A COMPREHENSIVE COMPUTER PROGRAM PACKAGE FOR SMALL SIGNAL STABILITY ANALYSIS OF POWER SYSTEMS

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Abstract – A powerful package of integrated programs for small signal stability analysis of large interconnected power systems is described. The package has extensive modelling capability and uses alternative eigenvalue calculation techniques, making it suitable for the analysis of a wide range of stability and control problems. Results of eigenvalue calculations for three power systems of differing size and complexity are presented and the accuracy, consistency and convergence of the alternative calculation methods are discussed.

Key Words – Power System Stability – Modal Analysis – Eigenvalues – Eigenvectors – Frequency Response – Power System Control – Large Systems

INTRODUCTION

Power system stability was first recognized as an important problem in the 1920s [1]. From the beginning, for convenience in analysis, gaining a better understanding of the nature of stability problems and developing solutions to the problems, it has been the usual practice to classify power system stability into two broad categories:

- Large disturbance stability; and
- Small disturbance stability.

The traditional large disturbance stability problem is related to the short term or transient period, which is usually limited to a few seconds following the disturbance. It is concerned with the system response to a severe disturbance, such as a transmission system fault. Much of the electric utility industry effort and interest related to system stability to date have been concentrated on the short-term response, and as a result the system is designed and operated so as to meet a set of reliability criteria concerning transient stability. Well established analytical techniques and computer programs exist for the analysis of transient stability. In recent years, the need for studying the response of the system for longer periods has been recognized, and the terms mid-term and long-term stability have been introduced. Analytical tools for studying these aspects of system stability are evolving.

Small disturbance or small signal stability is concerned with the system response to small changes and is a fundamental requirement for the satisfactory operation of power systems. Usually, the problem is one of ensuring sufficient damping of system oscillations. In general, the stability properties of the oscillatory modes do not depend on the size of the system disturbance and hence can be analysed by considering the system linearized about an equilibrium point represented by a steady state operating condition. This allows the use of powerful analytical methods applicable to linear systems to determine the stability characteristics which aid in the design of corrective controls. The analysis of small signal stability is not as wide spread as transient stability analysis. There are no standard study procedures or commonly accepted performance criteria with regard to small signal stability. Many utilities take small signal stability for granted and carry out no studies at all or depend on transient stability simulations to reveal problems related to small signal performance. This is primarily because, in the past, a system which remained stable for the first few seconds following a severe disturbance was sure to be stable for small perturbations about the post fault system condition. This is not true for present day systems. As power systems develop, the need to conduct small signal studies and to take measures to ensure that sufficient stability margins exist is being increasingly recognized, and a number of special purpose small signal stability programs have been developed [2-9].

One of the deterrents to the wide spread use of small signal stability programs has been their computational and modelling limitations. This situation has changed and recent developments in computational methods allow the analysis of large complex power systems.

In this paper, we will describe a comprehensive computer program package for the analysis of small signal stability, developed at Ontario Hydro with cofunding by EPRI

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METHODS OF SMALL SIGNAL STABILITY ANALYSIS

The small signal stability of a power system may be analysed using any of the methods applicable to linear systems. However, the modal analysis approach using eigenvalue techniques has many advantages.

Conventional eigenvalue programs form the state matrix of the system and then use a general purpose routine to compute all eigenvalues of the matrix. Most commonly, routines for eigenvalue calculation use the QR transformation method, originally developed by Francis [11]. The method is robust and converges rapidly, and is available in a number of very good general purpose commercial codes. The following are the strengths of conventional eigenvalue based small signal stability programs:

- All modes are clearly separated and identified by the eigenvalues.
- Mode shapes and the relationships between different modes and system variables or parameters are easily identified using eigenvectors.

However, the conventional eigenvalue based stability programs require large amounts of computer storage. The eigenvalues are computed from the system state matrix, which has no special structure that can be exploited so as to be able to analyse very large systems. This restricts the size of the system to a few hundred dynamic states.

To overcome the size limitations of the conventional eigenvalue programs, special techniques have been developed that focus on evaluating a selected subset of eigenvalues associated with the complete system response. One such technique is the AESOPS (Analysis of Essentially Spontaneous Oscillations in Power Systems) algorithm [4], developed under the EPRI project RP744-1.

It uses a novel frequency response approach to calculate the eigenvalues associated with the rotor angle modes. References [5] and [6] describe improved implementation of the AESOPS algorithm.

A number of other powerful methods for the computation of eigenvalues associated with a small number of selected modes of oscillation have been published in the literature on power system stability [7]-[9]. Reference [7] describes the application of two sparsity-based eigenvalue techniques: simultaneous iterations and modified Arnoldi method. The S-method described in [8] is suited for finding the unstable modes. The Selective Modal Analysis (SMA) approach described in [9] computes eigenvalues associated with selected modes of interest by using special techniques to identify variables that are relevant to the selected modes, and then constructing a reduced order model that involves only relevant variables.

Each of the methods described above has special features which make it attractive for a particular type of application. However, none of these methods satisfies all the requirements of small signal stability analysis of power systems. The best solution, therefore, is to use several techniques in a complementary manner.

SMALL SIGNAL STABILITY PROGRAM PACKAGE

Under a joint effort with EPRI (Project RP2447-1), Ontario Hydro has developed a comprehensive Small Signal Stability Program (SSSP) package which provides facilities for representing the system components with a wide range of models and for computing eigenvalues using several alternative techniques. In addition, the program package computes additional information about the system dynamic characteristics which enhance the overall understanding of the phenomena being investigated and assist in the design of corrective controls.

The SSSP package has two principal constituent programs:

- Multi-Area Small Signal stability program (MASS), and
- Program for Eigenvalue Analysis of Large Systems (PEALS).

MASS is ideally suited for detailed analysis of small to medium sized power systems and for use in the design of controls. PEALS, on the other hand, by limiting the eigenvalues calculated to those associated with a few selected modes, is able to determine the stability characteristics of very large systems with no compromises in modelling detail.

