Time Constants (Section 4.14.1)

In a linear static circuit with no capacitance, i.e., an R-L circuit, the transient currents decay with time according to

$$i(t) = i_0 e^{-t/T} \tag{1}$$

where i_0 is the initial current and T is the time constant.
For an R-L circuit, we may show that

$$i(t) = \frac{1}{R} e^{-t/(L/R)} \tag{2}$$

where we see that T=L/R. How do we think of T?

Let t=T and then we get that

$$i(t) = i_0 e^{-T/T} = i_0 e^{-1} = 0.368 i_0 \tag{3}$$

Thus we see that the time constant is:

1. The time in which the current decreases to 36.8% of its initial value OR
2. The time in which the current decrease equals 63.2% of its initial value.

A third way to understand T is that it is the time in which the current would decrease to zero if it continued to decrease at its initial rate of decrease. Figure 1 illustrates this.

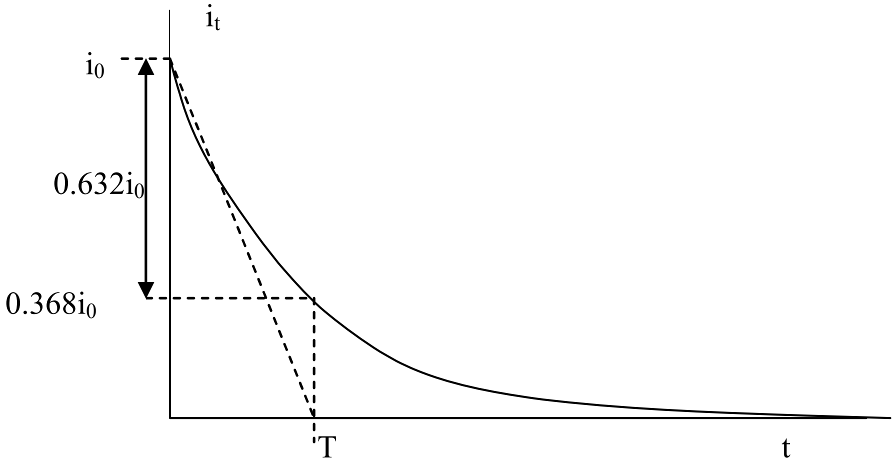

Fig. 1

So the time constant is a very good measure of the speed of the dynamics.
Low T ➔ fast dynamics.

For a salient pole machine, we have a time constant for each rotor circuit given as the ratio of some inductance to the circuit resistance.

We can get the time constants under one of two conditions:

1. Stator is open-circuited
2. Stator is short-circuited.

The procedure used in the text for developing these equations is as follows (see pp 124-125):

1. Write the voltage equation for the appropriate circuit. This equation will include a flux derivative. Use eq. (4.36') given below

$$\begin{bmatrix} v_d \\ v_q \\ -v_F \\ v_D = 0 \\ v_Q = 0 \\ v_G = 0 \end{bmatrix} = -\begin{bmatrix} r & 0 & 0 & 0 & 0 & 0 \\ 0 & r & 0 & 0 & 0 & 0 \\ 0 & 0 & r_F & 0 & 0 & 0 \\ 0 & 0 & 0 & r_D & 0 & 0 \\ 0 & 0 & 0 & 0 & r_Q & 0 \\ 0 & 0 & 0 & 0 & 0 & r_G \end{bmatrix} \begin{bmatrix} i_d \\ i_q \\ i_F \\ i_D \\ i_Q \\ i_G \end{bmatrix} + \begin{bmatrix} -\omega\lambda_q \\ \omega\lambda_d \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} \dot{\lambda}_q \\ \dot{\lambda}_d \\ \dot{\lambda}_F \\ \dot{\lambda}_D \\ \dot{\lambda}_Q \\ \dot{\lambda}_G \end{bmatrix}$$

   For example, the v_F and v_D equations is:

   $$v_F = r_F i_F + \dot{\lambda}_F \qquad (4.181a)$$

   $$v_D = 0 = r_D i_D + \dot{\lambda}_D \qquad (4.181b)$$

   We assume a step change is applied to the field winding (with the stator winding open or short-circuited, it is the only way we can provide an external forcing function.

2. Use eqt. 4.20' (see handout titled "macheqts") to replace fluxes with currents.

$$\begin{bmatrix}\lambda_0\\ \lambda_d\\ \lambda_q\\ \lambda_F\\ \lambda_D\\ \lambda_Q\\ \lambda_G\end{bmatrix}=\begin{bmatrix}L_0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & L_d & 0 & \sqrt{\frac{3}{2}}M_F & \sqrt{\frac{3}{2}}M_D & 0 & 0\\ 0 & 0 & L_q & 0 & 0 & \sqrt{\frac{3}{2}}M_Q & \sqrt{\frac{3}{2}}M_G\\ 0 & \sqrt{\frac{3}{2}}M_F & 0 & L_F & M_R & 0 & 0\\ 0 & \sqrt{\frac{3}{2}}M_D & 0 & M_R & L_D & 0 & 0\\ 0 & 0 & \sqrt{\frac{3}{2}}M_Q & 0 & 0 & L_Q & M_Y\\ 0 & 0 & \sqrt{\frac{3}{2}}M_G & 0 & 0 & M_Y & L_G\end{bmatrix}\begin{bmatrix}i_0\\ i_d\\ i_q\\ i_F\\ i_D\\ i_Q\\ i_G\end{bmatrix} \qquad (4.20')$$

