Time Constants (Section 4.14.1)

In a linear static circuit with no capacitance, i.e., an R-L circuit, the transient currents decay with time according to

$$i(t) = i_0 e^{-t/T} \tag{1}$$

where i_0 is the initial current and T is the time constant.
For an R-L circuit, we may show that

$$i(t) = \frac{1}{R} e^{-t/(L/R)} \tag{2}$$

where we see that $T=L/R$. How do we think of T?

Let $t=T$ and then we get that

$$i(t) = i_0 e^{-T/T} = i_0 e^{-1} = 0.368 i_0 \tag{3}$$

Thus we see that the time constant is:

1. The time in which the current decreases to 36.8% of its initial value;
2. The time in which the current decrease equals 63.2% of its initial value;
3. The time in which the current would decrease to zero if it continued to decrease at its initial rate of decrease.

Figure 1 illustrates these three ways of thinking about T.

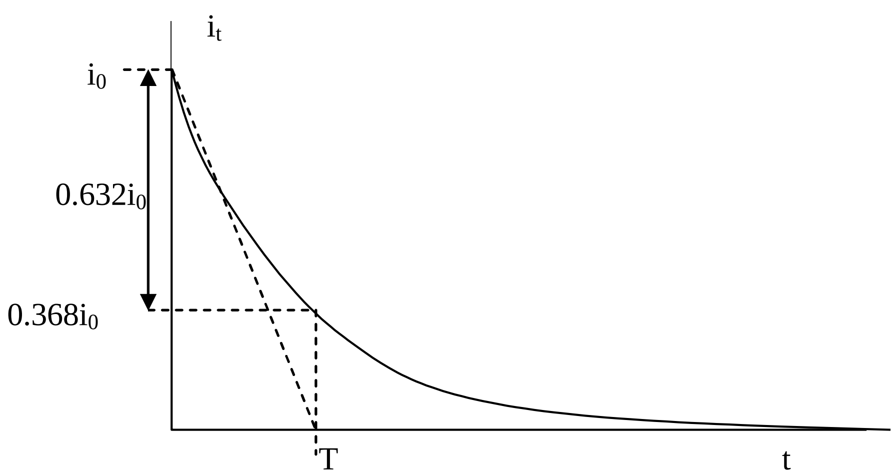


Fig. 1

So the time constant is a good measure of the speed of the dynamics. Low T ➔ fast dynamics.

For a salient pole machine, we have a time constant for each rotor circuit given as the ratio of some inductance to the circuit resistance.

We can obtain the time constants under one of two conditions:
1. Stator is open-circuited.
2. Stator is short-circuited.

The procedure used in VMAF for developing these equations is as follows (see pp 133-134). ***I will apply it to obtain the open circuit d-axis subtransient time constant***.

1. **Flux linkage voltage equations**: Write voltage equation for the appropriate circuit using flux derivatives using (4.36) below:

$$\begin{bmatrix} v_d \\ v_q \\ -v_F \\ v_G = 0 \\ v_D = 0 \\ v_Q = 0 \end{bmatrix} = -\begin{bmatrix} r & 0 & 0 & 0 & 0 & 0 \\ 0 & r & 0 & 0 & 0 & 0 \\ 0 & 0 & r_F & 0 & 0 & 0 \\ 0 & 0 & 0 & r_G & 0 & 0 \\ 0 & 0 & 0 & 0 & r_D & 0 \\ 0 & 0 & 0 & 0 & 0 & r_Q \end{bmatrix} \begin{bmatrix} i_d \\ i_q \\ i_F \\ i_G \\ i_D \\ i_Q \end{bmatrix} + \begin{bmatrix} -\omega\lambda_q \\ \omega\lambda_d \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} \dot{\lambda}_q \\ \dot{\lambda}_d \\ \dot{\lambda}_F \\ \dot{\lambda}_G \\ \dot{\lambda}_D \\ \dot{\lambda}_Q \end{bmatrix} \qquad (4.36)$$

   We assume a step change is applied to the field winding (with the stator winding open or short-circuited, it is the only way we can provide an external forcing function). We want to characterize the time constant of the D-winding. Therefore, we pull out of (4.36) the v_F and v_D equations:

$$v_F = r_F i_F + \dot{\lambda}_F \qquad (4.181a)$$

$$v_D = 0 = r_D i_D + \dot{\lambda}_D \qquad (4.181b)$$

2. **Replace fluxes with currents**: Use eq. (4.20) (see "macheqts") to replace fluxes with currents.

