

# Windings and Axes

## 1.0 Introduction

In these notes, we will describe the different windings on a synchronous machine. We will confine our analysis to two-pole machines of the salient pole rotor construction. Results will be generalizable because

- A machine with  $p > 2$  poles will have the same phenomena, except  $p$  times/cycle.
- Round rotor machines can be well approximated using a salient pole model and proper designation of the machine parameters.

We will also define an important coordinate frame that we will use heavily in the future.

## 2.0 Defined axes

The magnetic circuit and all rotor winding circuits (which we will describe shortly) are symmetrical with respect to the polar and inter-polar (between-poles) axes. This proves convenient, so we give these axes special names:

- Polar axis: Direct, or D-axis
- Interpolar axis: Quadrature, or Q-axis.

The Q-axis is  $90^\circ$  from the D-axis, but which way?

- Ahead?
- Or behind?

Correct modeling can be achieved either way, and some books do it one way, and some another. We will remain consistent with your text and choose the Q-axis to lag the D-axis by  $90^\circ$ .

Fig. 1 is from your text, and shows the Q-axis lagging the D-axis, consistent with our assumption.

Fig. 2 is from Kundur, and shows the Q-axis leading the D-axis, which we will NOT do.

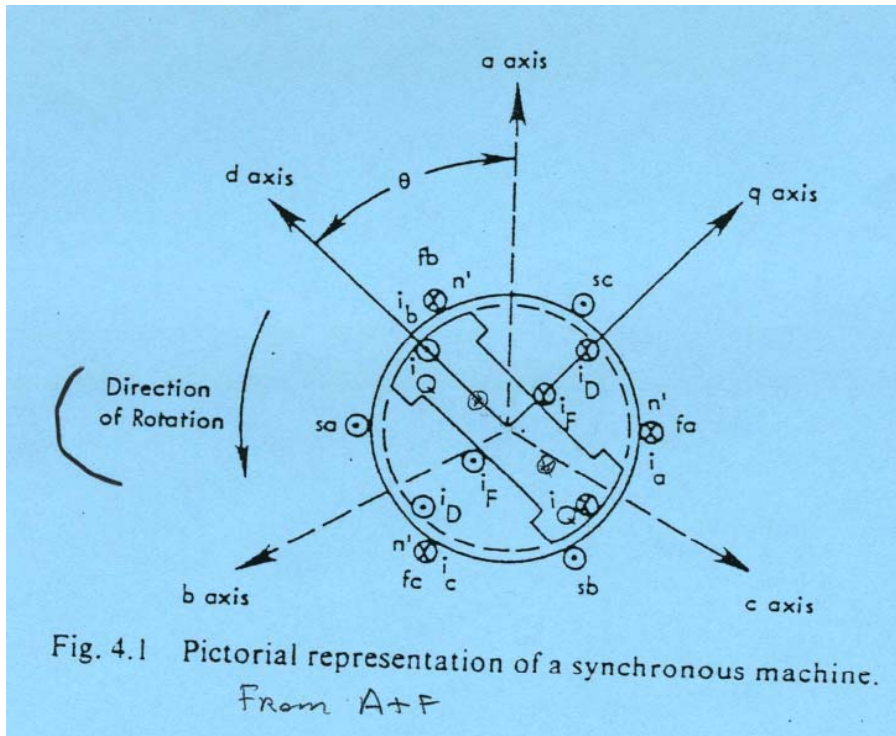
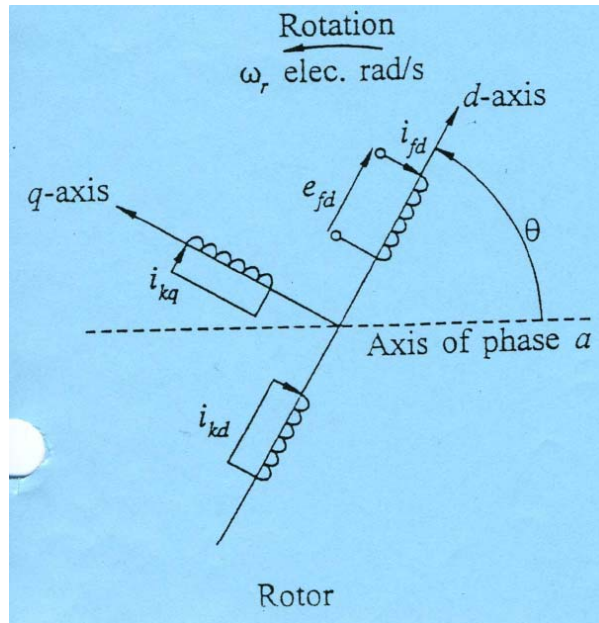