In addition, the package contains a Dynamic Data Bank Program which facilitates the storage and retrieval of data for dynamic devices, required for running MASS and PEALS. Currently a System Reduction Program is being developed as part of the SSSP package.

Modelling Capabilities of MASS and PEALS

An important requirement for any stability program is the detail and flexibility of representing power system components. The SSSP package has the ability to represent all standard models normally used in stability studies. In addition, provision is made for user-defined models to accommodate special modelling requirements. The following is a summary of modelling capabilities of MASS and PEALS.

Synchronous machines:

> Up to 3 rotor circuits may be represented in each axis. Saturation may be represented by separate characteristics in the d and q axis. Turbine-generator rotor may be represented as a multimass shaft system having up to 9 masses.

Excitation and Speed Governing Systems:

> All standard IEEE models may be represented. User-defined models allow representation of nonstandard models and special control schemes. Excitation systems may have up to 14 control inputs of which 4 can be associated with remote buses. Speed governors may have speed, power and one remote bus angle as control inputs.

Loads:

> Static loads may be represented as either constant impedance loads or nonlinear voltage dependent loads. Dynamic loads may be represented as induction or synchronous motor loads.

HVDC links:

> A two terminal DC link model with line dynamics and equidistant firing angle control is provided as a standard model. Modulation using signals from local and a remote AC bus voltage may be represented. The user-defined control model allows detailed representation of any specific two terminal DC installation, with up to 23 control inputs, 8 of which are associated with remote buses.

Static Var Compensators:

> Two options are provided, one accounts for the thyristor controlled reactor nonlinearities and the other assumes perfect linearity. The user-defined model allows representation of special controls, with up to 12 control inputs, 7 of which are associated with remote buses.

User-Defined Models

Facilities for representing controls associated with generating units, HVDC links and SVCs by user-defined models offer practically unlimited freedom in modelling these devices. User-defined models are constructed by interconnecting elementary blocks, defining their inputs in terms of system quantities and their outputs as a combination of the outputs from the elementary blocks. The blocks and their interconnections are specified by the user as part of the dynamic data and no additional programming is required. The menu of elementary blocks consists of normal linear system transfer function elements together with non-linear elements which are automatically linearized within the program. The non-linear elements vary with the type of device being modelled, for example excitation system user-defined models may use, as output devices, all the recommended IEEE exciters with the AVR specified by linear transfer function blocks. System inputs may be chosen from local and remote bus voltages and angles, line real and reactive powers, and line currents. Each user-defined device has a predetermined output. For example, the output of an excitation system is field voltage and that of a dc converter is firing angle.

Formulation of Device State Equations

The component connection modelling technique [10] is used for construction of the state equations for each device. The state matrix is assembled from the individual component models of the elements of the devices.

The linearized model for each dynamic device is expressed in the following form:

$$\dot{\mathbf{x}}_d = \mathbf{A}_d\mathbf{x}_d + \mathbf{B}_d\Delta\mathbf{v} \tag{1}$$

$$\Delta\mathbf{i}_d = \mathbf{C}_d\mathbf{x}_d - \mathbf{Y}_d\Delta\mathbf{v} \tag{2}$$

where

- $\mathbf{x}_d$ are the perturbed values of device state variables
- $\mathbf{i}_d$ is the current injection into the network from the device
- $\mathbf{v}$ is the vector of the network bus voltages
- Δ is a prefix representing perturbed values

In (1) and (2), $\mathbf{B}_d$ and $\mathbf{Y}_d$ have non-zero elements corresponding only to the dynamic device terminal voltage and any remote bus voltage used to control the device.

The formation of the state matrix for each user-defined control, and its integration with the state matrices of other devices is performed similarly to the formation used for standard models.

SPECIAL FEATURES OF MASS

The MASS program forms the state matrix of the interconnected power system and computes all eigenvalues of the matrix using the QR transformation method. It also computes right/left eigenvectors and uses these to compute:

- participation matrix;
- time response of generator quantities to step or impulse input; and
- steady-state frequency response to a sinusoidal input signal.

The right eigenvector gives the "mode shape", i.e. the relative activity of the state variables when a particular mode is excited. One problem in using the eigenvectors for identifying the relationship between the state and the modes is that the eigenvectors are dependent on units and scaling associated with the state variables. However, the participation matrix, which combines the right and left eigenvectors, provides a better indication of the association between the state variables and the modes [9]. We, therefore, use the angles of the eigenvector elements to give the phase relationships between state variables, and the magnitude of the participation matrix elements to give the net influence of a state variable on a mode.

The MASS program is capable of representing the power system either as a single area or as multiple areas. In the single area representation, the system is analysed as a whole. In the multiarea representation, the overall system is split into a number of areas whose modal characteristics are first analysed individually. The model of each area is reduced by mode elimination and the reduced area models are interconnected to compute eigenvalues of the interconnected system.

Formulation of the System State Matrix in MASS

The state equations of (1) and (2) for all the dynamic devices may be combined into the following form:

$$\dot{\mathbf{x}} = \mathbf{A}_D\mathbf{x} + \mathbf{B}_D\Delta\mathbf{v} \tag{3}$$

$$\Delta\mathbf{i} = \mathbf{C}_D\mathbf{x} - \mathbf{Y}_D\Delta\mathbf{v} \tag{4}$$

where $\mathbf{x}$ is the state vector of the complete system, and $\mathbf{A}_D$ and $\mathbf{C}_D$ are block diagonal matrices composed of $\mathbf{A}_d$ and $\mathbf{C}_d$ associated with the individual devices. $\mathbf{B}_D$ and $\mathbf{Y}_D$ are not block diagonal in general due to remote sensing. Note that the current injection vector $\Delta\mathbf{i}$ has non-zero elements only for dynamic device buses.

The interconnecting network is represented by the node equation:

$$\Delta\mathbf{i} = \mathbf{Y}_N\Delta\mathbf{v} \tag{5}$$

The elements of admittance matrix $\mathbf{Y}_N$ include the effects of voltage dependent nonlinear loads which, for small signal analysis, can be represented by equivalent shunt admittances [10].

