EQUIPMENT AND SYSTEM MODELING FOR LARGE-SCALE STABILITY STUDIES

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ABSTRACT

As systems have increased in size and complexity, and as the number of firm-power ties between systems have increased, the nature of the system stability problems has changed. Furthermore, there is also a demand for more accurate engineering analysis of these complex problems. Although much effort has been devoted to searching for alternate methods, the practical and effective method for large-scale studies is still the simulation of the system response, with time, following a prescribed disturbance. This emphasis on simulation necessarily requires system and equipment models which adequately represent their behavior over some time span, usually less than 20 seconds.

This paper describes the modeling used in a large-scale stability program which incorporates detailed models of synchronous machines, excitation systems, prime-movers, loads, relays, series capacitor protection devices, and DC systems and controls. The equations or block diagrams for the representation of the equipment are presented for those models which either have not been used previously in a large-scale program or which have been altered from that previously used.

I. INTRODUCTION

Power systems have grown in both size and complexity, and, consequently, the nature of system stability problems has changed. Furthermore, there is a demand for more accurate engineering analysis of the system stability problem. And to compound this situation, there is a trend toward the use of the control systems associated with the excitation system, the prime-movers, and the dc systems to improve the stability margins of power systems.

The practical and effective method for large-scale stability studies is still the simulation of the system response, with time, following a prescribed disturbance. This emphasis on simulation necessarily requires system and equipment models which adequately represent their behavior over some time span, usually less than 20 seconds. Furthermore, the representation of the automatic operation of equipment, such as relay action, may be a desirable part of such a simulation.

This paper describes the modeling used in a large-scale (2000 bus) stability program, which incorporates detailed models of synchronous machines, excitation systems, prime-movers, loads, relays, series capacitor protection devices, and dc systems and controls. Some of these models have been previously used and described. These include the representation of composite system loads[1] and the representation of dc systems and their controls.[1,2] The principal changes in these representations have been to make the composite system load characteristic a function of bus frequency as well as bus voltage, and the addition of a model of the dc system and controls which uses only algebraic equations to represent the equipment during a stability analysis. The representation of induction motor loads[3] has been included in this program in order to provide a model of large industrial loads when desired.

There are, however, models included in this program which either have not been used previously in a large-scale stability program, or which have been altered from those previously used. In this category are the models of the synchronous machines, the excitation systems, the prime-mover systems, and the simulation of automatic relay operation. The representation of relays has been incorporated in other programs,[4,5] but the specific assumptions or details may be different.

The representation of synchronous machines in stability programs should be as simple as possible to minimize computer costs. But, at the same time, there are situations where an accurate representation is required. Furthermore, there may be uncertainty as to whether a simpler model will be adequate. For these reasons, models of the synchronous machine have been included which involve the representation of between one and three rotor-iron (or amortisseur) circuits. These models are discussed in Section II and their equations are given in Appendix I.

The representations of the excitation systems have been slightly altered from those proposed by the IEEE Working Group,[6] and the current representations being used are described, along with the model provided for a supplementary stabilizing signal.

The prime-mover models provided are generally much more detailed than those previously used in a large-scale stability program. This is particularly true of the model of a steam turbine, which can include boiler effects and can also represent boiler controls. These models are discussed, and "typical" values for certain of the parameters are given for the steam turbine-boiler representation.

The special relay types represented in the program, and described here, include impedance relays, out-of-step relays, over-current relays, over-power relays, and under-frequency relays.

II. SYNCHRONOUS MACHINE MODELS

During any given system stability study, it is common to have synchronous machines in the system being represented by models of varying detail. For example, the machines near the disturbance may be represented

Paper 71 C 26-PWR-V-B, recommended and approved by the Power System Engineering Committee of the IEEE Power Society for presentation at the PICA Conference, Boston, Mass., May 24-26, 1971. Manuscript submitted January 6, 1971; made available for printing June 9, 1971.

by a very detailed model, whereas machines remote to the disturbance may be represented by the simplest model. Furthermore, the machines at intermediate locations may use still another model. Therefore, one requirement for a sophisticated stability program is that it have models available to the user which encompass the range of representation required.[7]

In the development of the models used in this program, it was decided to identify three basic models, and it was noted that by the proper selection of the numerical values used in these models, one could represent two additional models. The detailed equations of the models are described in Appendix I. In the order of increasing detail they are:

1. The classical transient stability model, which uses a constant voltage magnitude behind transient reactance (MODEL I). When represented by a Norton Equivalent, the transient reactance (X_d') is represented by a reactance to ground at the machine's terminal, and a current is injected at this node equal to ($\bar{e}'/X_d'$) where $\bar{e}'$ is the complex voltage behind X_d'.

2. The second model (MODEL II) allows the representation of the stator circuits and two rotor circuits. The rotor circuits are the field circuit in the direct axis and an iron circuit in the quadrature axis. This representation can be used to directly include a substantial portion of the damping effects which arise in a steam-turbine generator due to the presence of rotor iron currents.

By setting the quadrature-axis transient reactance (X_q') equal to the quadrature-axis synchronous reactance (X_q) in the equations, this model may also be used to represent the stator circuits and only the field circuit in the rotor. This particular representation has been widely used in stability studies, and is the simplest model which includes the direct representation of the effects of the excitation system behavior.

When represented by a Norton Equivalent, this model uses a reactance to ground equal to the transient reactance (X_d') and an injected current equal to ($\bar{e}'/X_d'$) where $\bar{e}'$ is the complex voltage behind X_d'. In this case, the magnitude and the angle of $\bar{e}'$ is a function of both the known internal conditions of the machine and the terminal conditions. Therefore, an iterative procedure is necessary to solve for the proper value of $\bar{e}'$.

3. The most detailed model (MODE I, III) represents the stator circuits and four rotor circuits. The rotor circuits are the field circuit, one amortisseur circuit in the direct axis, and two amortisseur circuits in the quadrature axis. In the case of a steam-turbine generator, these amortisseur circuits represent the rotor iron current effects.

This is the most detailed model that can be used where standard machine data are supplied by a manufacturer. Studies now underway suggest that this degree of detail of representation is sufficient for an accurate representation in a stability program.

The second amortisseur circuit in the quadrature axis is intended to represent a rotor iron circuit in a steam-turbine generator. In the case of a laminated rotor machine (such as a hydraulic turbine generator or synchronous motor), data for only one amortisseur circuit in the quadrature axis may be available. The representation for this type of machine may be made by setting the quadrature-axis transient reactance (X_q') equal to the quadrature-axis synchronous reactance (X_q).

