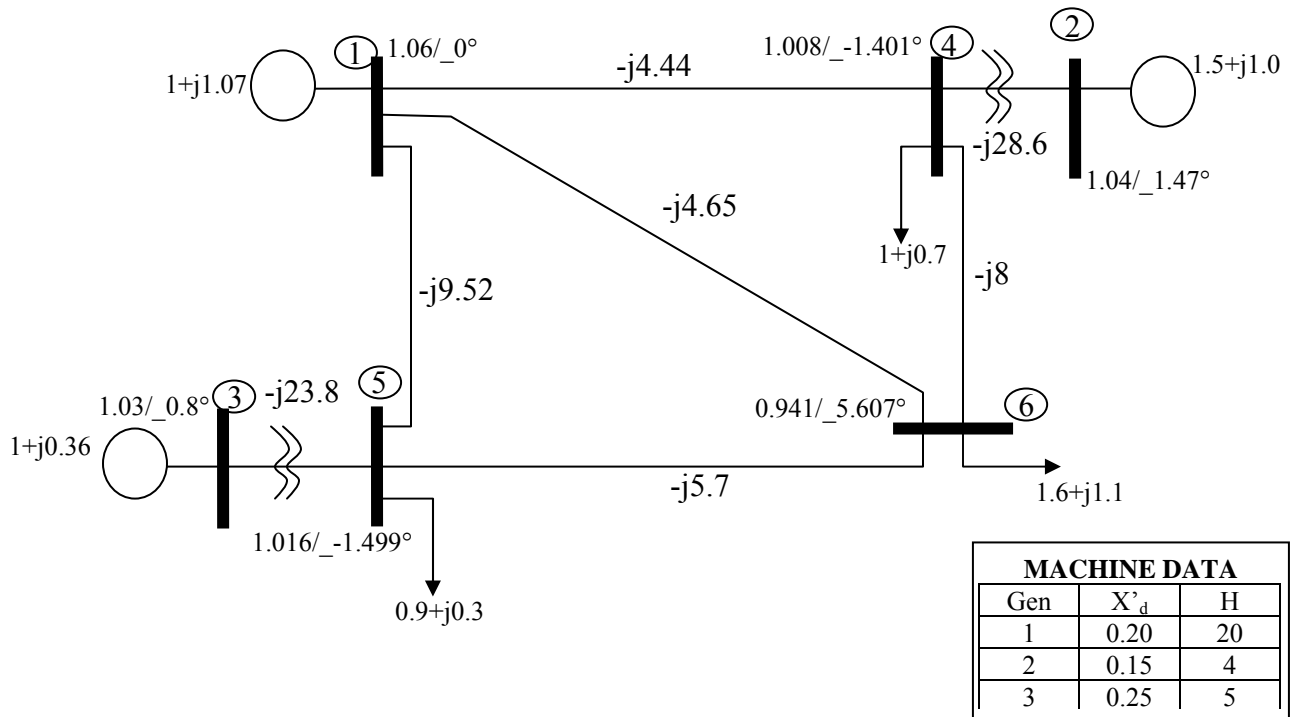


Work All Problems

1. Consider the six bus, 3 machine system given below. Bus numbers, branch admittances, loads, generation levels, and voltage phasors (in smaller fonts) are all provided in the figure. All data is provided in per-unit on a 100 MVA base.



- a. (5) Obtain the internal generator internal voltage for generation #1.

Solution:

$$I_{G1} = \frac{1 - j1.07}{1.06} = 0.9434 - j1.0094$$

$$\Rightarrow E'_{d1} = 1.06 \angle 0^\circ + (0.9434 - j1.0094)(j0.2) = 1.2619 + j0.1887 = 1.276 \angle 8.5^\circ$$

- b. (5) Determine the unreduced pre-fault Y-bus for purposes of use in a transient stability simulation. Any new buses should be numbered 7 or larger.