The overall state equation is formed by eliminating $\Delta\mathbf{i}$ and $\Delta\mathbf{v}$ from (3), (4) and (5), to give

$$\dot{\mathbf{x}} = [\mathbf{A}_D + \mathbf{B}_D(\mathbf{Y}_N + \mathbf{Y}_D)^{-1}\mathbf{C}_D]\mathbf{x} = \mathbf{A}\mathbf{x} \tag{6}$$

The state matrix $\mathbf{A}$ of the complete system when represented as a single area is, thus, given by:

$$\mathbf{A} = \mathbf{A}_D + \mathbf{B}_D(\mathbf{Y}_N + \mathbf{Y}_D)^{-1}\mathbf{C}_D \tag{7}$$

For multiarea representation the same general approach is used. Each area state equations are expressed in the form (3) and (4), and the eigenvalues and eigenvectors of the state matrix are determined with the interarea ties open. These are used to express the area state equations in the decoupled (modal) form by transformation of the state variables:

$$\mathbf{x}_a = \mathbf{U}_a\mathbf{z}_m \tag{8}$$

where

$\mathbf{x}_a$ are area state variables

$\mathbf{U}_a$ are eigenvectors of area state matrix

$\mathbf{z}_m$ are the new state variables related to area modes

The transformation of the area state equations into the modal form allows system reduction by mode elimination. The reduced state equations are then interconnected through tie-line node equations to give the reduced state matrix of the overall system.

SPECIAL FEATURES OF PEALS

PEALS has the ability to represent very large systems. Unlike MASS, it does not formulate the system state matrix. Instead, the state equations associated with the dynamic devices and the interconnecting network equations are organized so as to exploit sparsity and allow use of special techniques which evaluate eigenvalues associated with a selected subset of system modes.

At present, the following two alternative techniques are employed in PEALS to compute eigenvalues: AESOPS algorithm, and modified Arnoldi method (MAM).

Among the eigenvalue calculation methods applicable to very large systems, we found these two methods particularly attractive. They are quite efficient and complement each other in meeting the different requirements of PEALS applications.

The AESOPS algorithm computes only the rotor angle modes. One complex pair of eigenvalues and the corresponding speed components of the related eigenvector are computed at a time.

The modified Arnoldi method allows the determination of a small number (typically up to five) of system eigenvalues close to a specified eigenvalue. The method is a generalized Galerkin method and is similar to the Lanczos method, but more reliable [7]. It was first discussed in [12] based on the classical Arnoldi method, and its application to power system stability analysis was proposed in [7]. Unlike the AESOPS algorithm, it can compute eigenvalues associated with any system mode, not just the rotor angle modes.

In addition to eigenvalues, PEALS has facilities for computing the following:

- participation factors;
- network bus voltage change vector; and
- frequency response.

Bus voltage changes are helpful in identifying ideal locations for adding voltage control devices such as SVCs.

Implementation of AESOPS algorithm

The AESOPS algorithm is derived from the linearized equation of motion of a generator:

$$\frac{2H}{\omega_0}s\Delta\omega = \Delta T_m - \left[K_S(s)\frac{\Delta\omega}{s} + K_D(s)\Delta\omega\right] \tag{9}$$

The above equation recognizes that synchronizing and damping torque coefficients K_S and K_D are functions of complex frequency s. Rearranging (9)

$$\Delta T_m = \left[\frac{2H}{\omega_0}s + K_D(s) + \frac{K_S(s)}{s}\right]\Delta\omega \tag{10}$$

The eigenvalues of the system are given by the zeros of

$$\frac{2H}{\omega_0}s + K_D(s) + \frac{K_S(s)}{s} = 0 \tag{11}$$

It can be seen that, provided $\Delta\omega$ is not zero, the eigenvalues are those values of s which force ΔT_m to be zero. This is used by the AESOPS algorithm, which determines zeros of (11) by setting the complex speed deviation of the disturbed generator to ω_0+j0 rads/sec (or 1.0+j0 per unit). The external torque ΔT_m is determined in the solution process. As iterations converge, the magnitude of the external torque tends to zero and provides an indication of the accuracy of the eigenvalue.

The zeros of $\Delta T_m(s)$ may be determined by using the Newton's method. This requires the derivative of ΔT_m with respect to s. From equation (10)

$$\frac{\partial(\Delta T_m)}{\partial s} = \left[\frac{2H}{\omega_0} + \frac{\partial K_D(s)}{\partial s} + \frac{1}{s}\frac{\partial K_S(s)}{\partial s} - \frac{K_S(s)}{s^2}\right]\Delta\omega \qquad (12)$$

Substituting for K_S/s^2 from (11), and with K_D, $\partial K_D/\partial s$ and $\partial K_S/\partial s$ negligibly small, we have from (12)

$$\frac{\partial(\Delta T_m)}{\partial s} \approx \frac{4H}{\omega_0}\Delta\omega \qquad (13)$$

The Newton-Raphson method for the iterative solution of s is given by:

$$s_{n+1} = s_n - \left[\frac{\Delta T_m(s)}{\frac{\partial(\Delta T_m)}{\partial s}}\right]_{s=s_n} = s_n - \left[\frac{\Delta T_m(s)}{4H\frac{\Delta\omega}{\omega_0}}\right]_{s=s_n} \qquad (14)$$

For modes which involve many machines this makes too large of a change in the eigenvalue at each iteration. In such cases, $\partial K_S/\partial s$ and $\partial K_D/\partial s$ are not small. Therefore, an equivalent inertia is used so that kinetic energy associated with the change in speed at all machines in the system is equal to the equivalent inertia multiplied by the square of the speed change of the disturbed machine. The equivalent inertia is given by:

$$H_e = \sum_{i=1}^{N_m} H_i \frac{|\Delta\omega_i|^2}{\omega_\delta^2} \qquad (15)$$

with N_m = no. of machines.

Equation (14) is then modified to

$$s_{n+1} = s_n - \frac{\Delta T_m(s_n)}{4H_e} \qquad (16)$$

where, as discussed earlier, $\Delta\omega$ has been set to ω_0. The complex pair of eigenvalues are computed by solving the above equation iteratively. The torque ΔT_m necessary to keep the speed change of a chosen disturbed machine to ω_0 and the speed changes $\Delta\omega_i$ of all other machines depend on the system equations (1), (2) for each dynamic device and the network equations (5).