$$\begin{bmatrix} \lambda_0 \\ \lambda_d \\ \lambda_q \\ \lambda_F \\ \lambda_G \\ \lambda_D \\ \lambda_Q \end{bmatrix} = \begin{bmatrix} L_0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & L_d & 0 & kM_F & 0 & kM_D & 0 \\ 0 & 0 & L_q & 0 & kM_G & 0 & kM_Q \\ 0 & kM_F & 0 & L_F & 0 & M_R & 0 \\ 0 & 0 & kM_G & 0 & L_G & 0 & M_Y \\ 0 & kM_D & 0 & M_R & 0 & L_D & 0 \\ 0 & 0 & kM_Q & 0 & M_Y & 0 & L_Q \end{bmatrix} \begin{bmatrix} i_0 \\ i_d \\ i_q \\ i_F \\ i_G \\ i_D \\ i_Q \end{bmatrix} \quad (4.20)$$

For example, we see that

$$\lambda_F = \sqrt{\frac{3}{2}} M_F i_d + L_F i_F + M_R i_D \qquad (*)$$

$$\lambda_D = \sqrt{\frac{3}{2}} M_D i_d + M_R i_F + L_D i_D \qquad (**)$$

3. **Apply conditions**: Apply appropriate open circuit or short circuit conditions to simplify the equation. For example, if we are getting the open circuit time constants, then the stator windings are open circuited, and $i_d=0$. This causes $i_d=i_q=0$ and (*) and (**) become

$$\lambda_F = L_F i_F + M_R i_D \qquad (4.182a)$$

$$\lambda_D = M_R i_F + L_D i_D \qquad (4.182b)$$

Notice that from (4.182b), for a step change applied to the field voltage, CFLT indicates that $\lambda_D(0^+)=0$, which implies that

$$0 = M_R i_F + L_D i_D \Rightarrow i_F = \frac{-L_D}{M_R} i_D \qquad (4.183)$$

4. **Manipulate**: Differentiate (4.182a) and (4.182b), respectively,

$$\dot{\lambda}_F = L_F \dot{i}_F + M_R \dot{i}_D$$

$$\dot{\lambda}_D = M_R \dot{i}_F + L_D \dot{i}_D$$

and then substitute into (4.181a) and (4.181b), respectively. This results in

$$v_F = r_F i_F + L_F \dot{i}_F + M_R \dot{i}_D \qquad (4.184a)$$

$$0 = r_D i_D + M_R \dot{i}_F + L_D \dot{i}_D \qquad (4.184b)$$

Divide (4.184a) by L_F and (4.184b) by M_R to get

$$\frac{v_F}{L_F} = \frac{r_F}{L_F} i_F + \dot{i}_F + \frac{M_R}{L_F} \dot{i}_D \qquad (4.184c)$$

$$0 = \frac{r_D}{M_R} i_D + \dot{i}_F + \frac{L_D}{M_R} \dot{i}_D \qquad (4.184d)$$

Subtract (4.184c) from (4.184d) to get

$$\frac{r_D}{M_R} i_D - \frac{r_F}{L_F} i_F + \frac{L_D}{M_R} \dot{i}_D - \frac{M_R}{L_F} \dot{i}_D + \dot{i}_F - \dot{i}_F = -\frac{v_F}{L_F}$$

$$\frac{r_D}{M_R} i_D - \frac{r_F}{L_F} i_F + \left(\frac{L_D}{M_R} - \frac{M_R}{L_F} \right) \dot{i}_D = \frac{-v_F}{L_F}$$

Now replace i_F with (4.183) to get

$$\left(\frac{r_D}{M_R} + \frac{r_F}{L_F} \frac{L_D}{M_R} \right) i_D + \left(\frac{L_D}{M_R} - \frac{M_R}{L_F} \right) \dot{i}_D = \frac{-v_F}{L_F}$$

Now divide through by the coefficient of the derivative term:

$$\frac{\left(\frac{r_D}{M_R} + \frac{r_F}{L_F} \frac{L_D}{M_R} \right)}{\left(\frac{L_D}{M_R} - \frac{M_R}{L_F} \right)} i_D + \dot{i}_D = \frac{\frac{-v_F}{L_F}}{\left(\frac{L_D}{M_R} - \frac{M_R}{L_F} \right)}$$

Multiply top and bottom of the first term on the left-hand-side by M_R, and do the same to the right-hand-side, to get

$$\frac{\left(r_D + \frac{r_F L_D}{L_F} \right)}{\left(L_D - M_R^2 / L_F \right)} i_D + \dot{i}_D = \frac{-M_R v_F / L_F}{\left(L_D - M_R^2 / L_F \right)}$$

5. **Approximate and apply LaPlace**: Use the following information (in per-unit):
   - Damper circuits are very fast, because r_D and r_Q are large.
   - Field circuits are very slow, because r_F and r_G are small.

