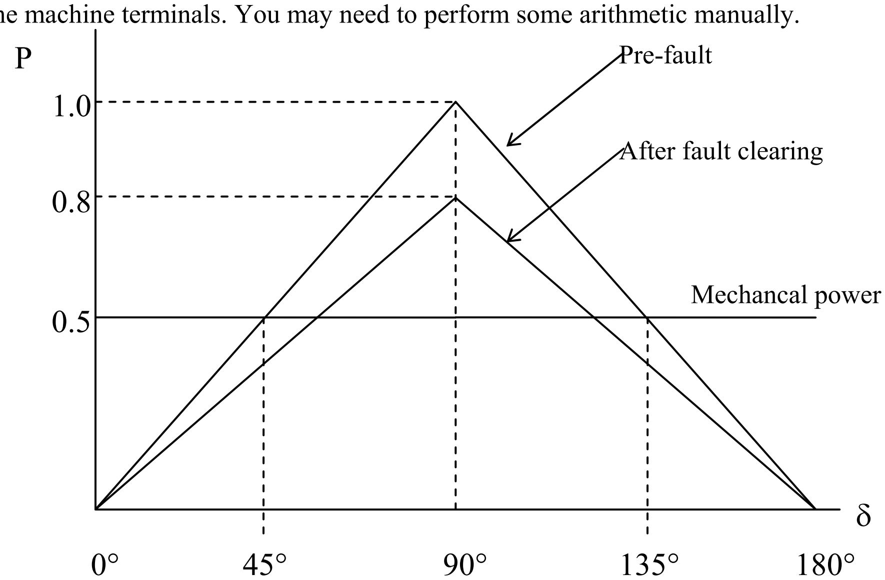
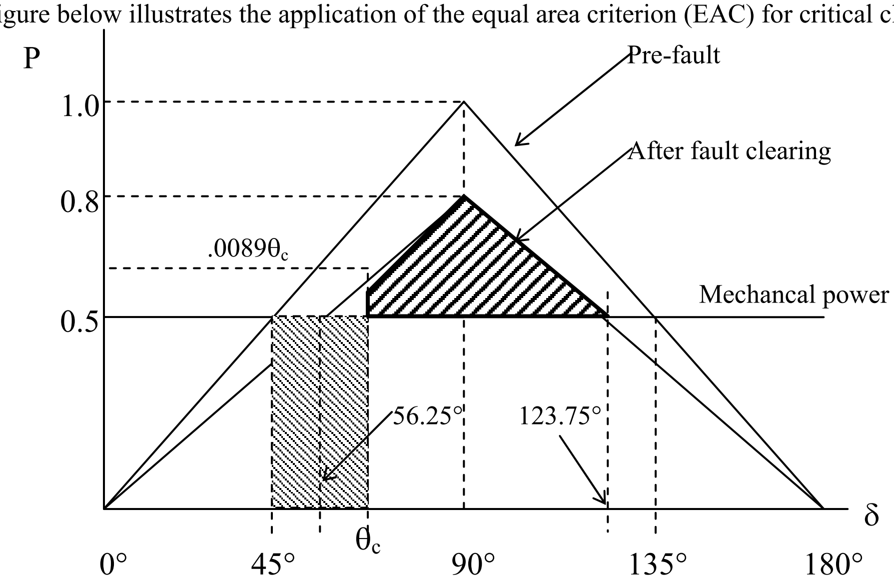
EE 554, Exam I, Spring 2009, Dr. McCalley
Time: 75 minutes, closed book, closed notes, calculators/computers not allowed

1. (20 pts) Consider the very rough approximations of the P-δ "swing curves" for a synchronous machine connected to an infinite bus in the following figure. The curves characterize the P-δ variation for pre-fault and after fault-clearing conditions. The generator mechanical power is 0.5 pu. Assuming these curves to be an accurate portrayal of the P-δ relation, compute the critical clearing angle for a 3-phase fault at the machine terminals. You may need to perform some arithmetic manually.