Fig. 1



### 3.0 Physical windings

There are typically 5 physical windings on a synchronous machine:

- 3 stator windings (a-phase, b-phase, and c-phase)
- 1 main field winding
- Amortisseur windings on the pole-faces

The stator windings and the field winding is familiar to you based on the previous notes. The amortisseur winding might not be, so we will take some time here to describe it.

Amortisseur means “dead.” So this winding is a dead winding under steady-state conditions. It is also frequently referred to as a “damper winding,” because, as the name suggests, it provides additional damping under transient conditions.

Amortisseur windings are not usually used on smooth-rotor machines, but the solid steel rotor cores of such machines provide paths for eddy currents and thus produce the same effects as amortisseur windings.

Amortissuer windings are often used in salient-pole machines, but even when not, eddy currents in the pole faces contribute the same effect, although greatly diminished.

Amortisseurs have a number of other good effects, as articulated by Kimbark in his Volume III book on synchronous machines.

Amortissuer windings are embedded in the pole-face (or “shoe” of the pole) and consist of copper or brass rods connected to end rings. They are similar in construction to the squirrel cage of an induction motor.

Figures 3 (from Sarma) and 4 (from Kundur) illustrate amortissuer windings. Note that they may be continuous (Fig. 3a and Fig. 4) or noncontinuous (Fig. 3b).