Solution:

First, we must convert the loads to constant admittance. Also, convert the generator transient reactances to admittances:

$$Y_4 = \frac{1 - j0.7}{1.008^2} = 0.9842 - j0.6889$$

$$Y_5 = \frac{0.9 - j0.3}{1.016^2} = 0.9534 - j0.6674$$

$$Y_6 = \frac{1.6 - j1.1}{0.941^2} = 1.8069 - j1.2423$$

$$Y_1 = \frac{1}{j0.2} = -j5$$

$$Y_2 = \frac{1}{j0.15} = -j6.667$$

$$Y_3 = \frac{1}{j0.25} = -j4$$

Now we can construct the Y-bus, although we must remember to include the loads and the generator transient reactances.

$$Y_1' = \begin{bmatrix} -j23.61 & 0 & 0 & j4.44 & j9.52 & j4.65 & j5 & 0 & 0 \\ 0 & -j35.267 & 0 & j28.6 & 0 & 0 & 0 & j6.667 & 0 \\ 0 & 0 & -j27.8 & 0 & j23.8 & 0 & 0 & 0 & j4 \\ j4.44 & j28.6 & 0 & .9842 - j41.729 & 0 & j8 & 0 & 0 & 0 \\ j9.52 & 0 & j23.8 & 0 & .953 - j40.26 & j5.7 & 0 & 0 & 0 \\ j4.65 & 0 & 0 & j8 & j5.7 & 1.81 - j14.94 & 0 & 0 & 0 \\ j5 & 0 & 0 & 0 & 0 & 0 & -j5 & 0 & 0 \\ 0 & j6.667 & 0 & 0 & 0 & 0 & 0 & -j6.67 & 0 \\ 0 & 0 & j4 & 0 & 0 & 0 & 0 & 0 & -j4 \end{bmatrix}$$

- c. (5) Determine the unreduced fault-on Y-bus given a fault is applied to bus 6.

Solution:

The Y-matrix is Y_2 is Y_1' with column 6 and row 6 eliminated.

- d. (5) What will be the dimension of the reduced fault-on Y-bus given a fault is applied to bus 6, and which buses will correspond to the remaining rows and columns?

Solution: The dimension will be 3x3 and the buses corresponding to the remaining rows and columns will be 7, 8, and 9 (the internal buses of the generators).

2. A "stiff" dynamical system is characterized by the following two differential equations.

$$\dot{x}_1 = 998x_1 + 1998x_2, \quad \dot{x}_2 = -999x_1 - 1999x_2$$

- a. (5) Develop the equations, as a function of time-step T, to use in performing numerical integration based on the forward (Euler) rule.

Solution:

Recall that

$$\underline{x}(kT) = \underline{x}(kT - T) + \int_{kT-T}^{kT} \underline{f}(\underline{x}(k\tau)) d\tau$$

where $\underline{f}(\cdot)$ is the right-hand-sides of the given derivative expressions.

The forward rule evaluates according to

$$\underline{x}(kT) = \underline{x}(kT - T) + T \underline{f}(\underline{x}(kT - T))$$

Applying this to the above problem, we obtain

$$\begin{bmatrix} x_1(kT) \\ x_2(kT) \end{bmatrix} = \begin{bmatrix} x_1(kT - T) + T(998x_1(kT - T) + 1998x_2(kT - T)) \\ x_2(kT - T) + T(-999x_1(kT - T) - 1999x_2(kT - T)) \end{bmatrix}$$

- b. (5) Identify a maximum step size for this system (Hint: absolute value of the coefficients of $x_1(kT-T)$ and $x_2(kT-T)$ must be < 1 for numerical stability.)

