A TOOL FOR SMALL-SIGNAL SECURITY ASSESSMENT OF POWER SYSTEMS

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Abstract - This paper presents a small-signal security assessment tool (SSAT), which integrates the latest in the development of computational algorithms with the analysis requirements from the industry. It provides advanced features for the small-signal security assessment as well as extensive capabilities for conventional small-signal stability analysis. The theoretical foundations of SSAT are described and its computational capabilities illustrated with numerical examples.

Keywords: *Small-signal stability, Eigenvalue, Modal analysis, Small-signal security, Dynamic security*

I. INTRODUCTION

Small-signal stability, along with transient and voltage stability, determines the dynamic characteristics of a power system [1]. The small-signal stability problem of a power system is usually one of insufficient damping of system oscillations and its analysis is normally based on the linearized system dynamic equations using modal (eigenvalue) analysis techniques.

Small-signal security assessment of a power system refers to the small-signal stability analysis of the system under a set of credible contingencies for a range of feasible operating conditions. The system is small-signal secure if the damping of all critical modes in the system are within a required threshold. Small-signal security assessment is the sibling of the other two forms of the dynamic security assessment, namely transient and voltage security assessment. Together, they determine the overall dynamic security status of the system.

With the growth of interconnected power systems, many problems related to small-signal stability have been reported, including major incidents [2]. Recently, as the power industry goes through the deregulation process and open access to the grid becomes possible, system security is often of critical concern, particularly for contingency and operating conditions that are subject to security constraints (including small-signal security) [3]. This is a direct consequence of the reduced generation reserve margins and increased power transactions in many of the systems. On the planning side, addition of new generations into existing power pools also requires extensive assessment of their impact on the damping of the systems [4].

The theory and analysis method for small-signal stability problem have been well established [1]. Algorithms applicable to the eigenvalue analysis of large systems are available [5,6,7,8]. Several computer programs based on these algorithms have been developed [2]. These theoretical and developmental breakthroughs in the past ten years have greatly helped the understanding and advancement of the small-signal stability analysis. They are now routinely used in the modal analysis of power systems and the control system design and tuning to improve system damping. Most of the currently available analysis tools, however, are designed mainly to perform eigenvalue analysis, without appropriate capabilities for security assessment. In addition, some of the tools are developed with specific modeling assumptions and requirements that may not be fully compatible with models used in the traditional transient stability analysis using the time-domain simulation approach. This can make it difficult to validate the small-signal analysis results.

This paper introduces a small-signal security assessment tool (SSAT), which is designed to integrate the latest in the development of computational algorithms with the small-signal security assessment requirements from the industry. The computation and study features of SSAT are described in details in the paper, illustrated by two numerical examples.

II. MOTIVATIONS OF THE DEVELOPMENT

The main driving force of the SSAT development is the demand from the power industry for an analysis tool that allows the extensive studies of a wide range of the small-signal stability problems, such as

- conventional eigenvalue analysis for identification of poorly damped modes and design of control systems to improve damping.
- security assessment capabilities to help determine planning and operating guidelines for systems with existing or potential damping problems.
- validation and tuning of generator and associated control models.
- potential on-line security assessment.

From the application point of view, there are three basic requirements for SSAT:

- *Modelling capability*:
  - compatibility with the standard models used in traditional transient stability analysis.
  - advanced user-defined modeling capabilities for representation of non-standard generator control devices, HVDC, and FACTS devices.

- *Computational features*:
  - basic eigenvalue solvers for different types of modes.
  - advanced security assessment capabilities for contingency analysis, transaction analysis, etc.

- *User friendliness*:
  - this requires a good user interface for input data processing, computation monitoring, and output result analysis. These issues are outside the scope of this paper and will be discussed in a separate paper.

As described earlier, very powerful algorithms exist for eigenvalue computations of large power systems. Therefore, SSAT is not focused on the further development of eigenvalue algorithms; rather, the main attention is to provide the required level of modelling support and to make available the necessary analysis capabilities. On the eigenvalue computation side, the state-of-the-art algorithms are employed with enhancements to achieve superior performance.

