation of such a case is shown in Fig. 8 below. The solid curve represents generators with fast high-gain excitation systems, but no PSS. The other two curves represent significantly fewer of such generators.

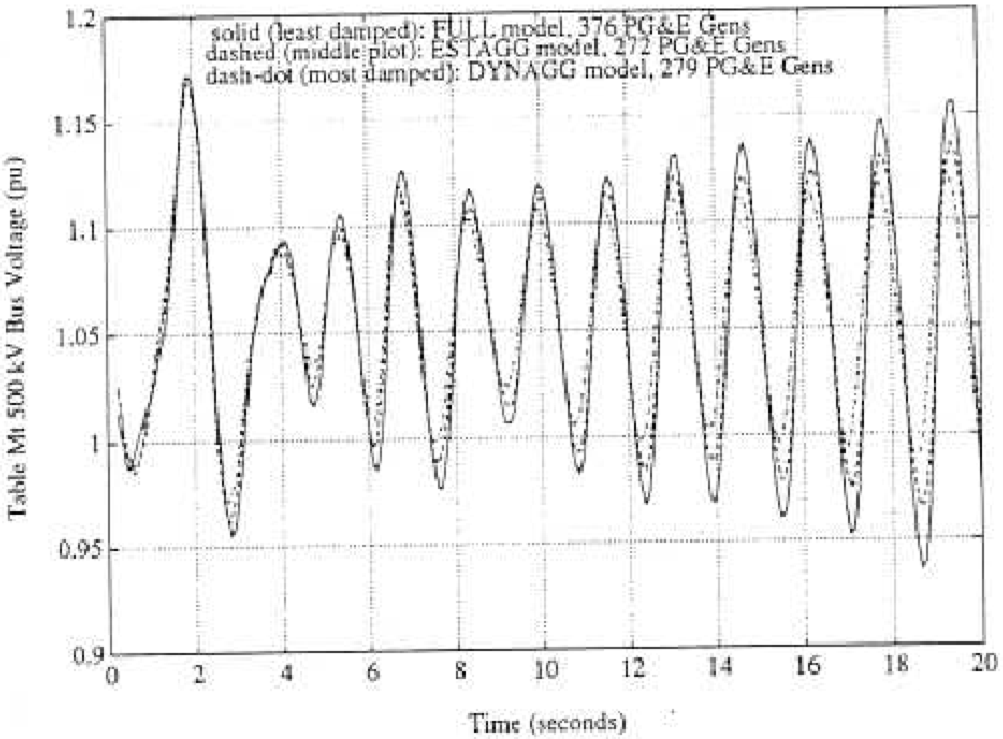

Fig. 8

Repeating our transfer function:

$$\Delta T_{exc} = \frac{-K_2K_3K_5K_A(1+j\omega_{osc}T_F)}{(\sigma_1 + j(\omega_1+\omega_{osc}))(\sigma_2 + j(\omega_2+\omega_{osc}))(\sigma_3 + j(\omega_3+\omega_{osc}))}(\Delta\delta)$$

Assuming $K_5<0$, the phase of ΔT_{exc} relative to $\Delta\delta$ is given by

$$\phi_{exc} = tan^{-1}\,\omega_{osc}T_F - \sum_{i=1}^{3} tan^{-1}\frac{\omega_i+\omega_{osc}}{\sigma_i}$$

Bounds for this phase are:

- $\phi_{exc} = -\sum_{i=1}^{3}\tan^{-1}\frac{\omega_i}{\sigma_i}$ at $\omega_{osc}=0$
- $\phi_{exc} = 90-270=-180$ at ω_{osc}=infinity.

For typical power system oscillation frequencies ranging from 0.2 Hz to 2 Hz, the range is –90 degrees to –20 degrees, and it tends to be closer to -90 degrees for high gain (large K_A) fast response (small T_A) static excitation systems.

So let’s assume that it is -80 degrees. In this case, our diagram will appear as in Fig. 9 below.

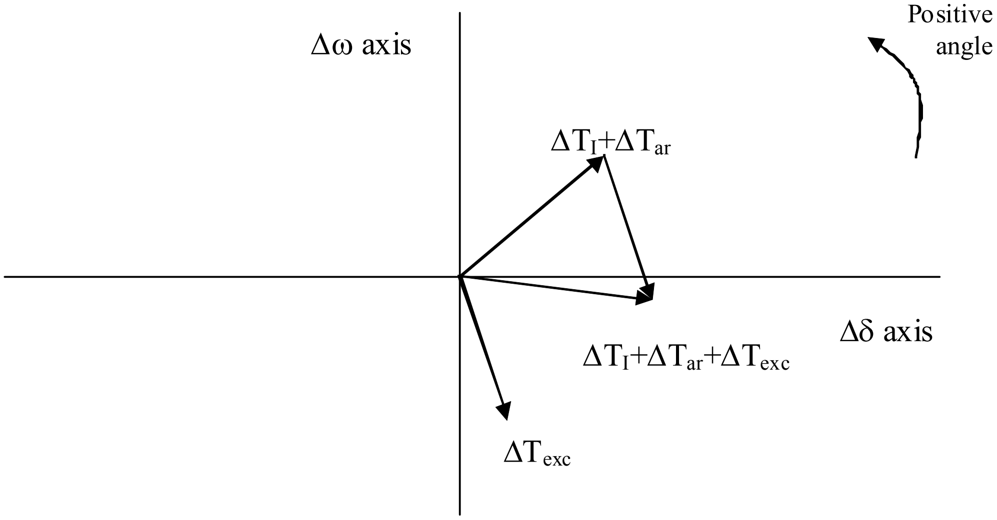

Fig. 9

and we see that the damping can go negative for fast (small T_A)-high gain (large K_A) excitation systems under heavy loading conditions! And this explains the effect observed in Fig. 8.