When represented by a Norton Equivalent, this model uses a reactance to ground equal to the subtransient reactance (X_d'') and an injected current equal to ($\bar{e}''/X_d''$) where $\bar{e}''$ is the voltage behind X_d''. The magnitude and the angle of $\bar{e}''$ is known from the solution of the machine's differential equations, and no iteration is necessary for this model.

In terms of the numbers of the various machine types that may be needed in a stability program, it is common for all of the machines represented in a study to be either MODEL I or MODEL II; whereas, it is likely that only a few MODEL III machines will be used in a study. Therefore, it is a common practice in stability programs to limit the number of MODEL III machines to some relatively small number, such as 25 machines.

The amount of machine data required increases as the detail of representation increases, and, generally, the computational costs also increase (particularly for MODEL III machines). But at the same time, the more detailed representations presumably permit a more accurate evaluation of system behavior and stability limits. Studies have been made in an attempt to define the effects of added detail in the representation (sometimes with conflicting conclusions), but there is a continuing need to define the simplest model which will meet the needs of system planners and operators in a given situation.

III. EXCITATION SYSTEMS

The basic representations for various types of excitation systems was proposed by an IEEE Working Group,[6] and its recommendations have been adopted for many stability programs. Generally, these have been satisfactory and flexible enough to represent a wide range of excitation system types. However, experience has indicated the need to modify the representation for Type I and to add the time constants T_{A1} and T_{A2} shown in Figure 1.

There are two principal reasons for making this change in the representation: (1) The analysis of some actual excitation systems has indicated that a more accurate representation of the excitation system behavior might require these additional constants. Therefore, where such tests have been made, this altered representation may be desirable. (2) In high initial response excitation systems, a reduction in transient gain may be physically attained by the use of series functions rather than by feedback functions. Therefore, to represent this type of equipment, the use of the function including T_{A1} and T_{A2} is desirable. One can, in effect, simulate the gain reduction by using only the feedback function, but this is undesirable if it can be avoided.

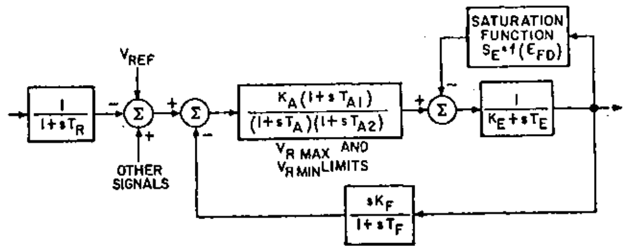

Fig. 1. Block diagram of excitation system

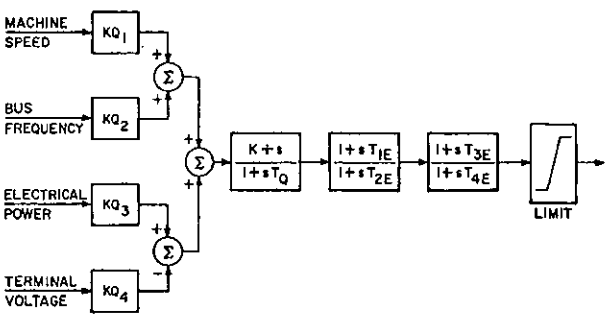

Fig. 2. Block diagram of stabilizer

The representation of supplementary stabilizing has not been standardized. Therefore, it is necessary to use as general a representation as possible. The representation shown in Figure 2 is intended to represent a general function, and inputs from machine speed, bus frequency, electrical power, and terminal voltage may be used separately or in combination.

IV. PRIME-MOVERS

The prime-mover models used in the program are more elaborate than those used in many large-scale stability programs. The use of such detailed models was prompted by two considerations: (1) In some studies, particularly those that involve the loss of generation or load, the response of the prime-movers have an important effect upon tie-line loading and frequency variations.[8] The effects which are now more directly modeled have been approximated before by empirical methods. (2) Surprisingly enough, there are times when it is more difficult to establish the proper data to be used with a simple model than with a more detailed model. Obviously, this is true only up to a certain point, but it has been noted that the manufacturers who must supply this data are more likely to be familiar with a representation where many of the important elements of the prime mover control system are separately identified. If only a simple representation is allowed, either the manufacturer must attempt to fit his transfer functions to the model or the program user must attempt it. Therefore, it seems that there may be actually an advantage in providing a certain amount of detail in a prime-mover representation.

The two principal types of prime-movers that will be discussed are the hydraulic turbine (and controls) and the steam turbine (and controls).

1. Hydraulic Turbine

The model of the hydraulic turbine and its controls are shown in Figure 3. If there is a surge tank associated with the hydraulic turbine, its representation is shown in Figure 4. These representations have been in use in the FACE Multi-Machine Power System Simulator Program,[9] and they include the effect of the surge tank and water-hammer effects in the penstock on either side of the tank, as well as speed and head effects on the turbine runner. The model is non-linear and can be used for a variety of studies. Each synchronous machine being represented by the program may have a hydraulic turbine represented with this degree of detail.

2. Steam Turbine

The representation of the steam turbine and its controls are shown in Figure 5. The model represents the effects of boiler pressure changes due to any changes in the steam flow which may occur in the turbine. If desired, a simplified representation of the combustion controls is available which includes a throttle pressure regulator control (Figure 6). Each synchronous machine being represented may have a steam-turbine represented with this degree of detail.

All values used with this model are in per-unit of the turbine rating. For cross-compound units, the combined rating of the high-pressure, intermediate-pressure, and low-pressure turbines is the base.

Governor dead-band may be represented, and a distinction can be made between a mechanical-hydraulic control system and an electro-hydraulic control system (as applied to General Electric turbines.)

The speed relay, valve, and steam bowl effects are separately represented by linearized models.[10] The limit on the rate-of-change of valve position may be represented as well as the limit representing the maximum valve position. When a change in the high-pressure turbine flow occurs, the effect upon the high-pressure turbine throttle pressure due to the change in the pressure drop and the boiler pressure may be modeled.

A reheater (or pipe volume) between the high and intermediate turbine and a reheater (or pipe volume) between the intermediate and low pressure turbines may be separately modeled. The intercept value position for each reheater is identified, but in this program these are maintained at a constant value. The functions $\frac{1 + sT_4}{1 + sT_5}$ and $\frac{1 + sT_6}{1 + sT_7}$ represent the effects of feed-water heaters.