III. SOLUTION METHODS

The solution methods are implemented in SSAT at two levels:

- The first level consists of basic computation algorithms; most of them are different types of eigenvalue solvers used by the security assessment options at the second level.

- The second level consists of various security assessment options.

This section is focused on the security assessment options, with a brief description of the eigenvalue solvers used at the first level.

3.1 Eigenvalue solvers

Three basic eigenvalue solvers are available in SSAT to meet different computation objectives:

- The classic QR method. This is used for computation of all system eigenvalues. In SSAT, the performance of this algorithm is enhanced by using the Intel Math Kernel Library (MKL) (available at http://developer.intel.com/vtune/perflibst/mkl/index.htm). MKL contains the BLAS routines for matrix-vector and matrix-matrix operations, fully optimized on the Intel Pentium-family microprocessors. Testing using SSAT has shown that its speed can be 100% faster than the code with the conventional optimization. This makes it feasible to analyze relatively large systems within reasonable computation time. Complete eigenvalues of systems with up to 3,000 dynamic states (representing approximately 300 generators with detailed models) have been computed on mid-range PCs with good speed performance and accuracy.

- Implicitly Restarted Arnoldi Method (IRAM) [9]. This is an enhanced version of the Modified Arnold Method (MAM). Instead of explicitly restarting an iteration using a trial vector in MAM, IRAM restarts an iteration by performing matrix transformations for the results obtained in the previous iteration. This reduces the number of time-consuming matrix-vector multiplication required in building the Hessenberg matrix and allows the use of a technique known as polynomial filtering to enhance the convergence. The resulting algorithm is faster and has better convergence. IRAM is used in SSAT,
  - to compute eigenvalues within a frequency and damping range, or close to a specified location
  - to determine small-signal stability index
  - as the base eigenvalue solver in eigenvalue sensitivity analysis, small-signal stability limit determination, and mode trace

- The enhanced AESOPS algorithm [8]. This is a type of selective eigenvalue analysis method and is found to be very effective in computing modes (mostly local) related to a specified generator in a large system model. This allows the efficient studies of local modes without the need to reduce the system model or to perform excessive mode searches. A brief description of the enhanced AESOPS algorithm used in SSAT is described in Appendix.

3.2 Small-signal security assessment

The small-signal security assessment options in SSAT are designed to address the following two key issues:

- Determination of the small-signal security status of a system at a specific operating point. This is measured with a small-signal security index.

- Determination of the system operating limit subject to the small-signal security constraint. This is performed using the transaction analysis with a specified small-signal security index threshold.

In both cases, a set of contingencies can be considered. The small-signal security assessment for a contingency applies to the post-contingency steady-state powerflow condition.

Small-signal security index

The small-signal security index of a system is defined as the damping ratio of the least stable rotor angle mode. This definition excludes the control modes that are mostly confined in the control systems of dynamic devices. Therefore they are normally not of concern for the system security.

To compute the small-signal security index, the following unit-circle (Mobius) transformation algorithm [7] is applied to the system state matrix A:

$$S = (A - s_1 I)^{-1}(A - s_2 I) \tag{1}$$

This transformation can map a constant damping ratio line on the complex plane into a circle of the unit radius, as shown in Figure 1.

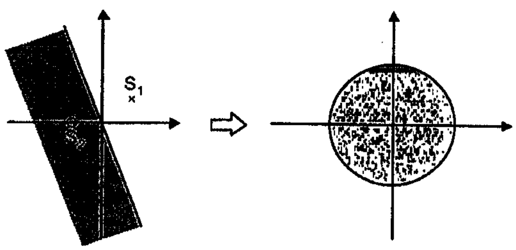

Figure 1 – Unit-circle mapping

Therefore, the determination of the least stable mode is converted to the computation of the mode with the largest modulus which can be done easily for large systems with a number of partial eigenvalue solvers. In the implementation, the computation is carried out for *n* modes of the largest modulus to ensure that at least one rotor angle mode is caught in the computation.