For example, we see that

$$\lambda_F=\sqrt{\frac{3}{2}}M_F i_d+L_F i_F+M_R i_D \qquad (*)$$

$$\lambda_D=\sqrt{\frac{3}{2}}M_D i_d+M_R i_F+L_D i_D \qquad (**)$$

3. Apply appropriate open circuit or short circuit conditions to simplify the equation. For example, if we are getting the open circuit time constants, then the stator windings are open circuited, and i_d=0. This causes (*) and (**) to become

   $$\lambda_F=L_F i_F+M_R i_D \qquad (4.182a)$$

   $$\lambda_D=M_R i_F+L_D i_D \qquad (4.182b)$$

   Notice that from (4.182b), for a step change applied to the field voltage, CFLT indicates that $\lambda_D(0^+)=0$, which implies that

   $$0=M_R i_F+L_D i_D \Rightarrow i_F=\frac{-L_D}{M_R}i_D \qquad (4.183)$$

4. Differentiate (4.182a) and (4.182b) and then substitute into (4.181a) and (4.181b), respectively. This results in

   $$v_F=r_F i_F+L_F\dot{i}_F+M_R\dot{i}_D \qquad (4.184a)$$

   $$0=r_D i_D+M_R\dot{i}_F+L_D\dot{i}_D \qquad (4.184b)$$

   Divide (4.184a) by L_F and (4.184b) by M_R to get

   $$\frac{v_F}{L_F}=\frac{r_F}{L_F}i_F+\dot{i}_F+\frac{M_R}{L_F}\dot{i}_D \qquad (4.184c)$$

   $$0=\frac{r_D}{M_R}i_D+\dot{i}_F+\frac{L_D}{M_R}\dot{i}_D \qquad (4.184d)$$

   Subtract (4.184c) from (4.184d) to get

$$\frac{r_D}{M_R} i_D - \frac{r_F}{L_F} i_F + \left(\frac{L_D}{M_R} - \frac{M_R}{L_F} \right) \dot{i}_D = \frac{-v_F}{L_F}$$

Now replace i_F with (4.183) to get

$$\left(\frac{r_D}{M_R} + \frac{r_F}{L_F} \frac{L_D}{M_R} \right) i_D + \left(\frac{L_D}{M_R} - \frac{M_R}{L_F} \right) \dot{i}_D = \frac{-v_F}{L_F}$$

Now divide through by the coefficient of the derivative term:

$$\frac{\left(\frac{r_D}{M_R} + \frac{r_F}{L_F} \frac{L_D}{M_R} \right)}{\left(\frac{L_D}{M_R} - \frac{M_R}{L_F} \right)} i_D + \dot{i}_D = \frac{\frac{-v_F}{L_F}}{\left(\frac{L_D}{M_R} - \frac{M_R}{L_F} \right)}$$

Multiply top and bottom of the first term on the left-hand-side by M_R, and multiply the right-hand-side by M_R to get

$$\frac{\left(r_D + \frac{r_F L_D}{L_F} \right)}{\left(L_D - M_R^2 / L_F \right)} i_D + \dot{i}_D = \frac{-M_R v_F / L_F}{\left(L_D - M_R^2 / L_F \right)}$$

5. Use approximations as necessary. The approximations employed are:
   - In computing transient quantities, the damper circuits are assumed to be infinitely fast, which makes $r_D=\infty$ or $r_Q=\infty$.
   - In computing subtransient quantities, the field circuits are assumed to be very slow, which makes $r_F=0$ or $r_G=0$.

   In our example, A&F (pg. 125) make the statement that "usually in pu $r_D>>r_F$ while L_D and L_F are of similar magnitude." This means $r_D>>r_F L_D/L_F$, and so the above becomes

$$\frac{r_D}{\left(L_D - M_R^2 / L_F \right)} i_D + \dot{i}_D = \frac{-M_R v_F / L_F}{\left(L_D - M_R^2 / L_F \right)}$$

   Rearranging, we obtain

$$\dot{i}_D + \frac{r_D}{\left(L_D - M_R^2 / L_F \right)} i_D = -v_F \frac{M_R / L_F}{\left(L_D - M_R^2 / L_F \right)} \qquad (4.186a)$$

   Now define

$$K_1 = \frac{r_D}{\left(L_D - M_R^2 / L_F\right)} \qquad K_2 = -v_F \frac{M_R / L_F}{\left(L_D - M_R^2 / L_F\right)}$$