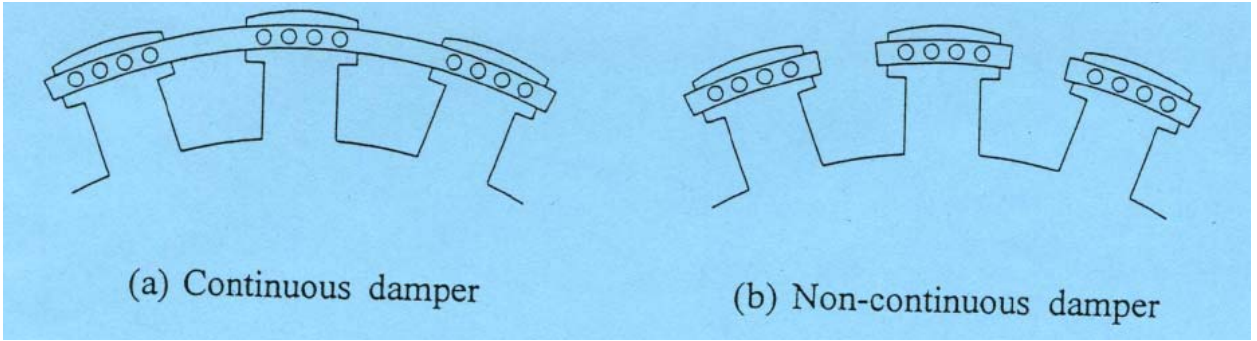


Fig. 3

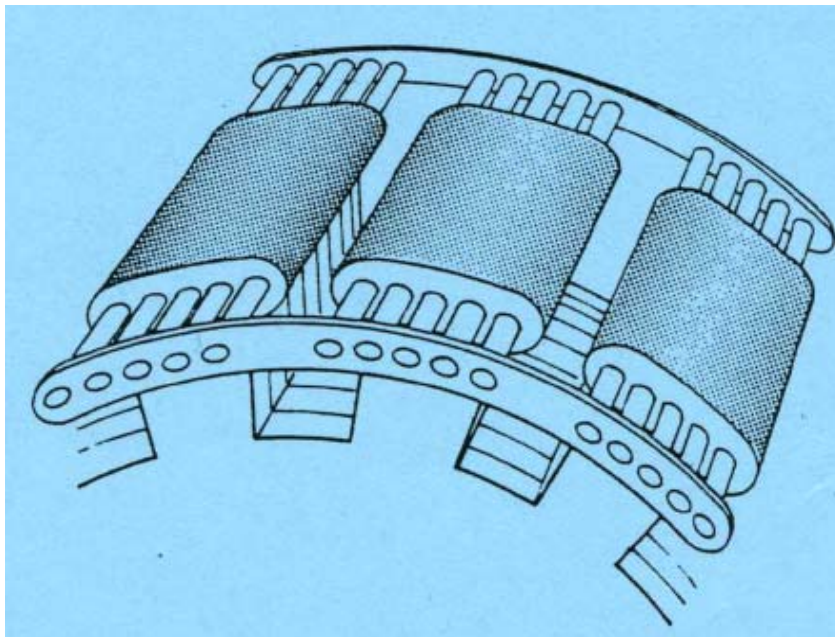


Fig. 4

## 4.0 Modeled windings and currents

Although there are typically 5 physical windings on a machine, we will model a total of 7, with associated currents as designated below.

- 3 stator windings:  $i_a, i_b, i_c$
- Field windings: There are 2: one physical; one fictitious
  - Main field winding: carrying current  $i_F$  and producing flux along the D-axis.
  - G-winding: carrying current  $i_G$  and producing flux along the Q-axis. This is the fictitious one, but it serves to improve the model accuracy of the round-rotor machine (by modeling the Q-axis flux produced by the eddy-current effects in the rotor during the transient period), and its presence does not affect the accuracy of the salient pole machine. NOTE: Text does not include this one (see pg 124). The G-winding is like the F-winding of the main field, except it has no source voltage in its circuit.

- Amortisseur winding: This one represents a physical winding for salient-pole machine with dampers, and a fictitious winding if not. Because these produce flux along both the D-axis and the Q-axis, we model two windings:
  - D-axis: amortisseur winding carrying current  $i_D$
  - Q-axis: amortisseur winding carrying current  $i_Q$

It is of interest to compare the F and G windings to the D and Q windings.

- Both the F and D produce flux along the D-axis, but D is “faster” (lower time constant) than F.
- Both the G and Q produce flux along the Q-axis, but Q is “faster” than G.

## 5.0 Flux linkages and currents

So we have seven windings (circuits) in our synchronous machine. The flux linkage seen by any winding  $i$  will be a function of

- Currents in all of the windings and
- Magnetic coupling between winding i and winding j, as characterized by  $L_{ij}$ , where  $j=1,\dots,7$ .

That is

$$\lambda_1 = \sum_{j=1}^7 L_{1j} i_j \quad (1)$$

For example, the flux linking the main field winding is:

$$\lambda_F = L_{Fa} i_a + L_{Fb} i_b + L_{Fc} i_c + L_{FF} i_F + L_{FD} i_D + L_{FQ} i_Q + L_{FG} i_G \quad (2)$$

Repeating for all windings results in Equation (4.11) in your text, with exception that your text does not represent the G-winding like we are doing here.

$$\begin{bmatrix} \lambda_a \\ \lambda_b \\ \lambda_c \\ \lambda_F \\ \lambda_D \\ \lambda_Q \\ \lambda_G \end{bmatrix} = \begin{bmatrix} L_{aa} & L_{ab} & L_{ac} & L_{aF} & L_{aD} & L_{aQ} & L_{aG} \\ L_{ba} & L_{bb} & L_{bc} & L_{bF} & L_{bD} & L_{bQ} & L_{bG} \\ L_{ca} & L_{cb} & L_{cc} & L_{cF} & L_{cD} & L_{cQ} & L_{cG} \\ \hline L_{Fa} & L_{Fb} & L_{Fc} & L_{FF} & L_{FD} & L_{FQ} & L_{FG} \\ L_{Da} & L_{Db} & L_{Dc} & L_{DF} & L_{DD} & L_{DQ} & L_{DG} \\ L_{Qa} & L_{Qb} & L_{Qc} & L_{QF} & L_{QD} & L_{QQ} & L_{QG} \\ L_{Ga} & L_{Gb} & L_{Gc} & L_{GF} & L_{GD} & L_{GQ} & L_{GG} \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \\ i_F \\ i_D \\ i_Q \\ i_G \end{bmatrix} \quad (3)$$