From (1), for any complex frequency s,

$$s\mathbf{x}_d = \mathbf{A}_d\mathbf{x}_d + \mathbf{B}_d\Delta\mathbf{v} \qquad (17)$$

Therefore,

$$\mathbf{x}_d = (s\mathbf{I} - \mathbf{A}_d)^{-1}\mathbf{B}_d\Delta\mathbf{v} \qquad (18)$$

Substituting into (2), we have the following expression for Δi_d in the s domain,

$$\Delta\mathbf{i}_d = \mathbf{C}_d(s\mathbf{I} - \mathbf{A}_d)^{-1}\mathbf{B}_d\Delta\mathbf{v} - \mathbf{Y}_d\Delta\mathbf{v} = -\mathbf{Y}_{de}(s)\Delta\mathbf{v} \qquad (19)$$

where

$$\mathbf{Y}_{de}(s) = [\mathbf{Y}_d - \mathbf{C}_d(s\mathbf{I} - \mathbf{A}_d)^{-1}\mathbf{B}_d] \qquad (20)$$

$$= [\mathbf{Y}_d - \mathbf{C}_d\mathbf{U}(s\mathbf{I} - \Lambda)^{-1}\mathbf{U}^{-1}\mathbf{B}_d] \qquad (21)$$

with

Λ = diagonal matrix of eigenvalues of the device

$\mathbf{U}$ = corresponding eigenvector matrix

Computation of $\mathbf{Y}_{de}(s)$ defined by (21) at any complex frequency is simplified by the fact that $(s\mathbf{I} - \Lambda)$ is diagonal.

For the disturbed machine, the dynamic equations, including the effect of the applied torque ΔT_m, may be written in the partitioned form as

$$\begin{bmatrix}\Delta\dot{\omega}\\ \Delta\dot{\delta}\\ \dot{\mathbf{x}}_r\end{bmatrix} = \begin{bmatrix}a_{11} & a_{12} & \mathbf{a}_{1r}\\ 1 & 0 & \mathbf{0}\\ \mathbf{a}_{r1} & \mathbf{a}_{r2} & \mathbf{A}_{rr}\end{bmatrix}\begin{bmatrix}\Delta\omega\\ \Delta\delta\\ \mathbf{x}_r\end{bmatrix} + \begin{bmatrix}b_1\\ 0\\ \mathbf{B}_r\end{bmatrix}\Delta v + \begin{bmatrix}\frac{\omega_0}{2H}\\ 0\\ \mathbf{0}\end{bmatrix}\Delta T_m \qquad (22)$$

$$\Delta\mathbf{i}_d = [\,c_1\ \ \mathbf{c}_2\ \ \mathbf{C}_r\,]\begin{bmatrix}\Delta\omega\\ \Delta\delta\\ \mathbf{x}_r\end{bmatrix} - \mathbf{Y}_d\Delta\mathbf{v} \qquad (23)$$

where $\mathbf{x}_r$ is a vector representing all state variables of the machine, except for $\Delta\omega$ and $\Delta\delta$. For the disturbed machine, $\Delta\omega$ is assumed to be equal to ω_0 and hence $\Delta\delta = \omega_0/s$.

From (22) and (23), we have the following expressions for ΔT_m and Δi_d in the s domain,

$$\Delta T_m = 2H\left[s - a_{11} - \frac{a_{12}}{s} - \mathbf{a}_{1r}(s\mathbf{I} - \mathbf{A}_{rr})^{-1}(\mathbf{a}_{r1} + \frac{\mathbf{a}_{r2}}{s})\right]$$

$$-\frac{2H}{\omega_0}\left[\mathbf{b}_1 + \mathbf{a}_{1r}(s\mathbf{I} - \mathbf{A}_{rr})^{-1}\mathbf{B}_r\right]\Delta\mathbf{v} \qquad (24)$$

$$\Delta\mathbf{i}_d = \mathbf{i}_{de}(s) - \mathbf{Y}_{de}(s)\Delta\mathbf{v} \qquad (25)$$

where

$$\mathbf{i}_{de}(s) = \left[\mathbf{c}_1 + \frac{\mathbf{c}_2}{s} + \mathbf{C}_r(s\mathbf{I} - \mathbf{A}_{rr})^{-1}(\mathbf{a}_{r1} + \frac{\mathbf{a}_{r2}}{s})\right]\omega_0 \qquad (26)$$

$$\mathbf{Y}_{de}(s) = \mathbf{Y}_d + \mathbf{C}_r(s\mathbf{I} - \mathbf{A}_{rr})^{-1}\mathbf{B}_r \qquad (27)$$

As in (21), computations associated with (24) and (25) are simplified by expressing $(s\mathbf{I} - \mathbf{A}_{rr})^{-1}$ in the diagonal form in terms of eigenvalues of $\mathbf{A}_{rr}$.

Combining the device equations (19), (25) and the network equations (5), we have

$$\mathbf{i}_{De}(s) = (\mathbf{Y}_N + \mathbf{Y}_{De}(s))\Delta\mathbf{v} \qquad (28)$$

where $\mathbf{Y}_{De}(s)$ is a block diagonal matrix of the device equivalent admittances $\mathbf{Y}_{de}(s)$ given by (21), and $\mathbf{i}_{De}(s)$ is the device current source vector with a non-zero value only for the disturbed machine given by (26).

The computation steps associated with the AESOPS algorithm are as follows:

1. With the initial value of complex frequency s equal to the specified estimate, compute $\mathbf{Y}_{de}(s)$, $\mathbf{i}_{de}(s)$ using (21), (26) and (27).
2. Solve (28) to compute bus voltage Δv.
3. Compute H_e and ΔT_m using (15) and (24).
4. Compute next estimate of s using (16).
5. If Δs is within tolerance, stop; otherwise go to 1.

Computational efficiency is achieved by using a very efficient sparsity based network solution technique for the solution of (28).