So what do we do about this?

Solution 1: Limit K_A to as high as possible without causing undamped oscillations. But high-gain, fast response excitation systems are good for transient (early-swing) instability! And so we would rather not do this.

Solution 2: Provide a supplementary torque component that offsets the negative damping torque caused by the excitation system.
➔Basically, the idea is to push (rotate forward) our torque vector back into the upper-right quadrant. Thus we need to phase-advance the torque vector by between 20 to 90 degrees. We will introduce a supplementary torque that does this, denoted by ΔT_{pss}, as indicated in Fig. 10 below.

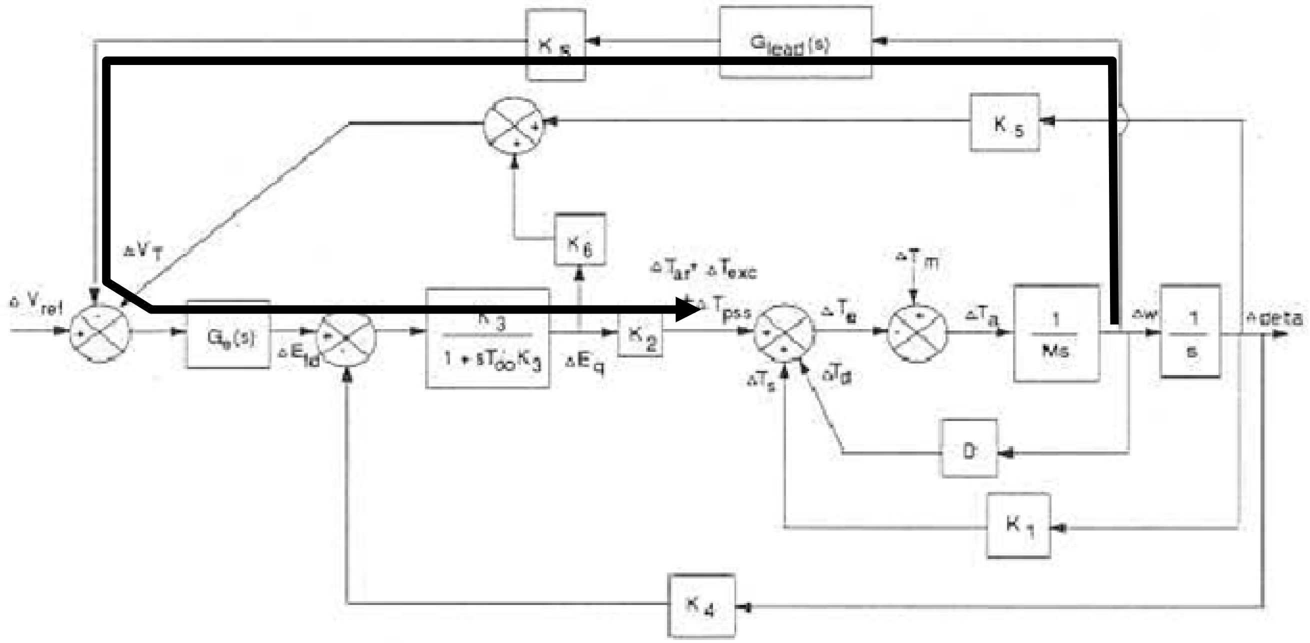

Fig. 10

The transfer function $K_S G_{lead}(s)$ is intended to provide the supplementary signal ΔT_{PSS} as illustrated in Fig. 11 below.