Note the blocks of the above matrix correspond to

- Lower right-hand 4×4 are rotor-rotor terms.
- Upper-left-hand 3×3 are stator-stator terms;
- Upper right-hand 3×4 are stator-rotor terms;
- Lower left-hand 4×3 are rotor-stator terms;

Your text summarizes the expressions for each of these groups of terms on pp. 86-87. I will expand on this summary in the next section.

## 6.0 Inductance blocks

### 6.1a Rotor-rotor terms: self inductances

Recall that the general expression for self-inductances is

$$L_{ii} = \frac{\lambda_i}{i_i} = \frac{N_i^2}{\mathcal{R}_i} \quad (4)$$

where  $\mathcal{R}_i$  is the reluctance of the path seen by  $\lambda_i$ .

At any given moment, the rotor windings see a constant reluctance path, independent of the “position” angle  $\theta$ . Thus, rotor winding self-inductances are constants, and

we define the following nomenclature, consistent with eq. (4.13) in your text.

- D-axis field winding  $L_{FF} = L_F$  (5)

- Q-axis field winding  $L_{GG} = L_G$  (6)

- D-axis amortisseur winding:  $L_{DD} = L_D$  (7)

- Q-axis amortisseur winding:  $L_{QQ} = L_Q$  (8)

Note your text's convention of using only a single subscript for constant terms.

### 6.1b Rotor-rotor terms: mutual inductances

Recall

$$L_{ij} = \frac{\lambda_i}{i_j} = \frac{N_i N_j}{\mathcal{R}_{ij}} \quad (9)$$

where  $\mathcal{R}_{ij}$  is the reluctance of the path seen by  $\lambda_i$  in linking with coil  $i$  or the path seen by  $\lambda_j$  in linking with coil  $j$  (either way – it is the same path!). Again, by similar reasoning as in section 6.1a, these mutual terms are constants (i.e., independent of  $\theta$ ).

Therefore, we have the following:

- F (field)–D (amort):  $L_{FD} = L_{DF} = M_R$  (10a)

- G (field)-Q (amort):  $L_{GQ} = L_{QG} = M_Y$  (10b)

But we have four other pairs to address:

- F (field)-G (field):  $L_{FG} = L_{GF} = 0$  (11a)

- F (field)-Q (amort):  $L_{FQ} = L_{QF} = 0$  (11b)

- G (field)-D (amort):  $L_{GD} = L_{DG} = 0$  (11c)

- D (amort)-Q (amort):  $L_{DQ} = L_{QD} = 0$  (11d)

But these pairs of windings are each in quadrature, so the flux from one winding does not link the coils of the other winding, as illustrated in Fig. 5. Therefore the above four terms are zero, as indicated in eqs (11a-11d).

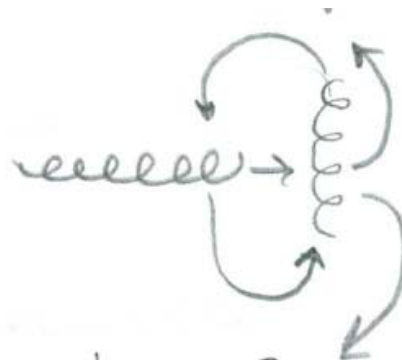


Fig. 5

## 6.2a Stator-stator terms: self inductances

We can derive these rigorously (see Kundur pp. 61-65) but the insight gained in this effort does not seem worth our time. Rather, we are better served by gaining a conceptual understanding of four ideas, as follows:

1. *Sinusoidal dependence on of permeance on  $\theta$* : Due to saliency of the poles (and to field winding slots in a smooth rotor machine), the path reluctance seen by the stator windings depends on  $\theta$ , as illustrated in Fig. 6.