Implementation of the modified Arnoldi method

The modified Arnoldi method is based on a reduction technique in which a general matrix A is reduced to an upper Hessenberg matrix by the recurrence:

$$h_{i+1,i}\mathbf{v}_{i+1} = \mathbf{A}\mathbf{v}_i - \sum_{j=1}^{i} h_{j,i}\mathbf{v}_j \qquad i = 1, \cdots, m \qquad (29)$$

where

$\mathbf{v}_1$ is an arbitrary starting vector with $\|\mathbf{v}_1\|_2 = 1$

$h_{j,i} = \mathbf{v}_j^H\mathbf{A}\mathbf{v}_i$ (superscript H means conjugate-transpose)

$h_{i+1,i}$ is a scaling factor to make $\|\mathbf{v}_{i+1}\|_2 = 1$

m is the prespecified order of the reduced Hessenberg matrix

Equation (29) can be rearranged and assembled for all m equations to give

$$\mathbf{A}\mathbf{V}_m = \mathbf{V}_m\mathbf{H}_m + h_{m+1,m}\mathbf{v}_{m+1}\mathbf{e}_m^T \qquad (30)$$

where

$$V_m = [\ v_1\ \cdots\ v_m\ ]$$

$$H_m = \begin{bmatrix} h_{1,1} & h_{1,2} & \cdot & h_{1,m} \\ h_{2,1} & h_{2,2} & \cdot & h_{2,m} \\ & \cdot & \cdot & \cdot \\ 0 & & h_{m,m-1} & h_{m,m} \end{bmatrix}$$

$$e_m^T = [\ 0\ \cdots\ 0\ 1\ ]$$

Note that H_m is an upper Hessenberg matrix. It can be shown that ideally the vector sequence v_i generated by (29) is orthonormal. Therefore, if m=N (the order of A), we have

$$h_{N+1,N} = 0$$

and equation (30) becomes

$$AV_N = V_N H_N \tag{31}$$

or A is exactly reduced to the upper Hessenberg matrix H_N whose eigenvalues are then the eigenvalues of A. On the other hand, based on an important feature of the method that the value of the subdiagonal elements of H_m, $h_{m+1,m}$, decrease very rapidly as m increases, a good approximation can be made for $m \ll N$ by dropping out the second term in (30):

$$AV_m \approx V_m H_m \tag{32}$$

In this case, the eigenvalues of the low order matrix, H_m, approximate a subset of the eigenvalues of A. In practice, m is usually not more than 30 even for a system having several thousand states. The corresponding eigenvectors of A are given by

$$W \approx V_m P \tag{33}$$

where P is the eigenvector matrix of H_m. In order to improve the accuracy of the eigenvalues of A, the above procedure can be iterated with a new starting vector v_1^{new} derived from V_m, for example, the linear combination of the columns of V_m:

$$v_1^{new} = \sum_{i=1}^{m} \alpha_i v_i \tag{34}$$

where the suitable coefficients α_i can be calculated from the modal information of H_m [7].

As noted in [12], equation (32) holds only if the vector sequence v_i is kept orthogonal at each step of calculation. In practice, and in common with the Lanczos method, the orthogonality is lost rapidly due to the round-off errors. The remedy is to introduce a reorthogonalization process after the calculation of each v_i from (29).

Another property of the method is that the eigenvalues of H_m in (32) converge to those eigenvalues of A which have largest (and smallest) modulus. Thus if eigenvalues of A around a specified point λ_t are desired, the following transformation

$$A_t = (A - \lambda_t I)^{-1} \tag{35}$$

can be used to magnify the eigenvalues of A close to λ_t, since

$$\lambda_{ti} = \frac{1}{\lambda_i - \lambda_t} \tag{36}$$

where λ_i is an eigenvalue of A, and λ_{ti} is the corresponding eigenvalue of A_t. Matrix A_t is matrix to which the method can be applied in order to find the set of eigenvalues of A close to λ_t, which is termed a shift point.

It should be pointed out that the only operation involving A is the matrix-vector multiplication Av_i in (29), or more practically the solution of the equation

$$(A - \lambda_t I)u_i = v_i \tag{37}$$

if the transformation (36) is used. This kind of calculation can readily be accomplished for the power system small signal model (3), (4) and (5). In fact, as can be verified from (7), equation (37) can be rewritten in terms of (3), (4) and (5) as

$$\begin{bmatrix} A_D - \lambda_t I & B_D \\ C_D & -(Y_N + Y_D) \end{bmatrix} \begin{bmatrix} u_i \\ q_i \end{bmatrix} = \begin{bmatrix} v_i \\ 0 \end{bmatrix} \tag{38}$$

where q_i an auxiliary vector. The solution of (38) to compute u_i involves three steps:

1. calculate $Y_{De}(\lambda_t) = Y_D - C_D(\lambda_t I - A_D)^{-1}B_D$ (39)
2. solve for q_i: $(Y_N + Y_{De}(\lambda_t))q_i = -C_D(\lambda_t I - A_D)^{-1}v_i$ (40)
3. calculate $u_i = (\lambda_t I - A_D)^{-1}(B_D q_i - v_i)$ (41)

It can be readily seen that the calculation of $Y_{De}(\lambda_t)$ in the first step is the same as that of $Y_{De}(s)$ of AESOPS algorithm in (28). The network solution in the second step is same as in (28), except that the current injection vector is different. In addition, the calculations involving $(\lambda_t I - A_D)^{-1}$ in the second and third steps are fast since the modal decoupling of A_D is already available from the first step as illustrated in (21). Therefore, the method can share with AESOPS much of the computer codes.

Once u_i is obtained, v_{i+1} and thus the related elements in H_m are formed with a prespecified order m. The QR transformation method is then employed to compute the eigenvalues and eigenvectors of H_m. If the changes in eigenvalues are outside a specified tolerance, a new starting vector v_1^{new} is formed using (34) for the next iteration.

It should be pointed out that the factorization of the network equivalent admittance matrix $Y_N+Y_{De}(\lambda_t)$ in (40) is performed only once for the whole calculation since λ_t is not changed during iterations. This makes the method generally faster than the AESOPS algorithm in which the factorization is performed in each iteration as s is updated.