   Reference to Example 4.1 (p. 107) indicates, in per-unit:

   r_D=0.0131, r_Q=0.054
   r_F=0.000742, r_G=0.00584

VMAF (pg. 134) make the statement that "usually in pu r_D>>r_F while L_D and L_F are of similar magnitude." This means r_D>>$r_F L_D/L_F$; it is also true that r_Q>>r_G while L_Q and L_G are of similar magnitude. Data for the pu inductances from Example 4.2 in VMAF (p. 112) are as follows:

L_F=1.65
L_D=1.605
L_G=1.76
L_Q=1.526

and so the above becomes

$$\frac{r_D}{\left(L_D - M_R^2 / L_F\right)} i_D + \dot{i}_D = \frac{-M_R v_F / L_F}{\left(L_D - M_R^2 / L_F\right)}$$

Rearranging, we obtain

$$\dot{i}_D + \frac{r_D}{\left(L_D - M_R^2 / L_F\right)} i_D = -v_F \frac{M_R / L_F}{\left(L_D - M_R^2 / L_F\right)} \qquad (4.186a)$$

Now define

$$K_1 = \frac{r_D}{\left(L_D - M_R^2 / L_F\right)} \qquad K_2 = -v_F \frac{M_R / L_F}{\left(L_D - M_R^2 / L_F\right)}$$

Then (4.186a) becomes

$$\dot{i}_D + K_1 i_D = K_2 \qquad (4.186b)$$

Using LaPlace transforms, we get

$$sI_D(s) + K_1 I_D(s) = K_2 / s$$

$$I_D(s)\left(s + K_1\right) = K_2 / s \qquad (4.186c)$$

$$I_D(s) = \frac{K_2}{s(s+K_1)}$$

Taking partial fraction expansion, we have:

$$I_D(s) = \frac{K_2}{s(s+K_1)} = \frac{K_2/K_1}{s} - \frac{K_2/K_1}{s+K_1} = \frac{K_2}{K_1}\left(\frac{1}{s} - \frac{1}{s+K_1}\right)$$

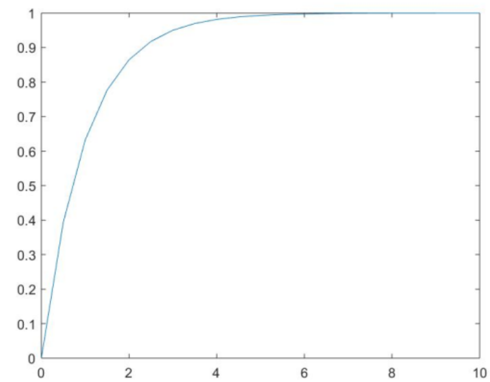
The inverse LaPlace transform is then

PFE: $$I_D(s) = \frac{K_2}{s(s+K_1)} = \frac{A}{s} + \frac{B}{(s+K_1)}$$

$A = sI_D(s)|_{s=0}$
$B = (s+K_1)I_D(s)|_{s=-K1}$

$$i_D(t) = \frac{K_2}{K_1}\left(1 - e^{-K_1 t}\right)u(t) \qquad (4.186d)$$

This shows that if we were to apply a step change in the field voltage ($v_F = V_F u(t)$) per bottom of p. 133), the current in the D-damper winding would rise in accordance with a time constant of K_1, similar to the function $y=(1-e^{-1t})u(t)$ as indicated below.



