Mechanical Dynamics, the Swing Equation, Units

1.0 Preliminaries

The basic requirement for generator operation is that they must remain "in synchronism." This means that all generators must have mechanical speeds so as to produce the same "electrical speed."

Electrical speed and mechanical speed are related as a function of the number of machine poles, *p*, or pole-pairs, *p/2*.

If *p=2*, as in Fig. 1, then there is one magnetic rotation for every one mechanical rotation, i.e., the stator windings see one flux cycle as the rotor turns once.

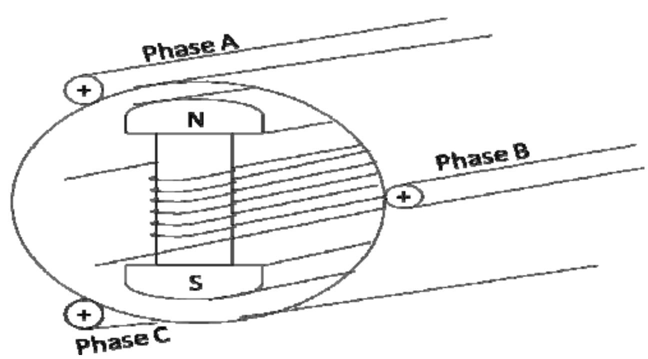


Fig. 1

If p=4, as in Fig. 2, there are two magnetic rotations for every one mechanical rotations, i.e., the stator windings see two flux cycles as the rotor turns once.

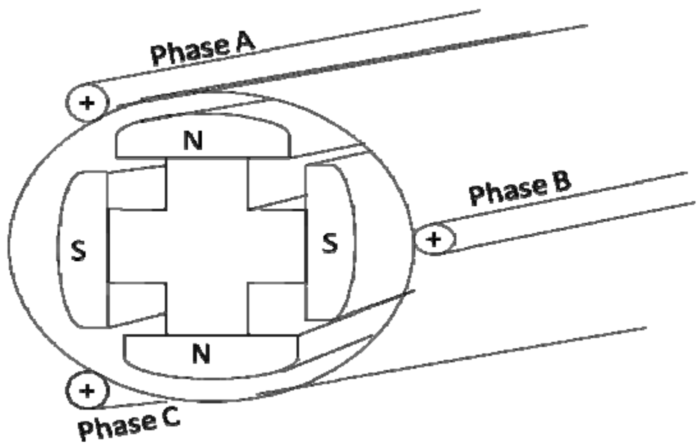


Fig. 2

Therefore, the electrical speed, ω_e, will be greater than (if $p \geq 4$) or equal to (if $p=2$) the mechanical speed ω_m according to the number of pole-pairs $p/2$, i.e.,

$$\omega_e = \frac{p}{2}\omega_m \quad (1)$$

The adjustment for the number of pole-pairs is needed because the electrical quantities (voltage and current) go through one rotation for every one magnetic rotation.

So to maintain synchronized "electrical speed" (frequency) from one generator to another, all generators must maintain constant mechanical speed. This does not mean all generators have the same mechanical speed, but that their mechanical speed must be constant.

All two-pole machines must maintain

$$\omega_m=(2/2)\omega_e=(2/2)377=377 rad/sec$$

We can also identify the mechanical speed of rotation in rpm according to

$$N_m = \omega_m \frac{\text{rad}}{\text{sec}} \times \frac{60\,\text{sec/min}}{2\pi\ \text{rad/rev}} \qquad (2)$$

Substituting for ω_m from (1), we get:

$$N_m = \omega_e \frac{2}{p} \frac{\text{rad}}{\text{sec}} \times \frac{60\,\text{sec/min}}{2\pi\ \text{rad/rev}} \qquad (3)$$

Using this expression, we see that a 2 pole machine will have a mechanical synchronous speed of 3600 rpm, and a 4 pole machine will have a mechanical synchronous speed of 1800 rpm.

2.0 Causes of rotational velocity change

Because of the synchronism requirement, we are concerned with any conditions that will cause a change in rotational velocity.

But what is “a change in rotational velocity”?

➔ It is acceleration (or deceleration).

What are the conditions that cause acceleration (+ or -)?

To answer this question, we must look at the mechanical system to see what kind of “forces” that are exerted on it.

Recall that with linear motion, acceleration occurs as a result of a body experiencing a “net” force that is non-zero. That is,

$$a = \frac{F}{m} \tag{4}$$

where a is acceleration (m/sec^2), F is force (newtons), and m is mass (kg). Here, it is important to realize that F represents the sum of all forces on the body. This is Newton’s second law of motion.

The situation is the same with rotational motion, except that here, we speak of torque *T* (newton-meters), inertia *J* (kg-m^2), and angular acceleration *A* (rad/sec^2) instead of force, mass, and acceleration. Specifically,

$$\mathrm{A} = \frac{T}{J} \tag{5}$$

Here, as with *F* in the case of linear motion, *T* represents the "net" torque, or the sum of all torques acting on the rotational body.