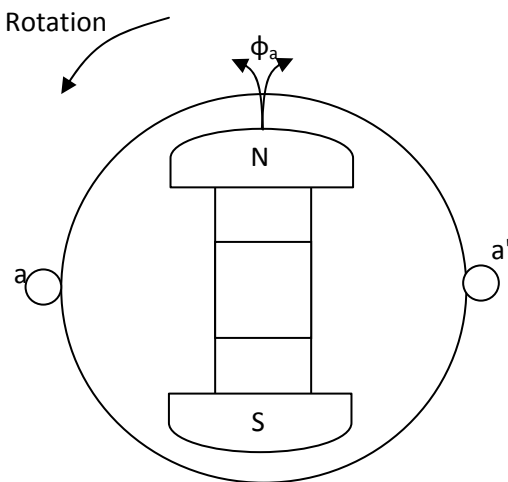


Fig. 6a:  $\theta=0^\circ$

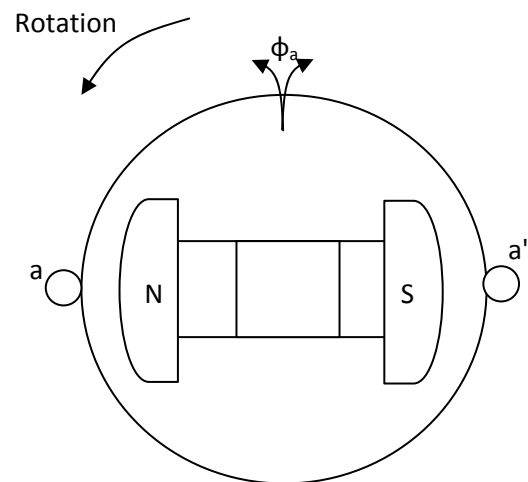


Fig. 6a:  $\theta=90^\circ$

From Fig. 6a, we observe that when  $\theta=0^\circ$ , the path of phase-a flux contains more iron than at any other angle  $0\rightarrow 180^\circ$ , and therefore the reluctance seen by the phase-a flux in this path is at a minimum, and permeance is at a maximum.

From Fig. 6b, we observe that when  $\theta=90^\circ$ , the path of phase-a flux contains more air than at any other angle  $0\rightarrow 180^\circ$ , and therefore the reluctance seen by the phase-a flux in this path is at a maximum, and permeance is at a minimum. This suggests a sinusoidal variation of permeance with  $\theta$ .

2. *Maximum permeance*: There will be a maximum permeance due to the amount of permeance seen by the phase-a flux at any angle. This will include the iron in the middle part of the rotor (indicated by a box in Figs. 6a and 6b), the stator iron, and the air gap. Denote the corresponding (maximum) permeance  $\mathcal{P}_s$ .

3. *Double angle dependence*: Because the effects described in 1 and 2 above depend on permeance (or reluctance), and not on rotor polarity, the maximum permeance occurs twice each cycle, and not once.

Taking (1), (2), and (3) together, we may write that

$$\mathcal{P} = \mathcal{P}_s + \mathcal{P}_m \cos 2\theta \quad (12)$$

4. *Inductance approximation*: Because  $L=N^2/\mathcal{R}=N^2\mathcal{P}$ , the self inductance of the a-phase winding can be written as

$$L_{aa} = L_s + L_m \cos 2\theta \quad (13)$$

Likewise, we will obtain:

$$L_{bb} = L_s + L_m \cos 2(\theta - 120) \quad (14)$$

$$L_{cc} = L_s + L_m \cos 2(\theta - 240) \quad (15)$$

Equations (13), (14), (15) are denoted (4.12) in your text.

### 6.2b Stator-stator terms: mutual inductances

We will identify 3 important concepts for understanding mutual terms of stator-stator inductances.