The combustion control representation includes a limit before the representation of the combustion elements. This is a limit which may exist in a given control scheme. In any case, it also affects the maximum power that the boiler may produce even after a long

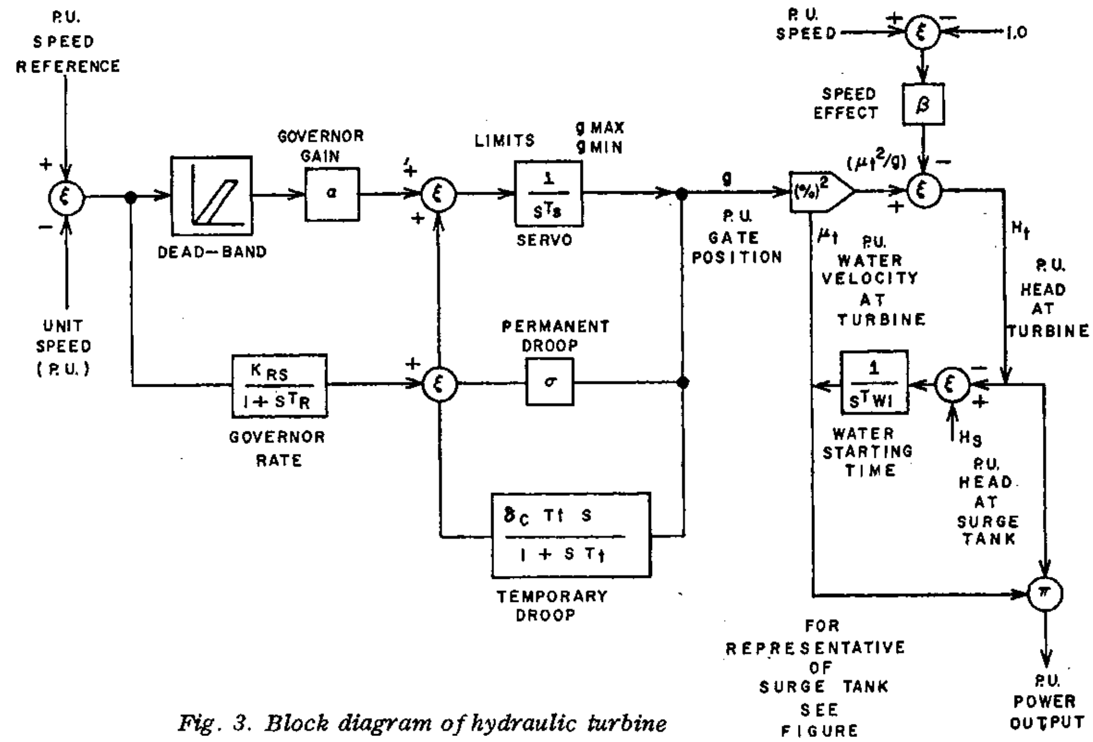

Fig. 3. Block diagram of hydraulic turbine

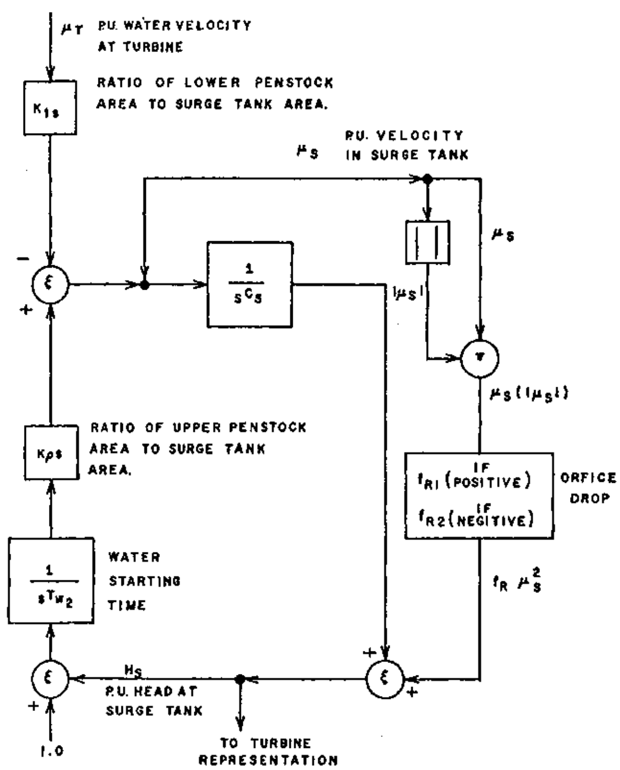

Fig. 4. Block diagram of surge tank representation

period of time. Therefore, if such a limit does not exist in the control, the value used should be the maximum available boiler output in per-unit of the combined turbine rating. This value would ordinarily neglect any effects which could not occur without manual intervention.

The values for the representation of the turbine are either obtained from the manufacturer for specific machines, or from typical values provided by the manufacturer. Since reheat time-constants depend upon the equipment used outside of the turbine, these are usually computed by the user.

In order to provide guidance to a user as to the typical numbers that might be used in the representation of the boiler and feed-water heater effects, the responses of a variety of boiler types were examined, and the following constants appear to represent average responses (see Figures 5 and 6 for the meaning of the symbols used):

K_2 = 0.08
T_D = 7 seconds for gas and oil-fired boilers
T_D = 25 seconds for fast responding coal-fired units
T_D = 60 seconds for slow responding coal-fired units
T_8 = 5 seconds for gas and oil-fired boilers
T_8 = 30 seconds for coal-fired units
T_4 = 29.4 seconds
T_5 = 30.0 seconds
T_6 = 29.4 seconds
T_7 = 30 seconds

For units less than 300 MW:

$K_3 = 0.5 \times 10^{-2}$

For units more than 300 MW:

$K_3 = 0.8 \times 10^{-2}$

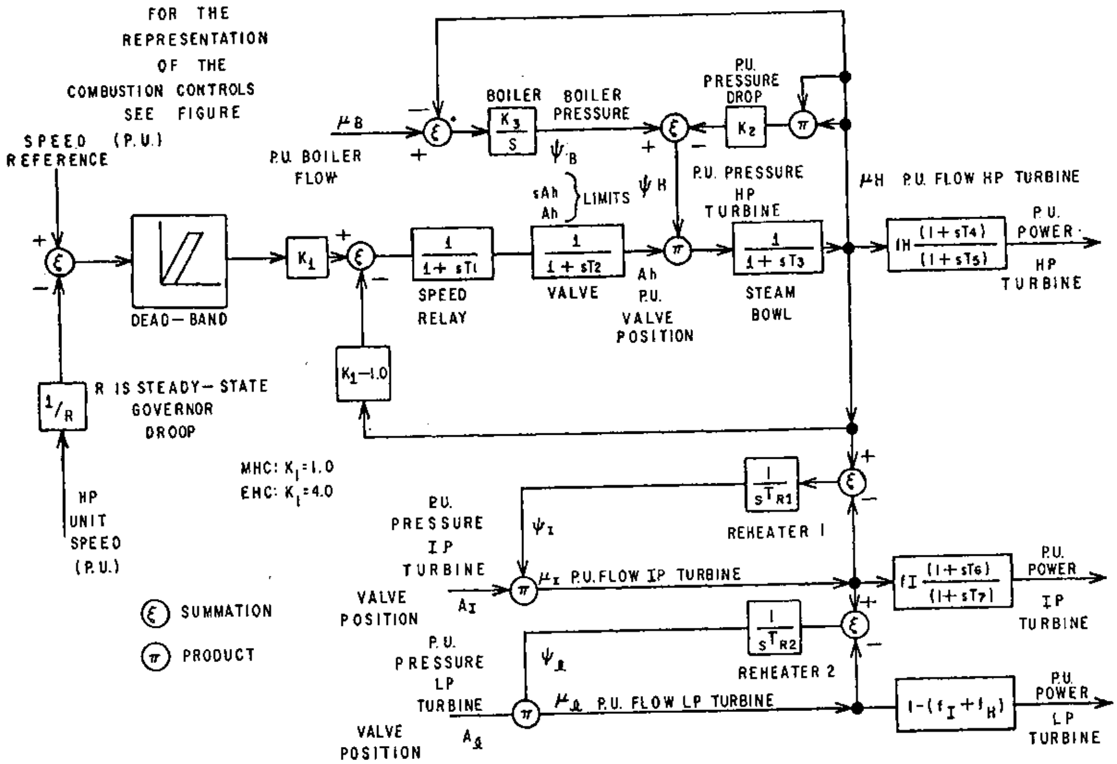

Fig. 5. Block diagram of steam turbine and boiler representation

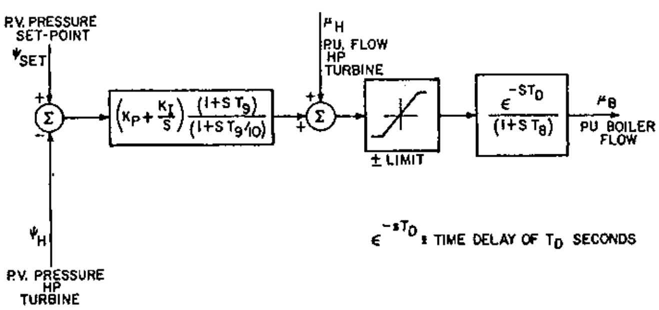

Fig. 6. Block diagram of boiler control

The pressure control settings are tabulated as follows:

	K_P	K_I	T_9
Oil, gas > 300 MW	3.33	.045	6
Oil, gas < 300 MW	5.0	.05	6
Fast coal > 300 MW	1.67	.02	36
Fast coal < 300 MW	3.33	.022	24
Slow coal > 300 MW	1.05	.0105	60
Slow coal < 300 MW	2.20	.011	36

Although the values given are considered to be typical, it should be recognized that there could be significant variations from these for a unit in a station.

There have been three types of responses included, one for gas and oil-fired boilers, one for fast coal and one for slow coal. However, since the effects of combustion control will not be felt for anything less than T_D seconds, a study which is concerned with a time period of less than 10 seconds will not be affected by these controls, and only the oil or gas fired units will be affected for time spans less than 25 seconds.

For once-through boilers, it has been noted that their responses can be quite similar to those of drum-type boilers if controlled in the boiler-follow mode. In actual practice, there will be a wide range of long term response characteristics of once-through boilers, depending upon the method of control; however, again, unless the investigation covers more than 10 to 20 seconds, these effects will be negligible.

While the effects of boiler controls may be neglected in a stability analysis, it should be noted that the effect of the boiler pressure change and pressure drop upon the throttle pressure is important when there has been a loss of generation and where the response of the system to this loss is important.

V. RELAY REPRESENTATIVES

There are four basic types of transmission line relay characteristics[11] which have been incorporated into the stability program. These are discussed in detail in the succeeding sections.

1. Impedance Relays

The impedance relay characteristic is in the form of a circle on the impedance plane (Figure 7). This circle is defined by its center coordinates (R_c, X_c) and its radius (r_1), and the relay is identified, by input, as being located at one end of a particular transmission line. The blocking relay characteristic is assumed to be a concentric circle, with a specified radius (r_2).

The apparent line impedance (Z_ℓ) is calculated at each time step, and when the impedance enters the outer circle, but not the inner circle, the blocking logic is initiated. If the impedance Z_ℓ stays in the outer circle for three cycles or more before entering the inner circle, tripping is blocked. If the impedance Z_ℓ leaves the outer circle, the relay is assumed to reset instantaneously.

If the impedance enters the interior circle, and if tripping is not blocked, the logic is initiated to trip the line. When Z_ℓ leaves the interior circle, if the time inside was less than one cycle, the relay is assumed to reset instantaneously. If the time was more than one cycle, the tripping is continued.

The choice of three cycles for the blocking relay and one cycle for the operation of the tripping relay is arbitrary and can be changed for all relays of this type.

The user specifies, as part of input, the total line tripping time (relay plus breaker time) and the dead-time between the tripping point and the time when line reclosing will be done. Single-shot reclosure is allowed.

This same relay may trip several additional lines, each with its own trip time and dead-time for reclosure.

2. Directional Over-current Relays

The directional element of an over-current relay is represented as shown in Figure 8. The angle of maximum torque (θ) is an input value. When the line current I_ℓ is in the tripping region, the overcurrent relay is allowed to operate.