Replacing K_1 and K_2, we obtain

$$i_D(t) = -v_F \frac{M_R / L_F}{\left(L_D - M_R^2 / L_F\right)} \frac{\left(L_D - M_R^2 / L_F\right)}{r_D}\left(1 - e^{-\frac{r_D}{\left(L_D - M_R^2/L_F\right)}t}\right)u(t)$$

$$= -v_F \frac{M_R / L_F}{r_D}\left(1 - e^{-\frac{r_D}{\left(L_D - M_R^2/L_F\right)}t}\right)u(t) \qquad (4)$$

Recall

$$i(t) = \frac{1}{R} e^{-t/(L/R)} \qquad (2)$$

where $T=L/R$, and so we see that $1/K_1$ is the time constant. We define this time constant as the open circuit direct-axis subtransient time constant, i.e.

$$\tau''_{d0} = \frac{1}{K_1} = \frac{L_D - M_R^2 / L_F}{r_D}$$

It’s name comes from the fact that

- it is computed when the stator windings are *open circuit*,

- it characterizes the behavior of the D-winding and is therefore a *subtransient* response.

Comments:

a. I have added a HW problem where you need to perform this same development except for the quadrature axis time constant.

b. On p. 134, VMAF writes the following: "When the damper winding is not available or after the decay of the subtransient current, we can show that the field current is affected only by the parameters of the field circuit, i.e.,

$$r_F i_F + L_F \dot{i}_F = V_F u(t) \tag{4.188}$$

The time constant of this transient is the d axis transient open circuit time constant τ'_{d0}, given by

$$\tau'_{d0} = \frac{L_F}{r_F} \tag{4.189}$$

Likewise, for round rotor machines (for which we need to model the G-winding), we can obtain the q-axis transient open circuit time constant τ'_{q0}, given by

$$\tau'_{q0} = \frac{L_G}{r_G} \qquad \text{(similar to 4.189)}$$

c. The time constants given using the Greek letter τ are in per-unit time, that is, they are related to time constants given using "t" (in seconds) according to

$$\tau = t\omega_B \Rightarrow \tau = t \times 377.$$

d. Most texts indicate time constants in seconds. These values must be normalized before using them in relations (4.189) or (similar to 4.189), e.g., see below example 4.2, p. 112.

Data in pu is as follows:

$L_d = 1.70$	$\mathrm{k}\,M_Q = 1.49$
$L_q = 1.64$	$r = 0.001096$
$L_F = 1.65$	$r_F = 0.000742$
$L_D = 1.605$	$r_D = 0.0131$
$L_Q = 1.526$	$r_Q = 0.0540$
$\mathrm{k}\,M_F = M_R = \mathrm{k}\,M_D = 1.55$	$H = 2.37\,\mathrm{s}$
$\ell_d = \ell_q = 0.15$	

And Example 4.7 indicates x'_q=0.38 pu which is for the same machine. Recall our development for L'_q in the last set of notes (SubtransientTransientLT) which resulted in

$$L'_q = L_q - \frac{{L_{AQ}}^2}{L_G} \qquad (4.180)$$

Solving (4.180) for L_G results in

$$L_G = \frac{{L_{AQ}}^2}{L_q - L'_q}$$

Recall from the "perunitization" notes, p. 30 that the mutual, L_{AQ}, is the difference between the self and the leakage, i.e.,

$$L_{qu}-l_{qu}=L_{Qu}-l_{Qu}=L_{Gu}-l_{Gu}=L_{AQ}$$

Substitution of this last expression for L_{AQ} in the L_G expression results in

$$L_G = \frac{\left(L_q - l_q\right)^2}{L_q - L'_q}$$

Now we may utilize the above Example 4.2 data to obtain:

$$L_G = \frac{\left(L_q - l_q\right)^2}{L_q - L'_q} = \frac{\left(1.64 - 0.15\right)^2}{1.64 - 0.380} = \frac{2.2201}{1.26} = 1.76$$

This value is given in the pu values computed via Example 4.1, which shows Example 4.2 is for the same machine. VMAF does not provide t'_{q0} or τ'_{q0} anywhere for the machine in these examples, and so I chose a value of 0.8sec that is typical of round-rotor machines based on my experience and review of various sources including, for example, Table 4.7 in VMAF, data in Kundur's book, and data in some WECC data sets I have. Then, the pu value of this would be

τ'_{q0}=0.8*377=301.6.

So then, $r_G=L_G/\tau'_{q0}=1.76/301.6=0.00583554$pu,
which again agrees with pu values computed for Example 4.1.
This value makes intuitive sense because it is an order of magnitude larger than r_F (and so it is faster than the field cct), but an order of magnitude lower than the damper values r_D and r_Q (and so it is slower than the damper winding circuits). This value, as computed here, was used in Ex. 4.2 (p. 112).