One of the direct applications of the small-signal security index is the ranking and identification of critical contingencies. This is illustrated by an example in Section V. This index is further used in the small-signal stability limit determination, as described in the following.

Small-signal stability limit determination

In the small-signal stability limit determination problem, a power transaction is scheduled in a system. The objective is to find if the transaction is secure for a given small-signal security index threshold, and if not secure, what is the secure power transfer level that does not violate the index threshold. This problem is very similar to the determination of the stability limits with the transient or voltage security constraint [3].

A power transaction is referred to as a specified power transfer arrangement between groups of generators or loads. The change of generation/load in one part of the system (sink) is specified and is to be balanced by a change of the generation/load in another part of the system (source).

The process to determine the small-signal stability limit for a power transaction (this is also referred to as transaction analysis) is as follows. Referring to Figure 2, the power transfer in the specified transaction is dispatched at P_0 (base), P_1, ..., P_s, P_u, according to certain rules (for example, using an iterative binary search method) and the corresponding small-signal security indices (ζ) are computed. The stability limit is found when the security index goes across the threshold value (ζ_t). Computationally, the stability limit can be determined when the difference between the last secure power transfer (P_s) and the first insecure power transfer (P_u) is small enough.

The above process is applied for all contingencies to find the final small-signal stability limit for the transaction.

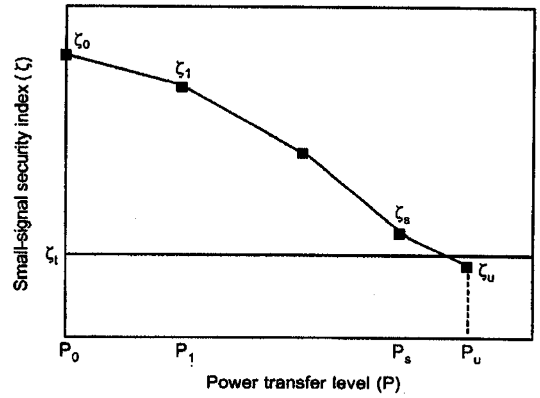

Figure 2 – Stability limit search

Mode trace

A variation of the small-signal stability limit determination problem is to trace a number of specified modes for a given transaction with a set of contingencies. This is useful if it is known that these modes are most critical to the system security even though they may not necessarily be the least stable modes for the particular operating condition. SSAT is able to handle this situation with the mode trace computation option. In this option, modes computed for the base case can be traced under different power transfers and/or with different contingencies. Two modes in two different system conditions are matched if

- the location of the dominant entries of the right eigenvectors are the same, and
- the 2-norm difference of the normalized right eigenvectors is small enough.

3.3 Contingency analysis

Contingency analysis is an important part of a security assessment process. In SSAT, all analysis options (except for analysis of the single-machine-infinite-bus configurations) can be applied to any number of contingencies consisting of combinations of the following disturbance types:

- Bus outage
- Branch outage
- Generator outage
- Load shedding

In a contingency analysis, an integrated powerflow solver solves the post-contingency powerflow after applying the disturbances to the network, and then the specified analysis option is performed for the post-contingency powerflow condition.

IV. CAPABILITIES OF SSAT

To fulfill different objectives in small-signal security assessment, the following computation options are available in SSAT:

(1) *Complete eigenvalue analysis.* This is to compute all modes of the system, with three possible configurations:
- The entire or a selected portion of the system
- The single-machine-infinite-bus (SMIB) simplification for all generators in a selected portion of the system
- A fully customizable SMIB system

(2) *Computation of modes closest to a specified value.* A specified number of modes of the system closest to a value (frequency and damping) anywhere on the complex plane can be computed.

(3) *Computation of modes within specified range.* All modes within a specified range (frequency/damping or real/imaginary) on the complex plane can be computed in SSAT. One application of this feature is to compute all modes within a specified frequency range with a damping threshold.

(4) *Computation of modes related to a generator.* This option computes all modes dominant in a specified generator. This is useful to find local (or interarea) modes related to the generator.