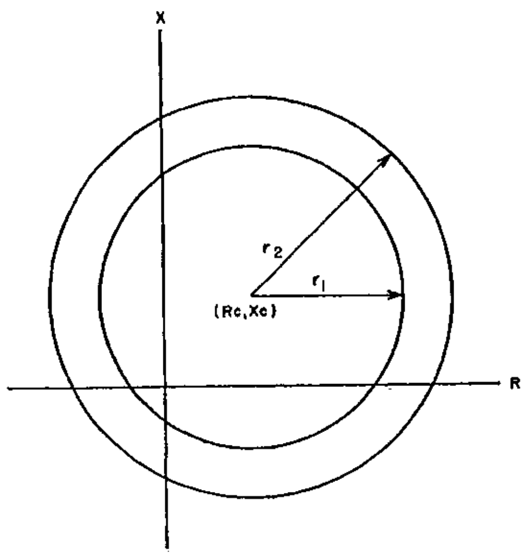

Fig. 7. Impedance relay characteristic

When I_ℓ is out of the tripping region, the relay is reset at a fixed rate (also given by input), or is held at the reset position if it has not moved.

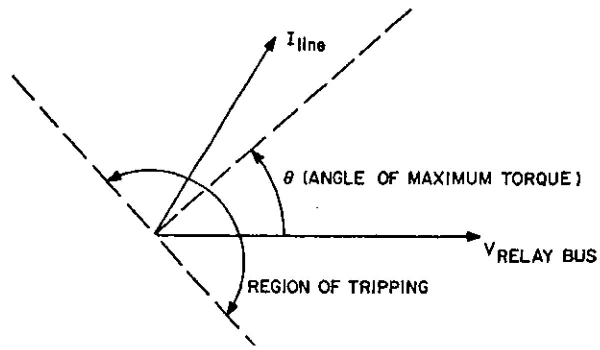

Fig. 8. Directional over-current relay characteristic

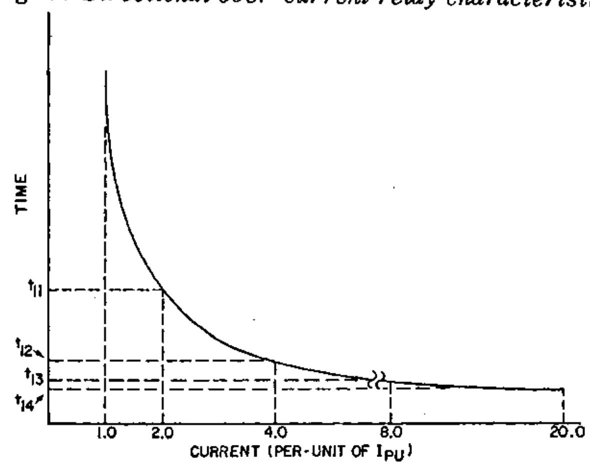

Fig. 9. Over-current relay time characteristic

The relay time-current characteristic (Figure 9) is curve fitted by a third-order polynominal. The specific time-dial setting for the relay is used to formulate this curve. The pick-up current (in per-unit on the system base) is given as input data, together with four relay operating times as functions of specific multiples of the pick-up current.

The following logic is used:

(1) If the current is in the proper direction, the relay operating time (t_c) is calculated from the polynominal fit of the current-time characteristic. If the line current is greater than 1.5 times the pick-up current, the change in the per-unit position of the relay (D_R) is:

$$\Delta D_R = \frac{\Delta t}{t_c}$$

$$D_R = D_R + \Delta D_R$$

When $D_R = 1.0$, line tripping is initiated, and the line is removed at a fixed time later (equal to the breaker time, and read in as part of the input).

If the direction of the current goes outside of the tripping region of the directional unit, or if the line current becomes less than 1.5 times the pick-up current, then the relay begins to reset:

$$\Delta D_R = \frac{-\Delta t}{t_{RESET}}$$

$$D_R = D_R + \Delta D_R$$

The action is hated when D_R reaches zero.

The tripping and relcosing logic for the line and for remote lines is identical to that for impedance relays.

3. Out-of-Step Relays

The relay characteristic for the Out-of-Step Relay is shown in Figure 10. The slopes and intercepts (on the R axis) of the two curves are given as input.

Once the starting zone is identified, the logic of the unit causes the line to trip if the apparent line impedance (Z_ℓ) goes from zones A to B to C. Also, if the impedance goes from zones C to B to A, the line is tripped. Otherwise, no tripping occurs.

The tripping and reclosing logic is the same as described for impedance relays.

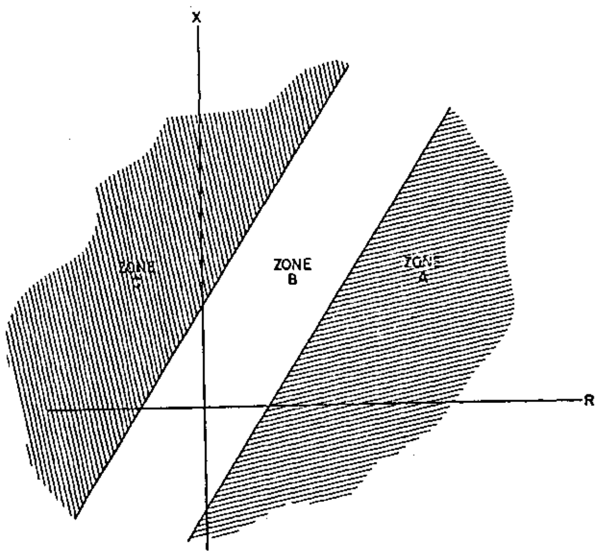

Fig. 10. Out-of-step relay characteristic

4. Over-Power Relays With Time-Delay

The transmission line power (P_ℓ) is compared with the relay pick-up power (with power direction considered as well as magnitude).

When P_ℓ exceeds the pick-up power, with the proper direction, the overpower relay begins to operate:

$$\Delta D_R = \frac{\Delta t}{t_{pu}}$$

where t_{pu} is the relay operating time (assumed constant).

$$D_R = D_R + \Delta D_R$$

and line tripping is initiated when $D_R = 1.0$.

Whenever P_ℓ is less than the pick-up power or reverses sign:

$$\Delta D_R = \frac{-\Delta t}{t_{RESET}}$$

The tripping logic is the same as for impedance relays.

For all relay types, the relay can be used to initiate the tripping of a number of remote lines, where each line has its individual breaker time and reclosing time.

In addition to the representation of conventional transmission line relays, the representation of the automatic operation of the series capacitor protective equipment can be useful. It is necessary to provide a current setting which will operate the protective gap in terms of the fundamental frequency symmetrical RMS current. When the line current exceeds this value, the shorting of the series capacitor is simulated. As part of input, the time between the gap closing and reinsertion is provided, and it is used to determine the time at which the capacitor is reinserted. Any number of removals and reinsertions is allowed.