#### 1. *Sign*:

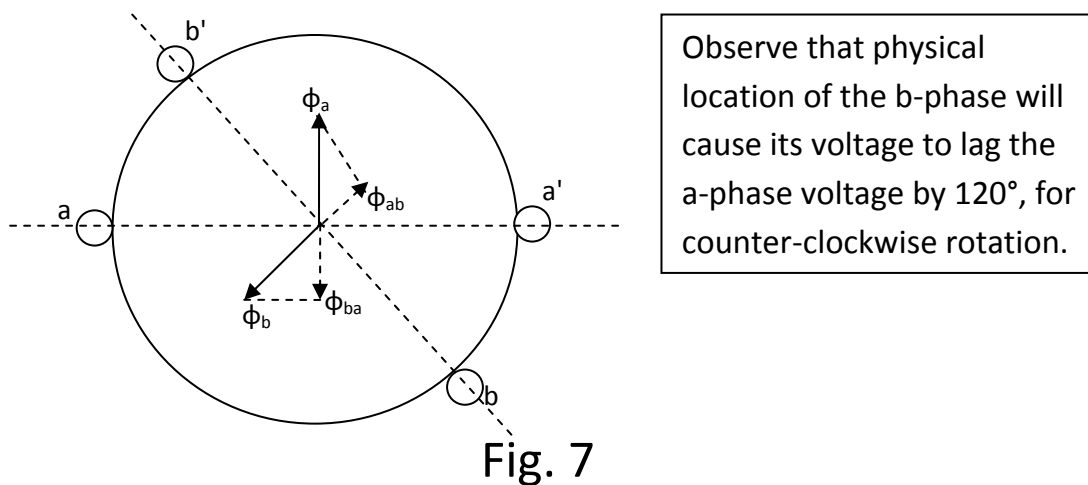
First, we need to remind ourselves of a preliminary fact:

➔ For any circuits  $i$  and  $j$ ,  $L_{ij}$  is positive if positive currents in the two circuits produce fluxes in the same direction.

With this fact, we can state important concept 1:

➔As a result of defined stator current directions, the stator mutual inductance is always negative.

To see this, we can observe that the flux produced by positive currents of a and b phases are in opposite directions, as indicated in Fig. 7.



Observe the following in Fig. 7:

- The component of flux from winding-a that links with winding-b,  $\phi_{ab}$ , is 180° from  $\phi_b$ .
- The component of flux from winding-b that links with winding-a,  $\phi_{ba}$ , is 180° from  $\phi_a$ .

The implications of the above 2 observations are that

➔ Mutually induced voltages are negative relative to self induced voltages.

➔ Mutual inductance is negative.

## *2. Function of position:*

### 2a. Maximum Permeance for Mutual Flux:

Recall that conditions where the amount of iron in the path is a maximum permeance (minimum reluctance) condition.

This condition for phase-a self-flux is  $\theta=0^\circ$ .

This condition for phase-b self-flux is  $\theta=-60^\circ$ .

Therefore the condition for maximum permeance for the mutual flux between phases a and b is halfway inbetween these two at  $\theta=-30^\circ$ .

### 2b. Periodicity of Permeance for Mutual Flux:

Starting at the maximum permeance condition, a rotation by  $90^\circ$  to  $\theta=60^\circ$  gives minimum permeance.

Starting at the maximum permeance condition, a rotation by  $180^\circ$  to  $\theta=150^\circ$  gives minimum permeance again.

The implication of these observations are that permeance, and therefore inductance, is a sinusoidal function of  $2(\theta+30^\circ)$ .

### 3. *Constant term:*

There is an amount of permeance that is constant, independent of rotor position. Like before, this is composed of the stator iron, the air gap, and the inner part of the rotor. We will denote the corresponding inductance as  $M_s$ .

From above 1, 2, and 3, we express mutual inductance between the a- and b-phases as

$$L_{ab} = -[M_s + L'_{ab} \cos 2(\theta + 30^\circ)] \quad (16)$$

One last comment: The amplitude of the permeance variation for the mutual flux is the same as the amplitude of the permeance variation for the self-flux, therefore  $L'_{ab}=L_m$ . And so the three mutual expressions we need are

$$L_{ab} = -[M_s + L_m \cos 2(\theta + 30^\circ)] \quad (17)$$

$$L_{bc} = -[M_s + L_m \cos 2(\theta - 90^\circ)] \quad (18)$$

$$L_{ca} = -[M_s + L_m \cos 2(\theta + 150^\circ)] \quad (19)$$

### 6.3 Stator-rotor terms

These are all mutual inductances. There are four windings on the rotor (F, G, D, and Q) and three windings on the stator (a, b, c phases). Therefore there are 12 mutual terms in all.