(5) *Response calculations.* Frequency or step (time) response computations from the linearzed system model (Single Input Multiple Outputs (SIMO) computation model) are provided in SSAT. This feature is implemented for control system design and tuning.

(6) *Sensitivity analysis.* Sensitivities of a mode with respect to any of the following operating conditions and system parameters can be computed in SSAT:
- Generator outputs
- Bus voltages
- Branch power flows
- Load powers
- Parameters in dynamic models

The sensitivities are obtained by the numerical perturbation method in which a sensitivity is determined by computing a pair of modes, one with the value in the base case (or a post-contingency case) and the other with a "perturbed" value.

(7) *Small-signal security index computation.* A small-signal security index (see Section 3.2 for discussions) can be computed for the base and all specified post-contingency conditions.

(8) *Small-signal stability limit determination.* This is also referred to as the transaction analysis. The definition and procedure of a transaction analysis are discussed in Section 3.2.

(9) *Mode trace.* This feature traces a number of specified modes for a set of contingencies and a given power transaction. Details are discussed in Section 3.2.

Modelling capability

SSAT supports power system network and dynamic device models commonly used in the transient stability analysis. It is made to be fully compatible with some of the largest system models, such as the eastern US-Canada interconnected model series compiled by the System Dynamics Database Working Group (SDDWG) of NERC. User-defined models are supported to represent non-standard models and HVDC/FACTS models.

V. NUMERICAL EXAMPLES

Two test systems shown in Table 1 are considered to illustrate the capabilities of SSAT.

Configuration	System 1	System 2
No. of buses	557	26,282
No. of generators	102	3,802
No. of states	1,062	37,563

Table 1 – Summary of the test systems

5.1 Test System 1

Figure 3 shows the critical portion of this system. Power from 10 units at buses 100, 108, and 109 is transferred through four 345-kV circuits (A, B, C, and D) to the rest of the system. It is known that this power transfer is subject to stability constraint. The transfer is at 3182 MW in the base case and under this condition an interarea mode is found at 0.837 Hz with a damping of 2.88%. In this mode, the portion of the system shown in Figure 3 swings against the rest of the system. Two problems are examined:

- This mode is traced for all N-1 and N-2 contingencies in the entire 345 kV network (total over 5,500 contingencies). This is to identify critical contingencies that may cause severe damping concerns.

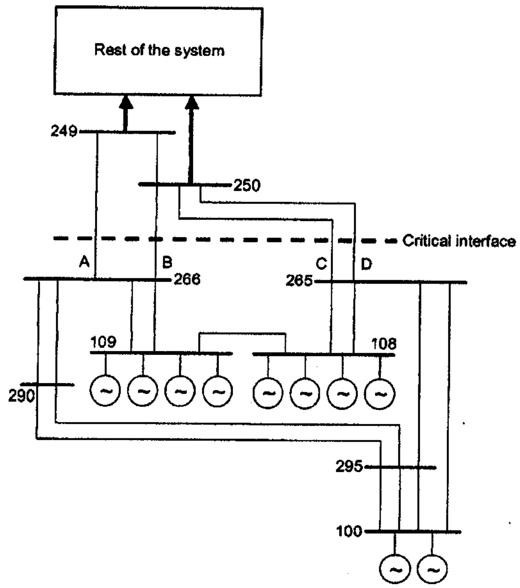

Figure 3 – Schematics of test system 1

- For a damping criterion of 3% to ensure the small-signal security, the maximum secure power transfer at the critical interface is to be found.

Contingency analysis

The interarea mode is traced for all post-contingency conditions. It is found that the damping of this mode would become less than 1% for 17 contingencies. Table 2 shows the 5 worst cases. Note that the damping of this mode goes to nearly zero for a couple of contingencies. This should cause concerns for the operation of the actual system.