5. Under-Frequency Relays

The representation of under-frequeny relays provides for the modeling of either a static type relay or an induction cup type relay. In either case, when the relay at a bus is represented, the bus frequency is calculated, and the load can be shed in three steps. Each of 200 buses may have an under-frequency relay with different characteristics, and different load shedding schedules. The breaker operating time is identified separately from the relay operating time.

For a static relay, the input data required is the relay trip time, the breaker trip time, the values of the three frequencies at which load shedding will be done, and the amount of bus load to be shed at each frequency. The principal consideration here is that if the frequency recovers, the correct decision will be made either to reset the relay (if the time since the relay began to operate is less than the relay operating time) or to proceed to trip the load as scheduled (if the time since the relay began to operate is more than the relay operating time but less than the combined breaker and relay time).

The representation of an induction cup type of under-frequency relay is more complicated. The representation assumes that there is a unique relay characteristic which describes the relation between the rate-of-change of frequency and the relay operating time. This characteristic is represented in the program by two straight-line segments. One extends from a point, where the maximum rate of frequency (MAXR) and minimum time (MINT) are defined, down to a "break point" with BRR and BRT defining the point. The second segment extends from the break point to the point that defines the minimum rate of frequency (MINR) and maximum time (MAXT).

When the bus frequency drops below a particular relay setting, the time (ST) is stored. Then, at subsequent times, where the frequency continues to be less than the set point, the average rate of change of frequency is calculated:

$$\text{AVR} = \frac{\text{RELAY SETTING} - \text{FREQUENCY}}{\text{TIME} - \text{ST}}$$

If AVR is greater than MAXR, then the shed time is the sum of MINT and the breaker time. If AVR is less than MINR, the shed time is the sum of MAXT and the breaker time. If the value of AVR is between MAXR and MINR, then the appropriate straight line segment must be identified, and the value of the relay operating time calculated. This time is added to the breaker time in order to calculate the shed time, and the shed time is recalculated at each time step.

If the bus frequency rises above the set point, the relay is assumed to reset instantaneously, unless the time since the relay began to operate is more than the relay operating time but less than the combined breaker and relay time. In that case, the load is tripped at the scheduled time.

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APPENDIX

MACHINE EQUATIONS

Each of the mathematical models used to represent a synchronous machine in a stability study consists of a set of simultaneous algebraic equations and a set of simultaneous differential equations. The phasor diagram associated with each representation is a way of describing the algebraic equations which relate the machine's internal and terminal conditions. In some cases, additional algebraic equations may exist. In all cases, the differential equations are expressed separately from the phasor diagram.

The equations which are not represented by the phasor diagram are described in this appendix for each of the representations considered.

A. MODEL I

Model I is the "Classical" stability representation for the synchronous machine (Figure 11). The only differential equation to be solved is the acceleration equation.

B. MODEL II

Model II is the representation of the machine which includes the field effects and represents the effect of a single iron circuit in the quadrature axis. In addition to the equations implied in the phasor diagram (Figure 12) other equations are:

$$\tilde{\bar{e}}' = \{\tilde{\bar{e}}'_q + j\,[\bar{e}'_d - (X'_q - X'_d)\,\bar{i}_q]\}\epsilon^{j\delta}$$

It should be noted that $\tilde{\bar{e}}'$ is a phasor, but $\bar{e}'_q$, $\bar{e}'_d$, and $\bar{i}_q$ are the magnitudes of these variables.

$$\frac{d\bar{e}'_q}{dt} = \frac{1}{T'_{do}}\,(\bar{E}_{fd} - \bar{E}_I)$$

$$\frac{d\bar{e}'_d}{dt} = \frac{1}{T'_{qo}} (-\bar{E}_{Ikq})$$

$$\bar{E}_{Ikq} = \bar{e}'_d + (\bar{X}_q - \bar{X}'_q)\,\bar{i}_q$$

$$\bar{E}_I = \bar{e}'_q + (\bar{X}_d - \bar{X}'_d)\,\bar{i}_d + \Delta\bar{E}_I$$

$$\frac{d^2\delta}{dt^2} = \frac{180\,f}{H} (\bar{T}_m - \bar{T}_e) - \bar{K}\frac{d\delta}{dt}$$

$\Delta\bar{E}_I$ = f (index of saturation)

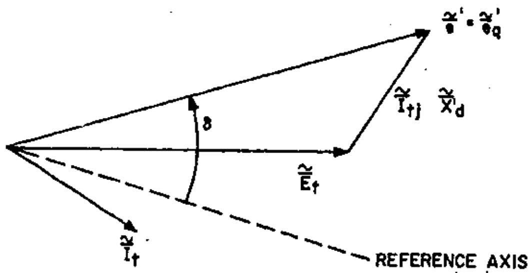

Fig. 11. Model I phasor diagram

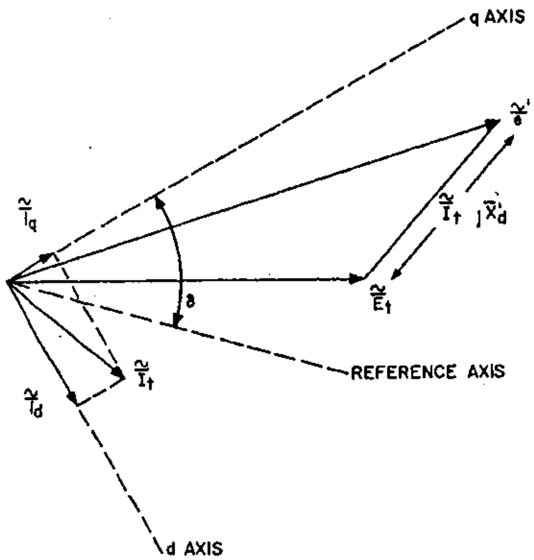

Fig. 12. Model II phasor diagram

C. MODEL III

Model IV is the representation of the machine which includes the field effects and the effects of amortisseur circuits in both axes. The equations given here (in addition to those of the phasor diagram (Figure 13)) are for the case where there is one amortisseur circuit in the direct axis and two amortisseur circuits in the quadrature axis, and $\bar{X}''_d$ equals $\bar{X}''_q$:

$$\bar{\psi}''_d = \bar{\psi}_{kd} + \left\{\frac{\bar{X}''_d - \bar{X}_1}{\bar{X}'_d - \bar{X}_1}\right\} (\bar{e}'_q - \bar{\psi}_{kd})$$

$$\bar{\psi}''_q = \bar{\psi}_{kq} + \left\{\frac{\bar{X}''_d - \bar{X}_\ell}{\bar{X}'_q - \bar{X}_\ell}\right\} (\bar{e}'_d - \bar{\psi}_{kg})$$

$$\frac{d\bar{e}'_q}{dt} = \frac{1}{T'_{do}} (\bar{E}_{fd} - \bar{E}_I)$$

$$\frac{d\bar{\psi}_{kd}}{dt} = -\left\{\frac{(\bar{X}''_d - \bar{X}_1)^2}{(\bar{X}'_d - \bar{X}''_d)\,T''_{do}}\right\} (\bar{i}_{kd})$$

$$\bar{E}_I = \bar{e}'_q + (\bar{X}_d - \bar{X}'_d)(\bar{i}_d - \bar{i}_{kd}) + \Delta\bar{E}_I$$

$$\bar{i}_{kd} = \frac{(\bar{X}'_d - \bar{X}''_d)}{(X'_d - X_1)^2} \left\{\bar{\psi}_{kd} - \bar{e}'_q + (\bar{X}'_d - \bar{X}_1)\,\bar{i}_d\right\}$$

$$\frac{d\bar{e}'_d}{dt} = \frac{1}{T'_{qo}} (-\bar{E}_{Iq})$$

$$\frac{d\bar{\psi}_{kq}}{dt} = \frac{1}{T''_{qo}} (-\bar{E}_{Ikq})$$

$$\bar{E}_{Iq} = \bar{e}'_d + (\bar{X}_q - \bar{X}'_q)\,\bar{i}_q - \frac{(\bar{X}_q - \bar{X}'_q)(\bar{X}'_q - \bar{X}''_d)}{(\bar{X}'_q - \bar{X}_\ell)^2}\,E_{Ikq}$$

$$\bar{E}_{Ikq} = \bar{\psi}_{kq} - \bar{e}'_d + (\bar{X}'_q - \bar{X}_\ell)\,\bar{i}_q$$

$$\frac{d^2\delta}{dt^2} = \frac{180\,f}{H} (\bar{T}_m - \bar{T}_e) - K\frac{d\delta}{dt}$$

$\Delta\bar{E}_I$ = f (index of saturation)

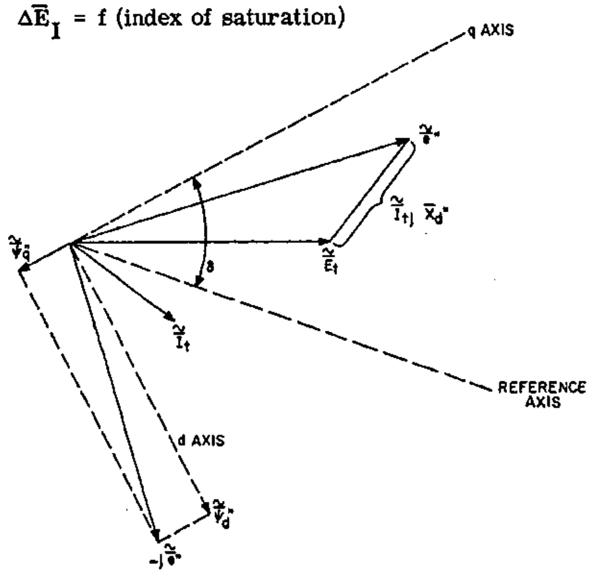

Fig. 13. Model III phasor diagram

APPENDIX II

SYMBOLS

δ	Angle between the q axis and the reference axis (degrees)
t	Time (seconds)
f	Base frequency (hertz)
H	Inertia constant (MW-seconds/MVA)
K	Damping coefficient
$\overline{T}_m$	Mechanical torque (per-unit)
$\overline{T}_e$	Electrical torque (per-unit)
$\overline{X}_1$	Stator leakage reactance (per-unit)
$\overline{X}''_d$	Direct-axis subtransient reactance (per-unit)
$\overline{X}'_d$	Direct-axis transient reactance (per-unit)
$\overline{X}_d$	Direct-axis synchronous reactance (per-unit)
$\overline{X}''_q$	Quadrature-axis subtransient reactance (per-unit)
$\overline{X}'_q$	Quadrature-axis transient reactance (per-unit)
$\overline{X}_q$	Quadrature-axis synchronous reactance (per-unit)
$\overline{X}_p$	Potier reactance (per-unit)
T''_{do}	Direct-axis subtransient open-circuit time-constant (seconds)
T''_{qo}	Quadrature-axis subtransient open-circuit time-constant (seconds)
T'_{do}	Direct-axis transient open-circuit time-constant (seconds)
T'_{qo}	Quadrature-axis transient open-circuit time-constant (seconds)
$\overline{i}_q$	Quadrature-axis component of stator current (per-unit)
$\overline{i}_d$	Direct-axis component of stator current (per-unit)
$\overline{i}_{kd}$	Direct-axis amortisseur current (per-unit)
$\overline{E}_{Ikq}$	Quadrature-axis amortisseur current (per-unit)
$\overline{E}_I$	Field current (per-unit)
$\overline{E}_{Iq}$	Quadrature-axis iron current
$\Delta\overline{E}_I$	Correct to field current for saturation (per-unit)
$\tilde{\overline{e}}'$	Phasor voltage behind transient reactance (per-unit)
$\tilde{\overline{e}}''$	Phasor voltage behind subtransient reactance (per-unit)
$\tilde{\overline{e}}'_q$	Field flux-linkages (per-unit)
$\tilde{\overline{e}}'_d$	Quadrature-axis iron-circuit flux-linkages (per-unit)
$\overline{E}_{fd}$	Field voltage (per-unit)
$\overline{E}_p$	Potier voltage (per-unit)
$\overline{\psi}_{kd}$	Direct-axis amortisseur flux-linkages (per-unit)
$\overline{\psi}_{kq}$	Quadrature-axis amortisseur flux-linkages (per-unit)
$\overline{\psi}''_d$	Direct-axis component of rotor subtransient flux-linkages (per-unit)
$\overline{\psi}''_q$	Quadrature-axis component of rotor subtransient flux-linkages (per-unit)

NOTE: Wherever a variable is a phasor, it may appear in two forms. For example, the per-unit field current may appear as a phasor, where the symbol $\tilde{\overline{E}}_I$ is used to designate this form; or, its per-unit magnitude may be used in an equation, where the symbol $\overline{E}_I$ is used to designate this form.