It is conceptually useful to remember that the torque on a rotating body experiencing a force a distance *r* from the center of rotation is given by

$$\vec{T} = \vec{r} \times \vec{F} \tag{6}$$

where $\vec{r}$ is a vector of length *r* and direction from center of rotation to the point on the body where the force is applied, $\vec{F}$ is the applied force vector, and the "×" operation is the vector cross product. The magnitudes are related through

$$T = rF \sin\gamma \tag{7}$$

where γ is the angle between $\vec{r}$ and $\vec{F}$. If the force is applied tangential to the body, then $\gamma=90°$ and $T=rF$.

Let's consider that the rotational body is a shaft connecting a turbine to a generator, illustrated in Fig. 3.

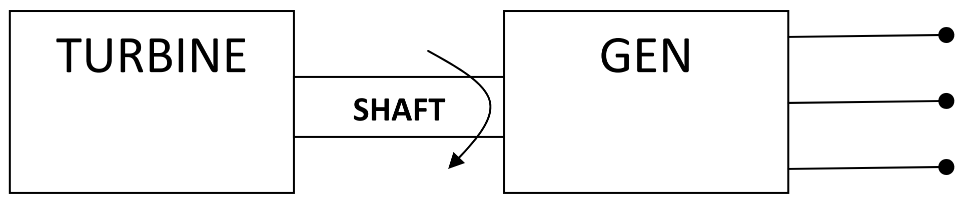


Fig. 3

For purposes of our discussion here, let's assume that the shaft is rigid (inelastic, i.e., it does not flex), and let's ignore frictional torques.

What are the torques on the shaft?

- From turbine: The turbine exerts a torque in one direction (assume the direction shown in Fig. 3) which causes the shaft to rotate. This torque is mechanical. Call this torque T_m.
- From generator: The generator exerts a torque in the direction opposite to the mechanical torque which retards the motion caused by the mechanical torque. This torque is electromagnetic. Call this torque T_e.

These two torques are in opposite directions. If they are exactly equal, there can be no angular acceleration, and this is the case when the machine is in synchronism, i.e.,

$$T_m = T_e \tag{8}$$

When (8) does not hold, i.e., when there is a difference between mechanical and electromagnetic torques, the machine accelerates (+ or -), i.e.,. it will change its velocity. The amount of acceleration is proportional to the difference between T_m and T_e. We will call this difference the accelerating torque T_a, i.e.,

$$T_a = T_m - T_e \tag{9}$$

The accelerating torque is defined positive when it produces acceleration in the direction of the applied mechanical torque, i.e., when it increases angular velocity (speeds up).

Now we can ask our original question (page 4) in a somewhat more rigorous fashion:

- Given that the machine is initially operating in synchronism ($T_m=T_e$), what conditions can cause $T_a \neq 0$?

There are two broad types of changes: change in T_m and change in T_e. We examine both of these carefully.

1. Change in T_m:
    a. Intentionally: through change in steam valve opening, with T_m either increasing or decreasing.
    b. Disruption in steam flow: typically a decrease in T_m causing the generator to experience negative acceleration (it would decelerate).
2. Change in T_e:
    a. Increase in load: this causes an increase in T_e, and the generator experiences negative acceleration.
    b. Decrease in load: this causes a decrease in T_e, and the generator experiences positive acceleration.

All of the above changes, 1-a, 1-b, 2-a, and 2-b are typically rather slow, and the generator's turbine-governor will sense the change in speed and compensate by changing the steam flow appropriately.

There is a third way that T_e can change, that is not slow.

c. Faults: We discuss this in the next section.

3.0 Generator under faulted conditions: qualitative

Consider the circuit of Fig. 4.

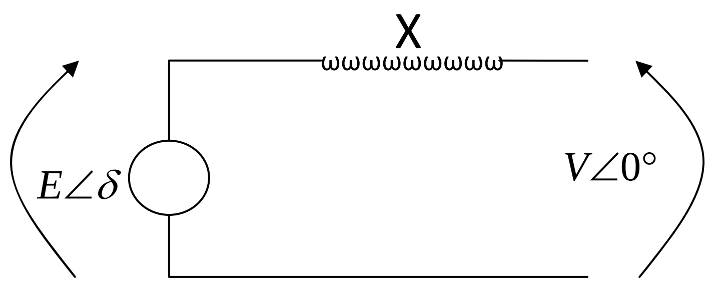


Fig. 4

Here, the voltage $E\angle\delta$ represents the internal voltage of a synchronous machine and the voltage $V\angle 0$ represents the terminal voltage of the machine. We are assuming balanced conditions and therefore we utilize the per-phase equivalent circuit for analysis of the three phase machine.

Assuming a round-rotor machine, we may apply $S = \bar{V}\,\bar{I}^*$, express $\bar{I}$ in terms of the two voltages using Ohm's law, and then take the real part to show that the steady-state real power supplied at the machine terminals is given by

$$P_e = \frac{EV}{X}\sin\delta \tag{10}$$

Let’s assume that a three-phase fault occurs at the machine terminals, so that V=0.