Contingency	Frequency (Hz)	Damping Ratio (%)
1	0.699	0.061
2	0.766	0.169
3	0.710	0.360
4	0.711	0.416
5	0.793	0.423

Table 2 – 5 worst contingencies for damping

Small-signal stability limit determination

As shown in the contingency analysis, the base case has very low damping under some contingencies and this indicates that the base power transfer at the critical interface would be insecure for the 3% damping criterion. To find the secure power transfer, a transaction to gradually reduce the power transfer at the critical interface is defined in which the generation from the 10 units is decreased while the generation from the rest of the system is increased. SSAT automatically dispatches the powerflow at the specified transfer levels according to this rule and traces the required mode for specified contingencies at each powerflow level.

For purpose of illustration, Figure 4 shows the computation results for the no-fault case and the two worst contingencies. It is clear that the 3% damping is satisfied for the no-fault case and the two contingencies when the transfer is reduced to 2445 MW. This is the required stability limit.

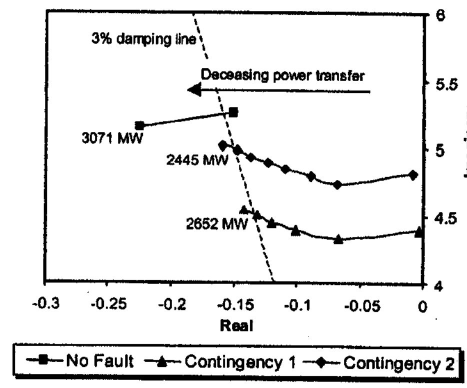

Figure 4 - Small-signal stability limit

It is interesting to note from Figure 4 that the power transfer is limited by contingency #2 which has slightly higher damping than contingency #1 at the base power transfer level. This indicates the highly complicated relationship between the power transfer and damping under various contingencies. It is therefore important to include sufficient number of critical contingencies in the stability limit determination.

5.2 Test System 2

This is a very large system model, representing the entire eastern US-Canada interconnected system. Two examples are shown:

- Computation of interarea modes
- Computation of a specified local mode

In both cases, the full system representation is kept, so no special model reduction is required to perform these computations.

Computation of interarea modes

The computation of interarea modes can be most efficiently done by using the computation option to calculate modes within a specified frequency range. In this example, all modes within the frequency range of 0.2 and 0.6 Hz with damping less than 10% are computed and Table 3 shows all 9 interarea modes found.

No.	Frequency (Hz)	Damping Ratio (%)
1	0.306	8.53
2	0.366	3.92
3	0.380	2.31
4	0.416	3.51
5	0.446	3.04
6	0.468	3.60
7	0.549	2.51
8	0.563	3.82
9	0.600	3.95

Table 3 – Interarea modes of Test System 2

Figure 5 shows the mode shape of the first mode (at 0.306 Hz). Each symbol in the figure represents the normalized right eigenvector entry for a generator. From this mode shape, it is seen that the generators in the eastern portion of the system have a large phase angle (close to 180°) against the generators in the western portion of the system. Therefore, this mode represents an east-west (interarea) oscillation in the system.

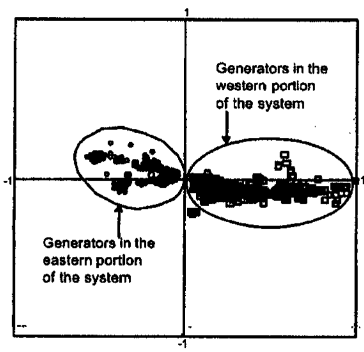

Figure 5 – Mode shape of the 0.306 Hz mode

Computation of a specified local mode

The objective of this example is to find the local mode at the Rush Island generating units in the Ameren UE area. This mode is the focus of several investigations [10] after the oscillation incident in 1992 involving the Rush Island units.

This mode occurred as a result of a contingency that effectively disconnected two of three circuits connecting the Rush Island units to the rest of the system. Under this condition, a local mode around 1 Hz at Rush Island may become poorly damped to cause sustained oscillations. To find this mode, the option in SSAT to compute modes related to a generator is used, after applying the contingency to the base case. This mode turns out to be at 1.28 Hz with a damping ratio of 4.19%. Figure 6 shows a time-domain simulation verification performed using the full nonlinear simulation in which one of the Rush Island unit speed is plotted. The simulation clearly shows an oscillation at about 1.3 Hz.