Application of similar procedures results in the expressions that Kundur calls the "classical expressions" given as follows (the VMAF equation number appears in the box to the right).

Without G-winding (salient pole machine):

OC/DA/T/TC: $$\tau'_{d0}=\frac{L_F}{r_F}$$ (D-axis field) (4.189)

OC/DA/ST/TC: $$\tau''_{d0}=\frac{L_D-(L_{AD})^2/L_F}{r_D}$$ (D-axis damper) (4.187)

OC/QA/ST/TC: $$\tau''_{q0}=\frac{L_Q}{r_Q}$$ (Q-axis damper) (4.193)

With G-Winding (round rotor machine):

OC/DA/T/TC: $$\tau'_{d0}=\frac{L_F}{r_F}$$ (D-axis field) (4.189)

OC/DA/ST/TC: $$\tau''_{d0}=\frac{L_D-(L_{AD})^2/L_F}{r_D}$$ (D-axis damper) (4.187)

OC/QA/ST/TC: $$\tau''_{q0}=\frac{L_Q-L_{AQ}^2/L_G}{r_Q}$$ (Q-axis damper) (4.192a)

OC/QA/T/TC: $$\tau'_{q0}=\frac{L_G}{r_G}$$ (Q-axis field) (4.192a)

In the last equation, L_G may be obtained as follows :

$$L_q'=\left[L_q-\frac{L_{AQ}^2}{L_G}\right]\Rightarrow L_G=\frac{L_{AQ}^2}{L_q-L_q'}=\frac{\left(L_q-\ell_q\right)^2}{L_q-L_q'}$$

In the above
OC : Open-circuit
DA : direct-axis
QA : quadrature axis
T : transient
ST : subtransient
TC : time constant

The short circuit time constants are as follows:

Without G-winding (salient pole machine)::

SC/DA/T/TC: $$\tau'_d = \tau'_{d0} \frac{L'_d}{L_d}$$ (D-axis field) (4.191)

SC/DA/ST/TC: $$\tau''_d = \tau''_{d0} \frac{L''_d}{L'_d}$$ (D-axis damper) (4.190)

SC/QA/ST/TC: $$\tau''_q = \tau''_{q0} \frac{L''_q}{L_q}$$ (Q-axis damper) (4.192b)

With G-Winding (round-rotor machine)::

SC/DA/T/TC: $$\tau'_d = \tau'_{d0} \frac{L'_d}{L_d}$$ (D-axis field) (4.191)

SC/DA/ST/TC: $$\tau''_d = \tau''_{d0} \frac{L''_d}{L'_d}$$ (D-axis damper) (4.190)

SC/QA/ST/TC: $$\tau''_q = \tau''_{q0} \frac{L''_q}{L_q}$$ (Q-axis damper) (4.192b)

SC/QA/ST/TC: $$\tau'_q = \tau'_{q0} \frac{L'_q}{L_q}$$ (Q-axis damper) (4.192b)

It is useful at this point to take note of the following from p. 132 of VMAF, where it says,

"Before we examine the *q* axis inductances, some clarification of the circuits that may exist in the *q* axis is needed. For a salient pole machine with amortisseur windings, a *q* axis damper circuit exists, but there is no other actual or effective *q* axis rotor winding. For such a machine the stator flux linkage after the initial subtransient dies out is determined by essentially the same circuit as that of the steady-state *q* axis flux linkage. Thus, for a salient pole machine, it is customary to consider the *q* axis transient inductance to be the same as the *q* axis synchronous inductance.

The situation for a round-rotor machine is different. Here the solid iron rotor provides multiple paths for circulating eddy currents, which act as equivalent windings during both transient and subtransient periods. Such a machine will have effective *q* axis rotor circuits that will determine the *q* axis transient and subtransient inductances. Thus, for such a machine, it is important to recognize that a *q* axis transient inductance (much smaller in magnitude than L_q) exists.

==

Another time constant used to characterize synchronous machines is the stator time constant, given by

$$\tau_a = \frac{(L'_d + L'_q)/2}{r}$$

Note that the text uses L_q in the above equation instead of L'_q (since $L_q = L'_q$ when the G-winding is not represented).

Table 4.3, pg. 135 in VMAF, provides a comparison of typical numerical range for time constants. Kundur also provides such a table, Table 4.2, pg. 150. Note transient T >> subtransient T.

Another way to get the time constants is to use the equivalent circuits.