Calculation of Eigenvectors in PEALS

In AESOPS, the speed deviations $\Delta\omega_i$ obtained in the last iteration directly give the speed components of the right eigenvector, and in MAM the complete right eigenvector is calculated by (33).

The left eigenvector is calculated in PEALS/AESOPS using the transposed system dynamic model of (3) and (4)

$$\dot{y} = A_D^T y + C_D^T \Delta u \tag{42}$$

$$\Delta J = B_D^T y - Y_D^T \Delta u \tag{43}$$

and the transposed network model of (5)

$$\Delta j = Y_N^T \Delta u \tag{44}$$

It can be readily shown that the AESOPS algorithm when applied to the above model will converge to a system eigenvalue and the corresponding left eigenvector. With the initial eigenvalue estimate equal to the correct value, one iteration is usually sufficient for the algorithm to converge to the left eigenvector.

The left eigenvector is calculated in PEALS/MAM by inverse iteration. It requires the solution of the transposed system of (38)

$$\begin{bmatrix} A_D^T - \lambda_t I & C_D^T \\ B_D^T & -(Y_N + Y_D)^T \end{bmatrix} \begin{bmatrix} y \\ q \end{bmatrix} = \begin{bmatrix} y_0 \\ 0 \end{bmatrix} \tag{45}$$

with the computed eigenvalue λ_t and an arbitrary vector y_0. Normally, one iteration is sufficient to give the left eigenvector.

RESPONSE CALCULATION IN MASS AND PEALS

In MASS both time and frequency response facilities are available which rely on modal decomposition. Provided that all eigenvectors are distinct, the state equations

$$\dot{x} = Ax + Bu \tag{46}$$

$$y = Cx + Du \tag{47}$$

may be diagonalized by the transformation

$$x = Uz \tag{48}$$

to give

$$\dot{z} = \Lambda z + U^{-1}Bu \tag{49}$$

$$y = CUz + Du \tag{50}$$

where Λ is the diagonal matrix of eigenvalues and U is the corresponding eigenvector matrix. Equation (49) and (50) may be easily solved for output y, for any given input u. Three types of response are calculated on request: impulse and step time response, and frequency response.

In PEALS, full modal analysis is not preformed and hence it is not available for time response calculations. Frequency response is, however, straightforward to calculate directly from the system equations in the form used by PEALS/AESOPS, with s replaced by $j\omega$ [5]. Because the network equations are retained explicitly in PEALS, it is as easy to apply disturbances to the network as it is to the dynamic devices. Similarly it is easy to monitor network quantities such as bus voltage and line power flow. This frequency response option is useful in the design of controls for HVDC links and SVCs.

STRUCTURE OF MASS/PEALS PACKAGE

Figure 1 shows the overall structure of the MASS/PEALS integrated package for eigenvalue calculations. It takes advantage of the common features between MASS and PEALS, and between PEALS/AESOPS and PEALS/MAM.

MASS has provision for representing up to 20 areas, with each area having a maximum of 250 dynamic devices and 500 system states. With the single area representation, it is limited to systems which may be represented by less than 500 dynamic states. PEALS is capable of simulating a system with up to 2,200 dynamic devices, with the maximum number of system states equal to 22,000. The network representation is common to both programs and is limited to 12,000 buses and 30,000 lines. The MASS/PEALS package requires a virtual memory of about 15.5 MB (VAX-VMS).

The overall package has a modular structure with flexibility to readily add new models and implement alternative solution techniques. Work currently underway for the enhancement of the package includes the implementation of multiterminal HVDC link model and facilities for computing system zeros and transfer functions. In addition, Selective Modal Analysis algorithm for computation of eigenvalues associated with interarea modes is being added in cooperation with the Instituto De Investigacion Tecnologica, Madrid, Spain.

In order to meet the broad requirements of power system stability analysis, the SSSP package has been made compatible with the new Extended Transient/Midterm Stability Program (ETMSP) being developed under the EPRI project RP1208-9.

APPLICATIONS OF MASS AND PEALS

MASS computes information about all system modes and, therefore, is ideally suited for analysing problems associated with local plant modes, control modes, torsional modes, and interarea modes of small or medium size systems. It is also useful for control design and identifying problems due to poor coordination of controls of different devices.

PEALS, on the other hand, computes information related to a

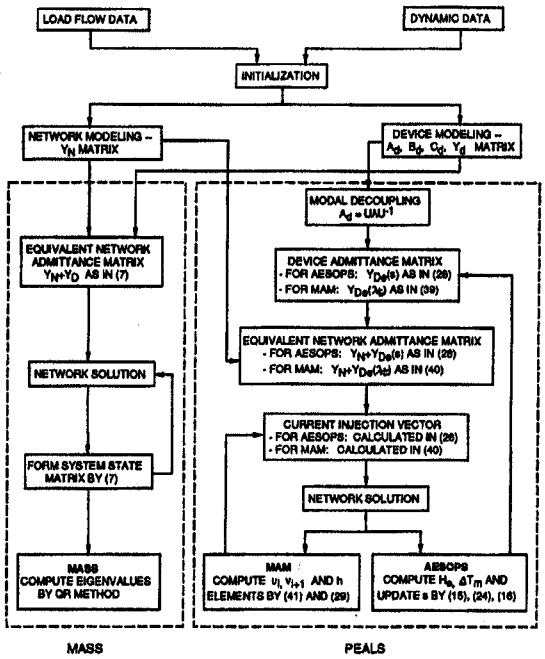

Figure 1 Structure of MASS/PEALS Package

selected subset of system modes. However, it can simulate very large systems and is therefore ideally suited for the analysis of interarea as well as local plant modes of large interconnected systems. Since full network solution is carried out as part of the eigenvalue calculation, PEALS also provides valuable information about the bus voltages. The frequency response option enables systematic control design techniques to be applied to enhance stability of inter-area modes.