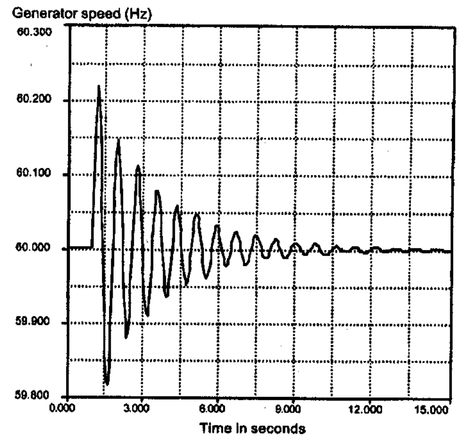

Figure 6 – Simulation verification of the Rush Island mode

The significance of this example is to show the capability of SSAT to selectively compute local modes in a large model. Using the usual eigenvalue analysis approach, this kind of computation would likely need preliminary model reduction work, or an extensive mode scan in the crowded local mode frequency range. Being able to directly locate the required mode with the base study model helps significantly improve the efficiency of the studies.

VI. CONCLUSIONS

This paper presents a tool (SSAT) for small-signal security assessment of power systems. It is developed as a result of the calls from the power industry for a program to meet the increasing need of system studies. The focus of this development has been to provide superior modelling support and capabilities for the security assessment, while taking advantages of the recent advancement in the basic computational algorithm development (such as eigenvalue solvers). The theoretical foundations of SSAT are described and its computational capabilities illustrated with numerical examples.

VII. ACKNOWLEDGEMENT

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Appendix – The Enhanced AESOPS Algorithm

The linearized power system dynamic model in the Laplace domain can be expressed in the following general form:

$$sx_1 = a_{11}x_1 + A_{12}X_2 + B_1V \quad \text{(A-1)}$$

$$sX_2 = A_{21}x_1 + A_{22}X_2 + B_2V \quad \text{(A-2)}$$

$$0 = C_1x_1 + C_2X_2 + DV \quad \text{(A-3)}$$

where $[x_1\ X_2^T]^T$ is the state vector and V is the voltage vector. x_1 is any state in the system. Eliminating X_2 and V from (A-1)-(A-3), we obtain

$$sx_1 = [a_{11} - B'D'^{-1}(s)C']x_1 \quad \text{(A-4)}$$

In the above, matrices B′, C′, and D′(s) can be easily deduced from (A-1)-(A-3). It is clear that any value of s satisfying (A-4) is an eigenvalue of the system. We now solve an eigenvalue λ from (A-4) using the Newton method. First notice that, after neglecting the higher order terms, $D'^{-1}(\lambda)C'$ can be expressed as

$$D'^{-1}(\lambda)C' \approx D'^{-1}(\lambda_k)C' + D'^{-1}(\lambda_k)I_0D'^{-1}(\lambda_k)C'(\lambda - \lambda_k) \quad \text{(A-5)}$$

where λ_k is an approximate value of the eigenvalue and $I_0 = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}$. Substituting (A-5) into (A-4) yields the following formula to iteratively compute an eigenvalue:

$$\lambda_{k+1} = \lambda_k + J^{-1}(\lambda_k)[a_{11} - B'D'^{-1}(\lambda_k)C' - \lambda_k] \quad \text{(A-6)}$$

where $J(\lambda_k) = 1 + B'D'^{-1}(\lambda_k)I_0D'^{-1}(\lambda_k)C'$. In this algorithm, $B'D'^{-1}(\lambda_k)$ and $D'^{-1}(\lambda_k)C'$ can be efficiently computed based on a sparse formulation, such as the one described in [1]. The algorithm (A-6) converges quadratically, and if the state x_1 is selected to be the speed of a generator, it has exhibited very good convergence to a mode that is mostly dominant in the generator.