Then derive the inductances in terms of the LaPlace variable “s” according to

$$L_d(s) = \frac{\lambda_d(s)}{i_d(s)}$$

$$L_q(s) = \frac{\lambda_q(s)}{i_q(s)}$$

I will not go through the development here, but you can find it on pp. 140-143 of Kundur’s text.

The denominator of the above expressions is the characteristic equation for the circuit. The roots of this equation are the inverse of the time constants.

This approach makes no approximations, and therefore Kundur refers to the resulting expressions for the parameters as the “accurate expressions.”

The relationship between our nomenclature and that used by Kundur is as follows:

Kundur➔VMAF

L_{ad}➔ L_{AD}

L_{fd}➔ l_F

R_{FD}➔ r_F

R_{1d}➔ r_D

L_{1d}➔l_D

L_l➔ l_d

You can review some of the data in appendix C of your text to see if it conforms to our conclusions about “fast” vs. “slow” circuits.

And you can check Kundur, page 153, for some comparative data for both salient pole and round-rotor machines, which I have copied out below.

Table 4.2

SALIENT Round

Parameter		Hydraulic Units	Thermal Units
Synchronous Reactance	X_d	0.6 - 1.5	1.0 - 2.3
	X_q	0.4 - 1.0	1.0 - 2.3
Transient Reactance	X'_d	0.2 - 0.5	0.15 - 0.4
	X'_q	-	0.3 - 1.0
Subtransient Reactance	X''_d	0.15 - 0.35	0.12 - 0.25
	X''_q	0.2 - 0.45	0.12 - 0.25
Transient OC Time Constant	T'_{d0}	1.5 - 9.0 s	3.0 - 10.0 s
	T'_{q0}	-	0.5 - 2.0 s
Subtransient OC Time Constant	T''_{d0}	0.01 - 0.05 s	0.02 - 0.05 s
	T''_{q0}	0.01 - 0.09 s	0.02 - 0.05 s
Stator Leakage Inductance	X_l	0.1 - 0.2	0.1 - 0.2
Stator Resistance	R_a	0.002 - 0.02	0.0015 - 0.005

Notes:
1. Reactance values are in per unit with stator base values equal to the corresponding machine rated values.
2. Time constants are in seconds.

Similar data is in Chapter 4 of Anderson & Fouad, p. 135: Note Table 4.3 comes from Kimbark, see next page of these notes.

Table 4.3. Typical Synchronous Machine Time Constants in Seconds

Time constant	Turbogenerators			Waterwheel generators			Synchronous condensers		
	Low	Avg.	High	Low	Avg.	High	Low	Avg.	High
τ'_{d0}	2.8	5.6	9.2	1.5	5.6	9.5	6.0	9.0	11.5
τ'_d	0.4	1.1	1.8	0.5	1.8	3.3	1.2	2.0	2.8
$\tau''_d = \tau''_q$	0.02	0.035	0.05	0.01	0.035	0.05	0.02	0.035	0.05
τ_a	0.04	0.16	0.35	0.03	0.15	0.25	0.10	0.17	0.30

Source: Reprinted by permission from *Power System Stability*, vol. 3, by E.W. Kimbark. © Wiley, 1956.

Table 4.4. Typical Turbogenerator and Synchronous Condenser Characteristics

Parameter	Generators		Synchronous condensers	
	Range	Recommended average	Range	Recommended average
Nominal rating	300–1000 MW	...	50–100 MVA	...
Power factor	0.80–0.95	0.90	...	...
Direct axis synchronous reactance x_d	140–180	160	170–270	220
Transient reactance x'_d	23–35	25	45–65	55
Subtransient reactance x''_d	15–23	20	35–45	40
Quadrature axis synchronous reactance x_q	150–160	155	100–130	115
Negative-sequence reactance x_2	18–20	19	35–45	40
Zero-sequence reactance x_0	12–14	13	15–25	20
Short circuit ratio	0.50–0.72	0.64	0.35–0.65	0.50
Inertia constant H, $\frac{(\text{kW}\cdot\text{s})}{(\text{kVA})}$ 3600 r/min	3.0–5.0	4.0	...	...
Inertia constant H, $\frac{(\text{kW}\cdot\text{s})}{(\text{kVA})}$ 1800 r/min	5.0–8.0	6.0	...	...

Source: From the 1964 National Power Survey made by the U.S. Federal Power Commission. USGPO.