Solution:

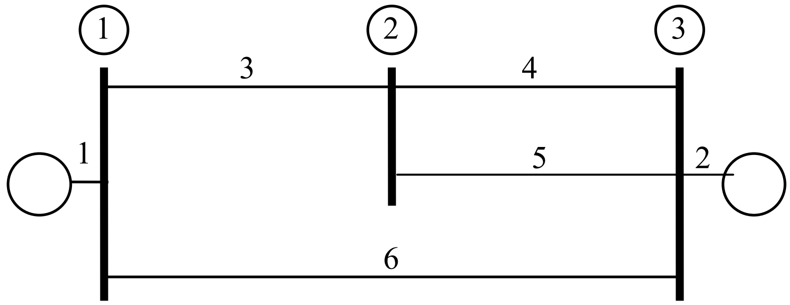
The equation of the after-fault clearing curve, for δ<90, is P=(0.8/90)δ=0.0089δ. The point of intersection between the P=0.5 line and the after fault-clearing curve is where 0.0089δ=0.5➔δ=56.25°. The maximum angle is 180-56.25=123.75°. Let's assume that the critical clearing angle θ_C, is greater than 56.25°. In that case, the figure below illustrates the application of the equal area criterion (EAC) for critical clearing angle:



EAC requires that (area of rectangle)=(area of big triangle)-(area of small triangle), i.e.,

$(\theta_c-45)(0.5)=1/2(123.75-56.25)(0.3)-1/2(\theta_c-56.25)(0.0089\theta_c-0.5)$ ➔ $\theta^2=4171.3$ ➔ **$\theta_c=64.586°$.**

2. (20 pts) The system shown below has two generators and three nodes. Generator and transmission line data are given below and the result of a load-flow study is also given. A three-phase fault occurs at the node 2 end of line 5 and is cleared by removing line 5.



(a) Identify the steps you would take to obtain the information necessary to *initiate* (but not perform) a transient instability simulation. Give sample calculations for each step (you need not make the calculations).

(b) Draw the pre-fault, fault-on, and post-fault *circuit diagram* associated with the above *one-line diagram*. Label each impedance appropriately (e.g., x'_{d1}, X_{T1}, X_3, etc).

SYSTEM DATA

Generator Data (in pu on generator MVA base)

Generator Number	x'_d (pu)	X_T^* (pu)	H (MWs/MVA)	Rating (MVA)
1	0.28	0.08	5	50
3	0.25	0.07	4	120

$*X_T$ = generator transformer reactance

Transmission Line Data (resistance neglected)

Line Number	3	4	5	6
	X_3	X_4	X_5	X_6
X pu on 100 MVA base	0.08	0.06	0.06	0.13

LOAD FLOW RESULT

Bus	Voltage		Load		Generator	
No.	mag, pu	angle°	MW	MVAR	MW	MVAR
1	1.030	0.0	0.0	0.0	30.0	23.1
2	1.018	–1.0	50.0	20.0	0.0	0.0
3	1.020	–0.5	80.0	40.0	100.0	37.8

The system base MVA is 100 MVA.

Solution:

(a) There are four basic steps to take:

i. Convert all data to a common 100 MVA base. This would include

- inertia data, as illustrated below.

$$H_1 = (5\ \text{MWs/MVA})\left(\frac{50}{100}\right) = 2.5\text{s}$$

$$H_3 = (4\ \text{MWs/MVA})\left(\frac{120}{100}\right) = 4.8\text{s}$$

- impedance data, as illustrated below:

$$x'_{d1} = 0.28\left(\frac{100}{50}\right) = 0.56 \qquad x'_{d3} = 0.25\left(\frac{100}{120}\right) = 0.2083$$

$$x_{t1} = 0.08\left(\frac{100}{50}\right) = 0.16 \qquad x_{t3} = 0.07\left(\frac{100}{120}\right) = 0.0583$$

ii. Convert the loads to equivalent passive impedance, as illustrated below (you would not need to make calculation, just show that you know how):