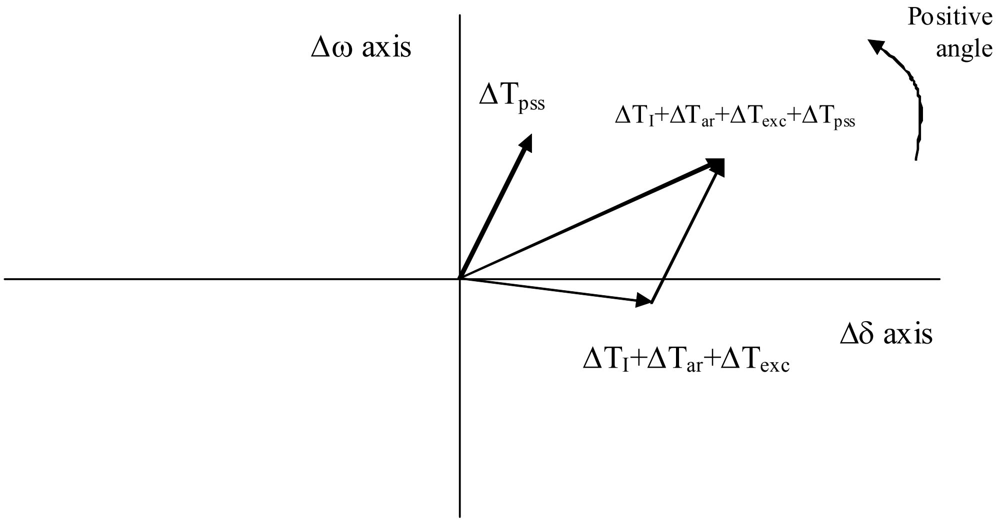

Fig. 11

We will take Δω as the feedback signal for our control loop to provide ΔT_{pss} (we could also use angle deviation, but speed deviation is easier to obtain as a control signal).

We may provide "shaping" networks to process the feedback signal in providing it with the proper amount of phase (lead or lag). For example (see Dorf, pg. 362-363), a network to provide phase lead is shown in Fig. 12.

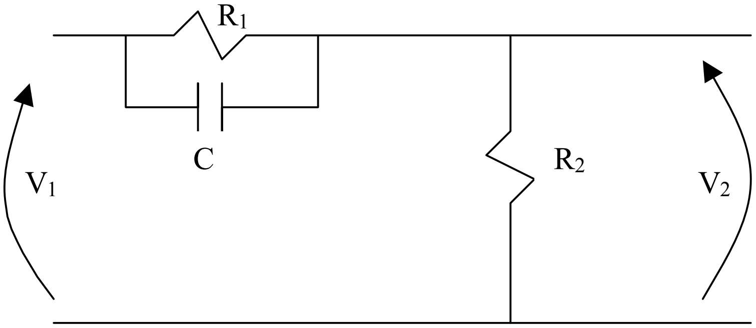

Fig. 12

(One can alternatively use digital signal processing techniques.)
In the phase lead network above, we get that

$$G_{lead}(s) = \frac{V_2(s)}{V_1(s)} = \frac{R_2}{R_2 + \left\{R_1 \frac{1}{Cs}\right\} \Big/ \left\{R_1 + 1/Cs\right\}} = \frac{1+\alpha\tau s}{\alpha(1+\tau s)}$$

where

$$\alpha = \frac{R_1 + R_2}{R_2}, \qquad \tau = \frac{R_1 R_2}{R_1 + R_2} C$$

Dorf shows that the maximum value of phase lead given by the above network occurs at a frequency of

$$\omega_m = \frac{1}{\tau\sqrt{\alpha}}$$

and the corresponding phase lead you get at this frequency is given by

$$\sin\phi_m = \frac{\alpha - 1}{\alpha + 1}$$

So the idea is that you can know the frequency ω_m that you want to provide the maximum phase lead. This is the frequency of your most troublesome electromechanical mode and is considered to be the PSS tuning mode.

Note from the above diagram that the supplementary signal ΔT_{PSS} is actually *lagging* $\Delta\omega$, so one might think that we should provide phase lag, not phase lead, to the input signal (which is actuated by $\Delta\omega$). This would in fact be the case if we could introduce the "shaped" signal (the output of G_{lead}) directly at the machine shaft.

However, this is not very easy to do because we cannot produce a mechanical torque directly from an electrical signal transduced from rotor speed.

In fact, the only place we can introduce an electrical signal is at the voltage regulator, i.e., the input to the excitation system, $G_e(s)$.

This causes a problem in that we now incur the phase lag introduced by $G_e(s)$ and the τ'_{d0} block, which is typically around ϕ_{exc}= -80 degrees as discussed previously.

So this means that we must think about it in the following way:

1. We start with the $\Delta\omega$ signal.
2. We introduce a phase lead of an amount equal to X. What is X?
3. We incur ~80 degrees of phase lag from ϕ_{exc}.
4. We provide ΔT_{pss} lagging $\Delta\omega$ by, say ~25 degrees. This means that X-80≈-25 degrees ➔ X=55 degrees.

Therefore X must be about 55 degrees. So we must provide an appropriate shaping network. This shaping network is referred to as $G_{lead}(s)$.

Therefore we can write

$$\sin 55 = \frac{\alpha - 1}{\alpha + 1}$$

And solve for α.

Then choose τ based on

$$\omega_m = \frac{1}{\tau\sqrt{\alpha}}$$

where $\omega_m = 2\pi(f_{osc})$, and f_{osc} is the frequency of the oscillation "problem mode."

Note the principle behind the power system stabilizer is to cancel the phase lags introduced by the excitation system with the right amount of lead compensation so that the torque exerted on the shaft by the excitation control effect will be in phase with speed deviation.

So the PSS introduces a supplementary signal into the voltage regulator with proper phase and gain adjustments to produce a component of damping that will be sufficient to cancel the negative damping from the exciters.