Then (4.186a) becomes

$$\dot{i}_D + K_1 i_D = K_2 \tag{4.186b}$$

Using LaPlace transforms, we get

$$\begin{aligned} &sI_D(s) + K_1 I_D(s) = K_2 \\ &I_D(s)\left(s + K_1\right) = K_2 \\ &I_D(s) = \frac{K_2}{s + K_1} \end{aligned} \qquad (4.186c)$$

Taking inverse LaPlace transform

$$i_D(t) = K_2 e^{-K_1 t}$$

Replacing K_1 and K_2, we obtain

$$i_D(t) = K_2 e^{-K_1 t} \tag{4}$$

Recall

$$i(t) = \frac{1}{R} e^{-t/(L/R)} \tag{2}$$

where T=L/R, and so we see that $1/K_1$ is a time constant. We define this time constant as the open circuit subtransient time constant, i.e.

$$\tau''_{d0} = \frac{1}{K_1} = \frac{L_D - M_R^2 / L_F}{r_D}$$

It's name comes from the fact that

- it is computed when the stator windings are *open circuit*,
- it characterizes the behavior of the D-winding and is therefore a *subtransient* response.

Application of similar procedures results in the expressions that Kundur calls the "classical expressions" given as follows:

Without G-winding (salient pole machine):

OC/DA/T/TC: $\tau'_{d0} = \dfrac{L_F}{r_F}$ (D-axis field)

OC/DA/ST/TC: $\tau''_{d0} = \dfrac{L_D - (L_{AD})^2 / L_F}{r_D}$ (D-axis damper)

OC/QA/ST/TC: $\tau''_{q0} = \dfrac{L_Q}{r_Q}$ (Q-axis damper)

With G-Winding (round rotor machine):

OC/DA/T/TC: $\tau'_{d0} = \dfrac{L_F}{r_F}$ (D-axis field)

OC/DA/ST/TC: $\tau''_{d0} = \dfrac{L_D - (L_{AD})^2 / L_F}{r_D}$ (D-axis damper)

OC/QA/ST/TC: $\tau''_{q0} = \dfrac{L_Q}{r_Q}$ (Q-axis damper)

OC/QA/T/TC: $\tau'_{q0} = \dfrac{L_Q - (L_{AQ})^2 / L_G}{r_G}$ (Q-axis field)

In the above
OC : Open-circuit
DA : direct-axis
QA : quadrature axis
T : transient
ST : subtransient
TC : time constant

The short circuit time constants are as follows:

Without G-winding (salient pole machine)::

SC/DA/T/TC: $\tau'_d = \tau'_{d0} \frac{L'_d}{L_d}$ (D-axis field)

SC/DA/ST/TC: $\tau''_d = \tau''_{d0} \frac{L''_d}{L'_d}$ (D-axis damper)

SC/QA/ST/TC: $\tau''_q = \tau''_{q0} \frac{L''_q}{L_q}$ (Q-axis damper)

With G-Winding (round-rotor machine)::

SC/DA/T/TC: $\tau'_d = \tau'_{d0} \frac{L'_d}{L_d}$ (D-axis field)

SC/DA/ST/TC: $\tau''_d = \tau''_{d0} \frac{L''_d}{L'_d}$ (D-axis damper)

SC/QA/ST/TC: $\tau''_q = \tau''_{q0} \frac{L''_q}{L_q}$ (Q-axis damper)

SC/QA/ST/TC: $\tau'_q = \tau'_{q0} \frac{L'_q}{L_q}$ (Q-axis damper)

Another time constant used to characterize synchronous machines is the stator time constant, given by

$$\tau_a = \frac{(L'_d + L'_q)/2}{r}$$

Note that the text uses L_q in the above equation instead of L'_q (since $L_q = L'_q$ when the G-winding is not represented).

Table 4.3, pg. 126 in your text, provides a comparison of typical numerical range for time constants. Kundur also provides such a table, Table 4.2, pg. 150. Note transient T >> subtransient T.

Another way to get the time constants is to use the equivalent circuits.

Then derive the inductances in terms of the LaPlace variable "s" according to

$$L_d(s) = \frac{\lambda_d(s)}{i_d(s)}$$

$$L_q(s) = \frac{\lambda_q(s)}{i_q(s)}$$

I will not go through the development here, but you can find it on pp. 140-143 of Kundur's text.

The denominator of the above expressions is the characteristic equation for the circuit. The roots of this equation are the inverse of the time constants.

This approach makes no approximations, and therefore Kundur refers to the resulting expressions for the parameters as the "accurate expressions."

The relationship between our nomenclature and that used by Kundur is as follows:

$L_{ad} \rightarrow L_{AD}$
$L_{fd} \rightarrow l_F$
$R_{FD} \rightarrow r_F$
$R_{1d} \rightarrow r_D$
$L_{1d} \rightarrow l_D$
$L_l \rightarrow l_d$