K. R. Padiyar and R. S. Ramshaw (University of Waterloo, Waterloo, Ont., Canada): In the section II of the paper, the author has presented three different models of the synchronous machine. It is of interest to note that the number of models given in this paper, is one less than the number presented earlier by the author in reference [7] of this paper. This shows that there is no basic difference between the various models presented. Actually, all these models can be derived from one common framework which does not put any restriction on the number of rotor circuits to be considered in the analysis. Present methods of analysis consider not more than two rotor circuits per axis and the different models apply for the particular cases of specified number of rotor circuits.

We have shown the general nature of the machine model (applicable to any arbitrary number of rotor circuits) in our paper presented in PICA-7 conference[1]. The 'dynamic saliency' is also taken into account in this approach.

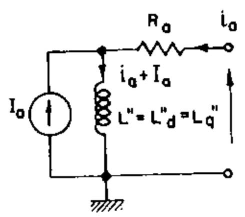

Fig. 1.

The author states that the Norton's equivalent circuit of the machine can be derived from the voltage source model given in the paper. This is not strictly correct, since the current source model shown in Fig. (1) and derived in reference [1] is the exact representation, while the voltage source model of the paper is not. The Thevenin's equivalent of the current source of Fig. (1) is given by

$$e_{Th} = L'' p\, I_a$$

where

$$I_a = I_d \cos\theta + I_q \sin\theta,\ I_d = \underline{c}_1^t\, \underline{\psi}_{rd},\ I_q = \underline{c}_2^t\, \underline{\psi}_{rq} + \xi i_q$$

$\underline{c}_1$ and $\underline{c}_2$ are constant vectors, $\underline{\psi}_{rd}$ and $\underline{\psi}_{rq}$ are respectively rotor direct axis and quadrature axis flux linkages.
Differentiating I_a, we obtain

$$e_{Th} = L'' \left(\frac{dI_d}{dt} \cos\theta + \frac{dI_q}{dt} \sin\theta\right) + L''\, \dot{\theta}(-I_d \sin\theta + I_q \cos\theta)$$

Ignoring the first term in comparison with the second term in the above equation and ignoring the variation of θ, we get the voltage source model of the paper.

Hence the voltage source model is inexact in comparison with the current source model and the correct conversion of the voltage source into a current source model

$$\left(I_N = \frac{1}{L''p}\, e\right)$$

will yield wrong results. However the transformation of the sources presented in the paper is valid only for steady-state conditions of the network at constant (nominal) frequency.

It would be interesting to know the integration method used by the author. The time-step for Runge-Kutta routines is determined by the smallest time constant of the system. Thus, the step size may be determined as much from the small regulator time constants as the damper circuit time constants.

Finally, we would like to compliment the author for this excellent work in the field of system modelling, which still remains a difficult task.

REFERENCE

[1] K. R. Padiyar and R. S. Ramshaw, "A Generalized Synchronous Machine Model for Large Scale Power System Dynamic Studies", Conference Proceedings, PICA-7, pp. 261-268, May 1971.

Manuscript received June 7, 1971.

C. C. Young: As system problems, the methods of analysis, and the characteristics of computing devices change, it is important to periodically examine the different models that one might use for system equipment, and to select those that are best suited for a particular situation. Therefore, I am always interested in discussions and technical papers on the subject of synchronous machine modeling for power system studies.

Although I have been directly involved in the modeling of synchronous machines in system studies for 20 years, I am continually amazed at just how many different ways one can write Park's equations. Professor Ramshaw and Mr. Padiyar have developed an interesting form of these equations that uses an injected current plus an inductance to ground to represent a machine. In comparing the discussor's model with other models which have been developed, there are two important facts to be recognized: First, if all of the various models start with Park's equations and then make the same simplifying assumptions, and if the mathematic process of manipulating the resulting equations are rigorous, then the models developed will all be equally accurate, regardless of how different they may be in their form. Considering the assumptions implicit in Park's equations, none of these models should be considered "exact", although they may be entirely adequate for a particular type of analysis.

Second, because of the practical problems of obtaining data and because of the computing costs that are involved in the stability analysis of a large power system, it is very important to avoid superfluous detail in the modeling of equipment which adds to the cost without contributing significantly to the accuracy of the results. Experience has indicated that it is not desirable and is not necessary to attempt to represent all of the synchronous machines in a system by a single, detailed model. Only a relatively few machines may actually require an accurate simulation. It is this situation that led us to define three distinct models of varying complexity.

The discussors seem to be confused about the differences between their model and the models described in the paper. Comparing the assumptions used in deriving these models (7) with the assumptions used by the discussors (12), they are identical except for two differences: (1) the discussors represent stator transients (fundamental and harmonic currents and fluxes arising from the presence of the $p\varphi d$, $p\varphi q$ terms in the stator voltage equations), and (2) the discussors represent the effect of speed upon the generated terminal voltage of of the machine. While there are some types of studies which require that one or both of these effects be represented, it has been found that these effects do not have to be directly represented in large-scale stability studies. (13,14) In fact, since the system connected to the machines will very rarely have its transients or speed effects represented, the representation of machine stator transients is usually inconsistent, and is therefore not as accurate a simulation as one might suppose. In any case, if the stator transients are represented, the size of the time step required for numerical integration is considerably smaller than that required by models where these effects are omitted. Therefore, the cost may become prohibitive for large system studies.

If the stator transients and speed effects are neglected, the machine stator equations become algebraic equations, and can be represented by phasors, as are the system equations. This leads to the forms shown in the paper. The Norton Equivalent discussed in the paper is an exact equivalent of the voltage form.

In the stability program, the methods used for numerical integration are predictor-corrector methods, and dual time steps are allowed where it is appropriate.

REFERENCES

[13] K. Prabhashankar, W. Janischewskyj, "Digital Simulation of Power System Dynamic Performance, Part I," IEEE Conference Paper 68CP94-PWR.

[14] D. W. Ewart, F. P. De Mello, "A Digital Computer Program for the Automatic Determination of Dynamic Stability Limits," IEEE Trans., Vol PAS 86, July, 1967, pp. 867-875.

Manuscript received July 8, 1971.