Note: All reactances in percent on rated voltage and kVA base. kW losses for typical synchronous condensers in the range of sizes shown, excluding losses associated with step-up transformers, are in the order of 1.2–1.5% on rated kVA base. No attempt has been made to show kW losses associated with generators, since generating plants are generally rated on a net power output basis and losses vary widely dependent on the generator plant design.

Table 4.5. Typical Hydrogenerator Characteristics

Parameter	Small units	Large units
Nominal rating (MVA)	0–40	40–200
Power factor	0.80–0.95*	0.80–0.95*
Speed (r/min)	70–350	70–200
Inertia constant H, $\frac{(\text{kW}\cdot\text{s})}{(\text{kVA})}$	1.5–4.0	3.0–5.5
Direct axis synchronous reactance x_d	90–110	80–100
Transient reactance x'_d	25–45	20–40
Subtransient reactance x''_d	20–35	15–30
Quadrature axis synchronous reactance x_q	...	...
Negative-sequence reactance x_2	20–45	20–35
Zero-sequence reactance x_0	10–35	10–25
Short circuit ratio	1.0–2.0	1.0–2.0

Source: From the 1964 National Power Survey made by the U.S. Federal Power Commission. USGPO.

Note: All reactances in percent on rated voltage and kVA base. No attempt has been made to show kW losses associated with generators, since generating plants are generally rated on a net power output basis and losses vary widely dependent on the generator plant design.

*These power factors cover conditions for generators installed either close to or remote from load centers.

And From Kimbark, p. 40 (note time constant data at bottom of table is used in A&F's table, given on previous page).

TYPICAL CONSTANTS OF THREE-PHASE SYNCHRONOUS MACHINES

(Adapted from Refs. 15, 34, and 41)

	Turbo-generators (solid rotor)			Water-Wheel Generators (with dampers)†			Synchronous Condensers			Synchronous Motors (general purpose)		
	Low	*Avg.*	*High*	*Low*	*Avg.*	*High*	*Low*	*Avg.*	*High*	*Low*	*Avg.*	*High*
Reactances in per unit												
x_d	0.95	1.10	1.45	0.60	1.15	1.45	1.50	1.80	2.20	0.80	1.20	1.50
x_q	0.92	1.08	1.42	0.40	0.75	1.00	0.95	1.15	1.40	0.60	0.90	1.10
x_d'	0.12	0.23	0.28	0.20	0.37	0.50‡	0.30	0.40	0.60	0.25	0.35	0.45
x_q'	0.12	0.23	0.28	0.40	0.75	1.00	0.95	1.15	1.40	0.60	0.90	1.10
x_d''	0.07	0.12	0.17	0.13	0.24	0.35	0.18	0.25	0.38	0.20	0.30	0.40
x_q''	0.10	0.15	0.20	0.23	0.34	0.45	0.23	0.30	0.43	0.30	0.40	0.50
x_p	0.07	0.14	0.21	0.17	0.32	0.40	0.23	0.34	0.45			
x_2	0.07	0.12	0.17	0.13	0.24	0.35	0.17	0.24	0.37	0.25	0.35	0.45
x_0*	0.01		0.10	0.02		0.21	0.03		0.15	0.04		0.27
Resistances in per unit												
r_a(d-c.)	0.0015		0.005	0.003		0.020	0.002		0.015			
r(a-c.)	0.003		0.008	0.003		0.015	0.004		0.010			
r_2	0.025		0.045	0.012		0.20	0.025		0.07			
Time constants in seconds												
T_{d0}'	2.8	5.6	9.2	1.5	5.6	9.5	6.0	9.0	11.5			
T_d'	0.4	1.1	1.8	0.5	1.8	3.3	1.2	2.0	2.8			
$T_d'' = T_q''$	0.02	0.035	0.05	0.01	0.035	0.05	0.02	0.035	0.05			
T_a	0.04	0.16	0.35	0.03	0.15	0.25	0.1	0.17	0.3			

*x_0 varies from about 0.15 to 0.60 of x_d'', depending upon winding pitch.

†For water-wheel generators without damper windings,

$$x_d'' = 0.85x_d', \qquad x_q'' = x_q' = x_q, \qquad x_2 = (x_d' + x_q)/2,$$

and x_0 is as listed.

‡For curves showing the normal value of x_d' of water-wheel-driven generators as a function of kilovolt-ampere rating and speed, see Ref. 54.