Load L2:

$$P_2 + jQ_2 = \frac{50 + j20}{100} = 0.5 + j0.2 = V_2 I_2^*$$

$$I_2 = \frac{S_2^*}{V_2^*} = \frac{0.5 - j0.2}{1.018\angle +1.0} = 0.5290\angle -22.801$$

$$Z_2 = \frac{V_2}{I_2} = \frac{1.018\angle -1.0}{0.5290\angle -22.801} = 1.9244\angle 21.801 = 1.7868 + j0.7147$$

$$Y_2 = Z_2^{-1} = 0.5195\angle -21.801 = 0.4825 - j0.1930$$

Load L3:

$$P_3 + jQ_3 = \frac{80 + j40}{100} = 0.8 + j0.4 = V_3 I_3^*$$

$$I_3 = \frac{S_3^*}{V_3^*} = \frac{0.8 - j0.4}{1.020\angle +0.5} = 0.8769\angle -27.065$$

$$Z_3 = \frac{V_3}{I_3} = \frac{1.020\angle -0.5}{0.8769\angle -27.065} = 1.1632\angle 26.565 = 1.0404 + j0.5202$$

$$Y_3 = Z_3^{-1} = 0.8597\angle -26.565 = 0.7689 - j0.3845$$

iii. Calculate the generator internal voltages and angles (you would not need to make calculation, just show that you know how).

Generator 1

$$E_1\angle\delta_1 = \boldsymbol{V}_{t1} + j0.72\boldsymbol{I}_{G1}$$

$$\boldsymbol{I}_{G1} = \frac{P_1 + jQ_1}{\boldsymbol{V}_{t1}^*} = \frac{0.300 - j0.231}{1.030\angle 0} = \frac{0.3786\angle -37.596}{1.03} = 0.3676\angle -37.596$$

$$E_1\angle\delta_1 = 1.03 + j0.72(0.3676\angle -37.596) = 1.1915 + j0.2097 = 1.2098\angle 9.982°$$

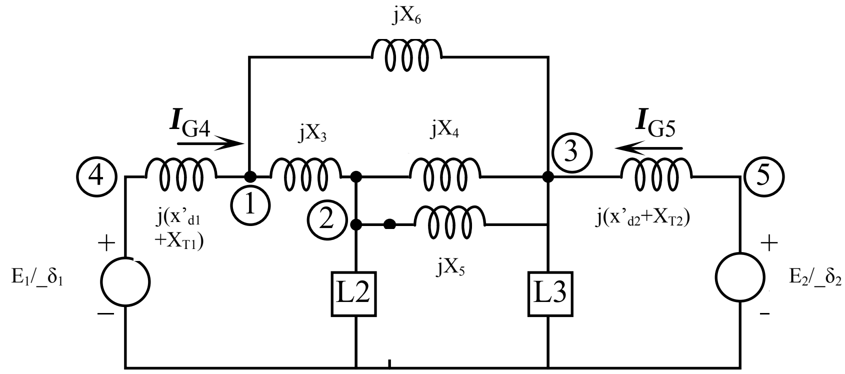
Generator 3

$$E_3\angle\delta_3 = \boldsymbol{V}_{t3} + j0.2666\boldsymbol{I}_{G3}$$

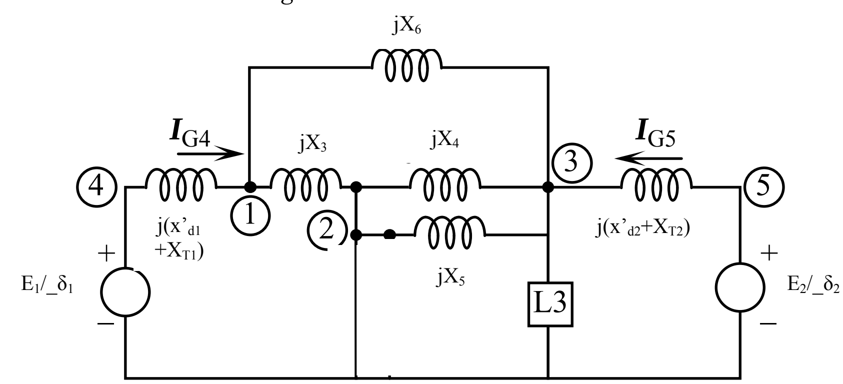
$$\boldsymbol{I}_{G3} = \frac{P_3 + jQ_3}{\boldsymbol{V}_{t3}^*} = \frac{1.000 - j0.378}{1.020\angle 0.5°} = \frac{1.0691\angle -20.707}{1.02} = 1.0481\angle -21.207$$

$$E_3\angle\delta_3 = 1.02\angle -0.5° + j0.2666(1.0481\angle -21.207) = 1.1211 + j0.2516$$
$$= 1.149\angle 12.649°$$