Rewriting the equations from part (c), we have

$$\begin{bmatrix} x_1(kT) \\ x_2(kT) \end{bmatrix} = \begin{bmatrix} x_1(kT - T)[1 + T998x_1] + T(1998x_2(kT - T)) \\ x_2(kT - T)[1 - T1999] + T(-999x_1(kT - T)) \end{bmatrix}$$

Note that if the absolute value of any coefficient on the right-hand-side is larger than 1, then the solution grows larger with each succeeding step. By inspection, the most limiting coefficient is in the second equation, as follows:

$$-1 < 1 - T(1999) < 1 \Rightarrow -2 < -T1999 \Rightarrow T < 2/1999 \approx 0.001$$

3. (15) Derive the quadrature axis transient inductance including the effects of the G-winding.

Solution: From eq. 4.20', we have that

$$\lambda_G = kM_G i_q + M_Y i_Q + L_G i_G$$

Let's assume there is no q-axis damper winding, so that $i_Q=0$. Then, at $t=0^+$,

$$\lambda_G(0^+) = 0 = kM_G i_q + L_G i_G \Rightarrow i_G = \frac{-kM_G}{L_G} i_q$$

Again, from 4.20', we have that

$$\lambda_q = L_q i_q + kM_Q i_Q + kM_G i_G$$

and with no damper, $i_Q=0$, and

$$\lambda_q = L_q i_q + kM_G i_G$$

Substitution for i_G from above expression yields: $\lambda_q = L_q i_q - kM_G \frac{kM_G}{L_G} i_q = \left(L_q - \frac{(kM_G)^2}{L_G} \right) i_q$

From this last expression we observe that $L'_q = L_q - \frac{(kM_G)^2}{L_G}$

Recalling that $L_{AQ}=kM_G$, we have that $L'_q = L_q - \frac{(L_{AG})^2}{L_G}$

4. Short-answer questions:

- a. (5) Prove that $\underline{Q} \underline{A} \underline{Q}^{-1} = \underline{A}$ if \underline{Q} is any $n \times n$ matrix and \underline{A} is an $n \times n$ diagonal matrix with equal elements on the diagonal.

Solution: $\underline{A} = k\underline{U}$ where k is the element along the diagonal and \underline{U} is the identity matrix. then $\underline{Q} \underline{A} \underline{Q}^{-1} = \underline{Q} k\underline{U} \underline{Q}^{-1} = k \underline{Q} \underline{U} \underline{Q}^{-1} = k \underline{Q} \underline{Q}^{-1} = k\underline{U} = \underline{A}$.

- b. (5) Prove that $\underline{Q} \underline{A} \underline{Q}^T = \underline{A}$ if \underline{Q} is any $n \times n$ orthogonal matrix and \underline{A} is an $n \times n$ diagonal matrix with equal elements on the diagonal.

Solution: $\underline{A} = k\underline{U}$ where k is the element along the diagonal and \underline{U} is the identity matrix. then $\underline{Q} \underline{A} \underline{Q}^T = \underline{Q} k\underline{U} \underline{Q}^T = k \underline{Q} \underline{U} \underline{Q}^T = k \underline{Q} \underline{Q}^T = k \underline{U} = \underline{A}$.

- c. (5) We know that $\lambda_{0dq} = P\lambda_{abc}$. Express $\dot{\lambda}_{0dq}$.

Solution:

$$\dot{\lambda}_{0dq} = \underline{P} \dot{\lambda}_{abc} + \dot{\underline{P}} \lambda_{abc}$$

- d. (5) Why is Park's transformation necessary in developing models of synchronous machines?

Solution: It converts time varying inductances to constant inductances. With it, the differential equations for modeling the synchronous machine would have state variables with time-varying coefficients.

- e. (5) Provide that if \underline{P} is orthogonal, then it is power invariant.