The programs complement each other for the solution of a wide range of system dynamic problems. They have been used at Ontario Hydro extensively for the following applications:

- Investigation of small signal stability problems;
- Design of power system stabilizers [13, 14];
- Selection of AVR and governor settings [2, 15];
- Analysis of interaction between torsional modes and excitation controls [2, 13], or speed governing system [16];

Reference [14] provides a detailed account of the application of MASS and PEALS for the design of power system stabilizers for a major nuclear station and illustrates the use of participation factors and frequency response characteristics for control design. Reference [13] shows comparisons of computed and measured on-line frequency and time responses for some of the stabilizer applications.

Currently, the SSSP package is being used by Ontario Hydro for a research project, funded by the Canadian Electrical Association, on low frequency interarea oscillations in large power systems.

RESULTS OF TEST CASES

Three systems are considered for verifying and comparing the three eigenvalue algorithms in SSSP package. The composition and size of these systems are summarized in Table 1. All three systems are representations of the eastern US/Canada interconnected system, with different degrees of detail and system reduction. It should be noted that because of size limitations MASS can analyse only the

first system, whereas PEALS using either AESOPS or MAM is able to analyse all three systems.

Table 1 – Description of Test Systems

	System 1	System 2	System 3
No. of Buses	1,368	3,732	10,546
No. of Generators			
– Detailed Model	18	725	1,036
– Classical Model	76	130	61
No. of DC Links	0	7	15
No. of SVCs	0	5	10
Total No. of States	366	7,195	13,475

The objective is to compare the results of eigenvalue calculations using the three methods as this is the difficult part in the analysis of complex systems. Results of other calculations such as eigenvectors and participation factors are not presented since they are fairly straightforward once the eigenvalues are computed. However, an example of frequency response calculation, using PEALS, is given for System 1.

System 1

This system was analyzed first using MASS to obtain all eigenvalues and participation factors from which three interarea modes are identified. Comparison was then made by calculating these interarea modes with both PEALS/AESOPS and PEALS/MAM. The results are summarized in Table 2.

Table 2 – Comparison of Results for System 1

Mode	MASS	PEALS/AESOPS	PEALS/MAM
1	-0.122±j1.650	-0.121±j1.650	-0.121±j1.650
2	-0.117±j3.073	-0.117±j3.073	-0.117±j3.073
3	-0.276±j5.212	-0.276±j5.212	-0.276±j5.212

A frequency response calculation was further carried out, using PEALS, for the change in speed of a large nuclear unit in this system to a change in mechanical torque, as shown in Figure 2. It can be seen from the figure that resonant frequencies at about 0.26 Hz, 0.49 Hz and 0.82 Hz are present in the selected machine, which correspond to three interarea modes in Table 2.

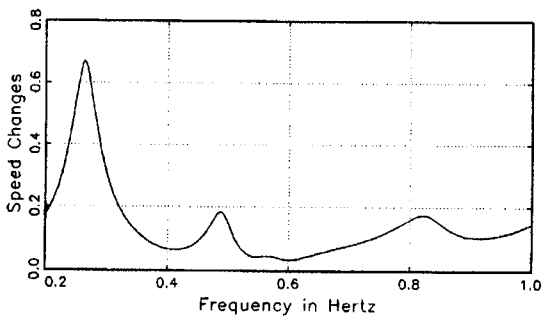

Figure 2 Frequency Response of System 1

System 2

PEALS/AESOPS and PEALS/MAM were employed to calculate two of the low frequency oscillatory modes. The results are summarized in Table 3.

Table 3 – Comparison of Results for System 2

Mode	PEALS/AESOPS	PEALS/MAM
1	-0.057±j3.886	-0.056±j3.886
2	-0.163±j4.317	-0.164±j4.318

System 3

One of the features of PEALS/MAM is the capability for scanning eigenvalues over a frequency range on the complex plane. This was used to search the frequency range of 0.1 Hz to 0.8 Hz with 15 shift points evenly placed within the range. For each shift point, 2 eigenvalues were calculated. Some of the eigenvalues computed were common to different shift points and in all 20 distinct pairs of eigenvalues were computed. From the participation factors nine of these were identified as being associated with interarea modes and these are summarized in Table 4.

Table 4 – Interarea Modes of System 3

Mode	Eigenvalue	Frequency	Damping Ratio
1	-0.050±j1.542	0.245	0.032
2	-0.057±j2.237	0.356	0.025
3	-0.265±j3.343	0.532	0.079
4	-0.047±j3.352	0.534	0.014
5	-0.261±j3.846	0.612	0.068
6	-0.150±j3.938	0.627	0.038
7	0.007±j4.206	0.669	-0.002
8	-0.072±j4.972	0.791	0.015
9	-0.083±j5.009	0.797	0.017

PEALS/AESOPS was also used to compute the first two modes in the above table. Table 5 gives the results of PEALS/AESOPS.

Table 5 – Results of PEALS/AESOPS for System 3

mode	Initial Estimate	Converged Value
1	0.0+j1.5	-0.051+j1.543
2	0.0+j2.3	-0.061+j2.240

In the above PEALS/AESOPS calculations, an error tolerance of 5×10^{-3} was used. We have found that this error tolerance is a practical compromise between computational speed and accuracy for very large systems. For the other cases and for the results of Table 4 an error tolerance of 10^{-4} was used. The PEALS/AESOPS results shown above can be seen to differ from the more accurate PEALS/MAM results by less than the tolerance.

CPU Time Comparison

The CPU times for the above eigenvalue calculations with PEALS are given in Table 6. The CPU times listed are in minutes on VAX8650-VMS. For each system, the computation conditions were made the same, i.e. both methods start from the same initial eigenvalue estimations (all within 10% of the true eigenvalues) with the same error tolerance (10^{-4} for system 1 and 2, and 5×10^{-3} for system 3). As a reference, the MASS run for system 1 requires 11 minutes to compute all eigenvalues and the associated participation factors.

Table 6 – CPU Time Comparison

System	No. of Modes Found	PEALS/AESOPS	PEALS/MAM
1	3	61	28
2	2	50	34
3	2	184	154

General Comments

From Table 2 to 5, it is clear that all three methods give practically identical results for the different systems considered. This is particularly reassuring in view of the very complex systems analyzed and the widely differing theories on which the eigenvalue calculations are based.