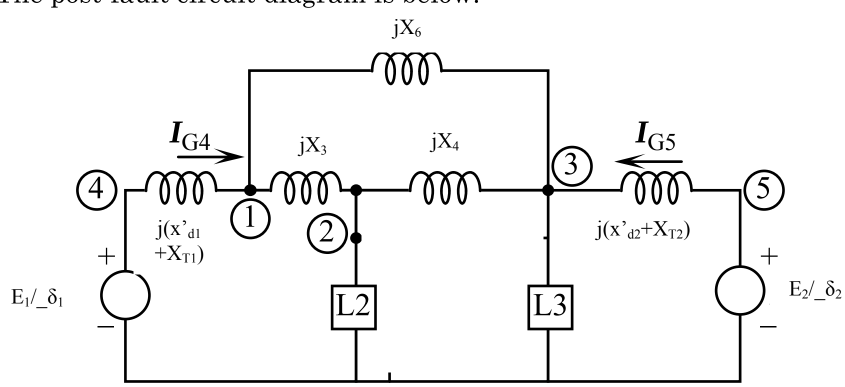
(b) The pre-fault circuit diagram is below. Note that loads are normally of very large impedance relative to all other circuit elements are therefore are effectively open circuits, but in this case, they are not and so should be included in the circuit diagrams.



The fault-on circuit diagram is below.



The post-fault circuit diagram is below.



3. (12 pts) A power system has only two machines, each of inertia constant $M_i=2H_i/\omega_{Re}$, i=1,2. Define the torque angles of each machine as δ_i, i=1,2. Show that the equations of motion for such a case can be written as one swing equation of exactly the same form as that of a single-machine-infinite-bus system, where the equivalent machine inertia is given by

$$M = \frac{M_1 M_2}{M_1 + M_2}$$

and the equivalent machine torque angle is $\delta_{12} = \delta_1 - \delta_2$.

Solution

For a 2 machine system, we can write

$$M_1 \ddot{\delta}_1 = P_{m1} - P_{e1} = P_{a1}$$

$$M_2 \ddot{\delta}_2 = P_{m2} - P_{e2} = P_{a2}$$

Now, let

$$\delta_{12} = \delta_1 - \delta_2$$

$$\ddot{\delta}_{12} = \ddot{\delta}_1 - \ddot{\delta}_2$$

Then

$$\ddot{\delta}_{12} = \ddot{\delta}_1 - \ddot{\delta}_2 = \frac{P_{a1}}{M_1} - \frac{P_{a2}}{M_2} = \frac{M_2 P_{a1} - M_1 P_{a2}}{M_1 M_2}$$

Multiply both sides by $\frac{M_1 M_2}{M_1 + M_2}$ to get

$$\frac{M_1 M_2}{M_1 + M_2}\ddot{\delta}_{12} = \frac{M_2 P_{a1} - M_1 P_{a2}}{M_1 + M_2} = \frac{M_2 P_{m1} - M_2 P_{e1} - M_1 P_{m2} + M_1 P_{e2}}{M_1 + M_2}$$

$$= \frac{M_2 P_{m1} - M_1 P_{m2}}{M_1 + M_2} - \frac{M_2 P_{e1} - M_1 P_{e2}}{M_1 + M_2}$$

Now, for a one-machine-infinite bus system we write

$$\overline{M}\ddot{\delta}_{12} = \overline{P}_m - \overline{P}_e$$

where

$$\overline{M} = \frac{M_1 M_2}{M_1 + M_2} \qquad \overline{P}_m = \frac{M_2 P_{m1} - M_1 P_{m2}}{M_1 + M_2} \qquad \overline{P}_e = \frac{M_2 P_{e1} - M_1 P_{e2}}{M_1 + M_2}$$

4. (14 pts) A two-bus system is characterized by the following equations:

$$G_1(\theta, V, \lambda) = 0.4\lambda - 2V\sin\theta = 0$$

$$G_2(\theta, V, \lambda) = 0.1\lambda + 2V^2 - 2V\cos\theta = 0$$

In performing a step of the continuation power flow, the tangent vector is found to be $\begin{bmatrix} d\theta \\ dV \\ d\lambda \end{bmatrix} = \begin{bmatrix} 0.2500 \\ -0.1152 \\ 1 \end{bmatrix}$, and the "predicted" point is identified as $\begin{bmatrix} \theta^{(2,p)} \\ V^{(2,p)} \\ \lambda^{(2,p)} \end{bmatrix} = \begin{bmatrix} 0.2686 \\ 0.8990 \\ 1.2 \end{bmatrix}$.