Solution: If the transformation is orthogonal ($\underline{P}^{-1}=\underline{P}^T$), then the power calculation, which is $p = \underline{v}_{abc}^T \dot{\underline{i}}_{abc}$, is also given by $p = \underline{v}_{0dq}^T \dot{\underline{i}}_{0dq}$. This can be proven (see eq. 4.10 in text) since, from eqs. 4.3 and 4.7, $\underline{P}^{-1} \underline{v}_{0dq} = \underline{v}_{abc}$ and $\underline{P}^{-1} \dot{\underline{i}}_{0dq} = \dot{\underline{i}}_{abc}$, we may write:

$$p = \underline{v}_{abc}^T \dot{\underline{i}}_{abc} = \left(\underline{P}^{-1} \underline{v}_{0dq} \right)^T \left(\underline{P}^{-1} \dot{\underline{i}}_{0dq} \right)$$

Recalling that $(\underline{a}\underline{b})^T = \underline{b}^T \underline{a}^T$, the above is:

$$\begin{aligned} p &= \underline{v}_{abc}^T \dot{\underline{i}}_{abc} = \underline{v}_{odq}^T \left(\underline{P}^{-1} \right)^T \left(\underline{P}^{-1} \dot{\underline{i}}_{0dq} \right) = \underline{v}_{odq}^T \left(\underline{P} \right) \left(\underline{P}^{-1} \dot{\underline{i}}_{0dq} \right) \\ &= \underline{v}_{odq}^T \dot{\underline{i}}_{0dq} \end{aligned}$$

- f. (5) For what type of machine do we typically model the G-circuit, and what does this circuit represent?

Solution: The G-circuit is used in modeling the round-rotor machine because the solid iron rotor provides multiple paths for circulating eddy currents which act as equivalent windings during both transient and subtransient time periods (see page 124 of the text). The G-circuit represents the flux produced by these currents along the q-axis during the transient period.

- g. (5) At a certain level of flux linkage λ_{AD} , the ratio of
 [the current that would produce λ_{AD} under no saturation effects to
 the current that produces λ_{AD} with saturation effects]
 is 0.8. Find the corresponding value necessary to provide as input data for a commercial grade time domain simulation program.

Solution:

i_{M0} : current that would produce λ_{AD} if no saturation effects

i_{MS} : current that produces λ_{AD} with saturation effects

So $K_S = i_{M0}/i_{MS}$ is what we are given in the problem statement. $S = 1/K_S - 1 = 1/0.8 - 1 = 0.25$.

- h. (15) Indicate whether you think the following data is correct or incorrect, and support your answer using arguments based on flux paths and reluctance.
- i. Hydroelectric unit: $X_d = X_q$. *Incorrect. The steady-state quadrature flux path occurs largely in the interpolar regions and the steady-state direct axis flux path occurs largely in the polar regions, implying that the quadrature flux has significantly more air in its path than the direct axis flux, and therefore quadrature flux path reluctance (inductance) is higher (lower) than direct axis flux path reluctance, so $L_d > L_q \rightarrow X_d > X_q$.*
 - ii. Coal-fired unit: $X''_d > X'_d$. *Incorrect. Because the percentage of the path in air (and thus the reluctance) gets increasingly greater as we move from steady-state to transient to subtransient conditions, due to effects of CFL, the inductance gets smaller with each figure, implying $L_d > L'd > L''d \rightarrow X_d > X'd > X''d$.*
 - iii. Hydroelectric unit: $X_q = X'_q$. *Correct. The percentage of the path in air (and thus the reluctance) is the same for the steady state and transient periods (because no G-winding).*
- i. (5) Power networks are R-L-C circuits. Phasor-representation of voltages and currents are appropriate under steady-state analysis of an R-L-C circuit, but not for transient analysis of an R-L-C circuit. Provide justification of why we can use phasor representation of network voltages and currents in a transient stability simulation.

Solution: We justify the use of phasor-representation of network voltages and currents in a transient stability simulation by assuming that the electrical transients in the network are very fast relative to the electro-mechanical transients of the synchronous machine. Therefore, the use of phasor-representation of network voltages and currents provides a "pseudo-steady-state" analysis of the network where the voltages are varying slowly enough so that each time step may be assumed as a new steady-state network solution.