From the CPU time comparison, AESOPS/MAM is seen to be faster than PEALS/AESOPS. The convergence of the AESOPS algorithm is very sensitive to the chosen initial estimate and error tolerance. The computation time shown in Table 6 for PEALS/AESOPS assumes good initial estimates and may be much larger if this is not the case. The modified Arnoldi method, on the other hand, has been observed to converge with very small error tolerance, within two

iterations provided less than five eigenvalues are computed per shift point.

The two alternative eigenvalue techniques employed in PEALS program have different convergence properties. The AESOPS algorithm converges only to those rotor angle modes associated with the disturbed generator and inherently selects a system location which includes that machine. This selection process can be useful when it is required to track the changes in a specific mode with changes in system conditions or control parameters. The modified Arnoldi method, however, converges to those system eigenvalues closest to a chosen shift point so that it is able to compute all eigenvalues within a specific frequency range.

SUMMARY

Small signal analysis using modal analysis techniques provides valuable information about the inherent dynamic characteristics which influence system stability. Such information is required in the economic design and control of the present day complex interconnected power systems. This paper has described a small signal stability program package based on eigenvalue techniques suitable for analysing a wide range of power system stability and control problems which addresses this requirement. The package has facilities for representing all standard models normally used in power system stability studies. In addition, provision is made for representing the controls associated with generating units, HVDC converters and SVCs by user-defined models. This offers practically unlimited freedom in modelling these devices, making SSSP particularly suited for developing new controls for enhancement of power system stability.

The SSSP package has a modular structure providing the flexibility to readily implement alternative solution techniques. At present, the package has facilities for computing eigenvalues using the QR transformation, the AESOPS algorithm and the modified Arnoldi method. Work is currently underway to implement the Selective Modal Analysis technique. These techniques may be used in a complementary manner so as to satisfy differing requirements of small signal stability analysis.

In addition to eigenvalues, the SSSP package computes eigenvectors, participation factor, frequency response, linear time response, and network bus voltage vectors. These provide valuable information required for identifying the sources of stability problems and developing corrective controls.

The overall package includes a System Reduction Program and a Dynamic Data Bank Program. Further, the SSSP package has been made compatible with the Extended Transient/Midterm Stability Program, thus forming a comprehensive stability analysis package.

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REFERENCES

[1] C.P. Steinmetz, "Power Control and Stability of Electric Generating Stations", AIEE Trans, vol. XXXIX, Part II, pp 1215, July-December 1920.

[2] P. Kundur and P.L. Dandeno, "Practical Application of Eigenvalue Techniques in the Analysis of Power System Dynamic Stability Problems", Proc Fifth Power System Computation Conference, Cambridge, England, September 1975.

[3] G. Gross, C.F. Imparato and P.M. Look, "A Tool for the Comprehensive Analysis of Power System Dynamic Stability", IEEE Trans, vol. PAS-101, pp 226-234, January 1982.

[4] R.T. Byerly, R.J. Benon and D.E. Sherman, "Eigenvalue Analysis of Synchronizing Power Flow Oscillations in Large Electric Power Systems", IEEE Trans, vol. PAS-101, pp 235-243, January 1982.

[5] N. Martins, "Efficient Eigenvalue and Frequency Response Methods Applied to Power System Small-Signal Stability Studies", IEEE Trans, vol. PWRS-1, pp 217-225, February 1986.

[6] D.Y. Wong, G.J. Rogers, B. Porretta and P. Kundur, "Eigenvalue Analysis of Very Large Power Systems", IEEE Trans, vol. PWRS-3, pp 472-480, May 1988.

[7] L. Wang and A. Semlyen, "Application of Sparse Eigenvalue Techniques To The Small Signal Stability Analysis of Large Power Systems", Proc. of the Sixteenth PICA conference, Seattle, Washington, pp 358-365, May 1989.

[8] N. Uchida and T. Nagao, "A New Eigen-Analysis Method of Steady-State Stability Studies for Large Power Systems: S Matrix Method", IEEE Trans, vol. PWRS-3, No. 2, pp 706-714, May 1988.

[9] G.C. Verghese, I.J. Perez-Arriaga, and F.C. Schweppe, "Selective Modal Analysis with Applications to Electric Power Systems Part I: Heuristic Introduction, Part II: The Dynamic Stability Problem", IEEE Trans, vol. PAS-101, No. 9, pp 3117-3134, September 1982.

[10] EPRI Report EL-5798, "The Small Signal Stability Program Package", vol.1, Final Report of Project 2447-1, Prepared by Ontario Hydro, May 1988.

[11] J.G. Francis, "The Q-R Transformation - A Unitary Analogue to the LR Transformation", Part 1 and 2, Comp. Journal, 4, 1961/62, pp 256-271 and 332-345.

[12] Y. Saad, "Variations on Arnoldi's Method for Computing Eigenelements of Large Unsymmetric Matrices", Linear Algebra and Its Applications, vol. 34, pp 184-198, June 1981.

[13] P. Kundur, D.C. Lee, and H.M. Zein El-Din, "Power System Stabilizers for Thermal Units: Analytical Techniques and On-Site Validation", IEEE Trans PAS, vol. 100, pp 81-95, January 1981.

[14] P. Kundur, M. Klein, G.J. Rogers and M. Zywno, "Application of Power System Stabilizers for Enhancement of Overall System Stability", IEEE Trans. vol. PWRS-4, no. 2, pp 614-626, May 1989.

[15] P.L. Dandeno, P. Kundur, and J.P. Bayne, "Hydraulic Unit Dynamic Performance Under Normal and Isolated Conditions - Analysis and Validation", IEEE Trans PAS, vol. 97, pp 2134-2143, 1978.

[16] D.C. Lee, R.E. Beaulieu, and G.J. Rogers, "Effects of Governor Characteristics on Turbogenerator Shaft Torsionals", IEEE Trans PAS, vol. 104, pp 1255-1261. June 1985.

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