Central idea: Recall that for stator-stator mutuals,

- windings were locationally fixed, and...
- the path of mutual flux was fixed, but...
- the rotor moves within the path of mutual flux, and for this reason, the path permeance varied.

Now, in this case, for stator-rotor terms (all mutuals),

- the rotor winding locations vary and the stator winding locations are fixed, and so...
- the path of mutual flux varies, and for this reason...
- the path permeance varies.

To illustrate, consider the permeance between the a-phase winding and the main field winding (F).

- When the main field winding and the stator winding are aligned, as in Fig. 8a, the permeance is maximum, and therefore inductance is maximum.

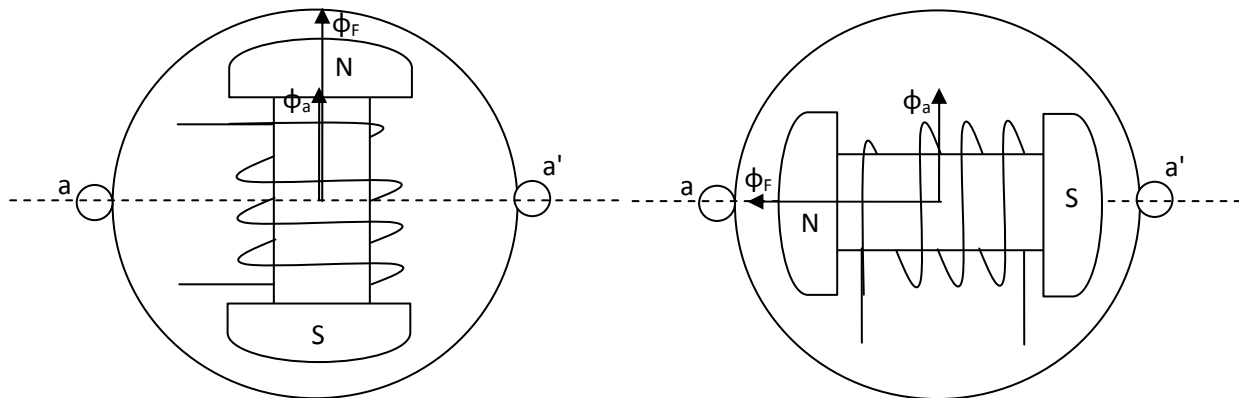


Fig. 8a

Fig. 8b

- When the main field winding and the a-phase stator winding are  $90^\circ$  apart, as in Fig. 8b, there is no linkage at all, and inductance is zero.
- When the rotor winding and the a-phase stator winding are  $180^\circ$  apart, as in Fig. 9, the permeance is again maximum, but now polarity is reversed.

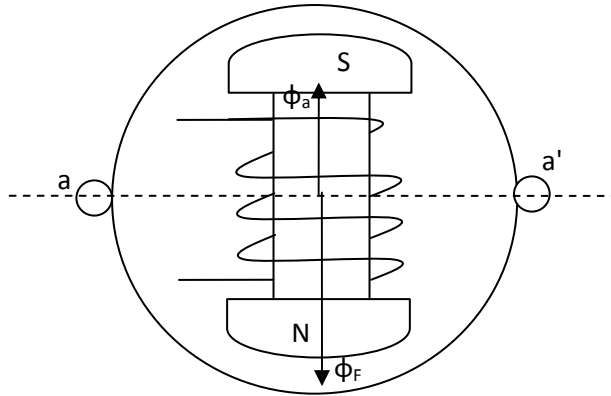


Fig. 9

This discussion results in a conclusion that the mutual inductance between a-phase winding and the main field winding should have the form:

$$L_{aF} = M_F \cos \theta \quad (20a)$$

The D-axis damper (amortisseur) winding is positioned concentric with the main field winding, both producing flux along the D-axis. Therefore, the reasoning about the mutual inductance between the a-phase winding and the D-axis damper winding will be similar to the reasoning about the mutual inductance between the a-phase winding and the main field (F) winding, leading to

$$L_{aD} = M_D \cos \theta \quad (21a)$$

Now consider the mutuals between the a-phase winding and the windings on the q-axis, i.e., the G-winding and the Q damper (amortisseur) winding.