(a) Provide equations necessary to identify a "corrected" point based on the perpendicular intersection method.

(b) Describe application of the Newton-Raphson method to solve these equations. Your description need not utilize numerical values, but it should convince me that you know how to implement it.

Solution:

$$\underline{0} = \underline{G}(\underline{y}^{(i+1)}, \lambda^{(i+1)}) \quad \Rightarrow \quad \begin{array}{l} 0.4\lambda - 2V\sin\theta = 0 \\ 0.1\lambda + 2V^2 - 2V\cos\theta = 0 \end{array}$$

$$\left\{\underline{y}^{(i+1)} - \underline{y}^{(i+1,p)}\right\} \bullet \underline{t} = 0 \Rightarrow$$

$$\begin{bmatrix}\theta - 0.2686 & V - 0.899 & \lambda - 1.2\end{bmatrix} \bullet \begin{bmatrix} 0.2500 \\ -0.1152 \\ 1 \end{bmatrix} = 0$$

Performing the multiplication associated with the last equation results in

$$0.25\theta - 0.0712 - 0.1152V + 0.1036 + \lambda - 1.2 = 0 \Rightarrow 0.25\theta - 0.1152V + \lambda - 1.1676 = 0$$

Writing the three equations together

$$0.4\lambda - 2V\sin\theta = 0$$

$$0.1\lambda + 2V^2 - 2V\cos\theta = 0 \qquad (*)$$

$$0.25\theta - 0.1152V + \lambda - 1.1676 = 0$$

These equations may then be solved iteratively using the following relation:

$$\underline{x}^{(1)} = \underline{x}^{(0)} + \Delta\underline{x}^{(0)} = \underline{x}^{(0)} - \underline{J}^{-1}\underline{f}(\underline{x}^{(0)}) \qquad (**)$$

where $\underline{x}^{(1)}$ is the new point, $\underline{x}^{(0)}$ is the old point, $\underline{f}(\underline{x})$ are the three expressions on the left-hand-side of (*) above, and J is the Jacobian matrix of $\underline{f}(\underline{x})$. Equation (**) is iterated until the maximum absolute value of $\underline{f}(\underline{x})$ is within a σ of 0, where σ is a small number.

5. Answer the below questions:
   a. (15 pts) Indicate, for each of the below data changes, whether you would expect the system to move <u>closer to</u>, <u>further from</u>, or <u>not move</u>, with respect to the MW-loading "distance" to the voltage collapse point.
      i. Load characteristics changed from 100% constant impedance to 50% constant current, 50% constant power.

         Closer to.

      ii. Q_{min} of all generators is doubled (e.g., -50 MVAR is changed to -100 MVAR).

         Not move.

      iii. A series capacitor is inserted into a heavily loaded transmission circuit

         Further from.

      iv. A shunt reactor is switched in at a high voltage bus having a voltage of 0.94 pu.

         Closer to.

      v. Remove the regulating capability of some LTC's feeding some heavy loads modeled as 100% constant impedance,

         Further from.

   b. (15 pts) Consider a small ball sitting in the bottom of a rounded bowl. If you give the ball a push, it will climb the side of the bowl. It might go over the edge, and it might not. Identify what in this ball-bowl experiment is analogous to the following aspects of the power system stability problem:

i. The stable equilibrium point and the unstable equilibrium point.

   The SEP is the bottom of the bowl. The UEP is the top of the edge.

ii. The severity and duration of a fault.

   The force of the push.

iii. During the “first-swing” when the generator is experiencing acceleration.

   During the push.

iv. During the “first-swing” when the generator is experiencing deceleration.

   After the push, when the ball is going up the side of the bowl.

v. The critical stability case, when speed is just zero at the unstable equilibrium point.

   When the push is just enough for the ball to reach the edge of the bowl but not go over.

c. (4 pts) Assume a reasonable H-constant for a synchronous machine rated at 500 MVA. Based on your assumption, compute the MW-sec of this machine.

   Assume H=3➔ MW-sec=3*500=1500 MW-sec.