The only difference in reasoning about these mutuals and the mutuals between the a-phase winding and the windings on the d-axis (the F-winding and the D damper winding) is that the windings on the q-axis are 90° behind the windings on the d-axis. Therefore, whereas the a-phase/d-axis mutuals were cosine functions, these mutuals will be sine functions, i.e.,

$$L_{aQ} = M_Q \sin \theta \quad (22a)$$

$$L_{aG} = M_G \sin \theta \quad (23a)$$

Summarizing stator-rotor terms for all three phases, we obtain the equations on the next page.

$$L_{aF} = M_F \cos \theta \quad (20a)$$

$$L_{bF} = M_F \cos(\theta - 120^\circ) \quad (20b)$$

$$L_{cF} = M_F \cos(\theta - 240^\circ) \quad (20c)$$

$$L_{aD} = M_D \cos \theta \quad (21a)$$

$$L_{bD} = M_D \cos(\theta - 120^\circ) \quad (21b)$$

$$L_{cD} = M_D \cos(\theta - 240^\circ) \quad (21c)$$

$$L_{aQ} = M_Q \sin \theta \quad (22a)$$

$$L_{bQ} = M_Q \sin(\theta - 120^\circ) \quad (22b)$$

$$L_{cQ} = M_Q \sin(\theta - 240^\circ) \quad (22c)$$

$$L_{aG} = M_G \sin \theta \quad (23a)$$

$$L_{bG} = M_G \sin(\theta - 120^\circ) \quad (23b)$$

$$L_{cG} = M_G \sin(\theta - 240^\circ) \quad (23c)$$

## 7.0 Summary

Summarizing all of our needed equations:

Rotor-rotor self terms: 5, 6, 7, 8

Rotor-rotor mutuals: 10a, 10b, 11a, 11b, 11c, 11d

Stator-stator self terms: 13, 14, 15

Stator-stator mutuals: 17,18, 19

Rotor-stator mutuals: 20a, 20b, 20c, 21a, 21b, 21c, 22a, 22b, 22c, 23a, 23b, 23c

Counting the above equations, we see that we have 28. But let's look back at our original flux linkage relation (3):

$$\begin{bmatrix} \lambda_a \\ \lambda_b \\ \lambda_c \\ \lambda_F \\ \lambda_D \\ \lambda_Q \\ \lambda_G \end{bmatrix} = \begin{bmatrix} L_{aa} & L_{ab} & L_{ac} & L_{aF} & L_{aD} & L_{aQ} & L_{aG} \\ L_{ba} & L_{bb} & L_{bc} & L_{bF} & L_{bD} & L_{bQ} & L_{bG} \\ L_{ca} & L_{bc} & L_{cc} & L_{cF} & L_{cD} & L_{cQ} & L_{cG} \\ \hline L_{Fa} & L_{Fb} & L_{Fc} & L_{FF} & L_{FD} & L_{FQ} & L_{FG} \\ L_{Da} & L_{Db} & L_{Dc} & L_{DF} & L_{DD} & L_{DQ} & L_{DG} \\ L_{Qa} & L_{Qb} & L_{Qc} & L_{QF} & L_{QD} & L_{QQ} & L_{QG} \\ L_{Ga} & L_{Gb} & L_{Gc} & L_{GF} & L_{GD} & L_{GQ} & L_{GG} \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \\ i_F \\ i_D \\ i_Q \\ i_G \end{bmatrix} \quad (3)$$

We have 49 terms! Where are the other 21 equations?

Note because  $L_{ij}=L_{ji}$ , the inductance matrix will be symmetric. Of the 49 terms, 7 are diagonal. The other 42 terms are off-diagonal and are repeated twice. So we are “missing” the 21 equations corresponding to the off-diagonal elements for which we did not provide equations. But we do not need to, since those “missing” equations for the off-diagonal elements  $L_{ij}$  are exactly the same as the equations for the off-diagonal elements  $L_{ji}$ .

We will look closely at this matrix in the next set of notes.