Then clearly, by (10), P_e=0.

Recall that torque and power are related by

$$T_e = \frac{P_e}{\omega_m} \tag{11}$$

And so if P_e=0, it must be the case also that T_e=0.

By (9), then $T_a=T_m$, which means that **<u>all</u>** mechanical torque is being used to accelerate the machine. This is a very severe situation in that the machine will accelerate at a very high rate.

Of course, faults at the machine terminals are very rare (although they do occur occasionally). Most faults are not so severe in that they occur somewhere in the network rather than at the machine terminals. But even for network faults, the voltage V at the machine terminals is reduced in magnitude, causing P_e and therefore T_e to reduce, causing an imbalance between T_m and T_e and therefore a non-zero accelerating torque T_a.

There are two main influences on the amount of overspeed seen by a synchronous generator under faulted conditions.

- The amount of reduction in T_e: The greater is the electrical distance between the fault point in the network and the machine terminals, the less will be the reduction on V, and consequently, the less will be the reduction on P_e (see (10)) and also T_e (see (11)). The fault location is something we cannot control of course. But there is another way to prevent reduction in V, and that is through excitation control. Today's excitation systems are very fast responding so that terminal voltage reduction is sensed and field current is boosted within just a few cycles following a faulted condition.
- Minimize the amount of time that T_e is reduced: This is achieved by removing the faulted condition very quickly. EHV protection systems are typically able to sense and clear a fault within 4 cycles (4/60=.0667 seconds).

This discussion shows that the mechanical dynamics associated with the acceleration of the generator is intimately related to the effect on T_e of the fault. Such effects can only be properly ascertained by analysis of the network before, during, and after the faulted condition.

In the next section, we will therefore derive the relationship between the mechanical dynamics and the electric network.

4.0 Derivation of swing equation

We begin with (5), repeated here for convenience.

$$A = \frac{T}{J} \tag{5}$$

where we recall that T is the "net" torque on the rotating body.

We will write the angular acceleration in terms of the angle θ, which is here defined as the "absolute angle," in radians. It gives the position of a tic-mark on the shaft relative to a fixed point on the generator frame as illustrated in Fig. 5.

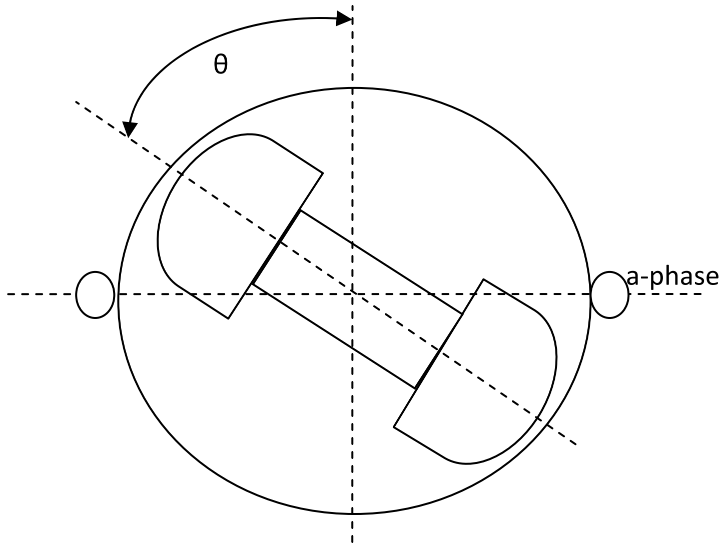


Fig. 5

We can express angular acceleration as $A=\ddot{\theta}=d^2\theta/dt^2$, i.e., angular acceleration is the 2nd time derivative of θ. Noting that T_a is the "net" torque on the turbine-generator shaft, we have

$$\ddot{\theta}=\frac{T_a}{J} \tag{12}$$

Here, J is the moment of inertia of the combined turbine-generator set, in kg-m^2.

We also define ω_R as the rated mechanical angular velocity of the shaft, in rad/sec and note that

$$\omega_R=\frac{\omega_e}{p/2} \tag{13}$$

where *p* is the number of poles, as before.

This allows us to define a synchronously rotating reference frame as:

$$\theta_{ref} = \omega_R t + \alpha \tag{14}$$

where *α* is the initial angle at *t=0* and allows us to position our reference frame wherever it might be convenient for us. We will position it so that it is numerically equal to the angle of the magnetomotive force (mmf) corresponding to the a-phase terminal voltage $v_a(t)$ ($v_a(t)$ is the time-domain representation corresponding to the phasor we have called $\overline{V}$)[1]. This mmf is composed of the mmf produced by the rotor and the mmf produced by all three phase currents (typically called the mmf of armature reaction).

Note from (14), that

$$\dot{\theta}_{ref} = \omega_R \tag{15}$$

[1] We assume here that the a-phase terminal voltage differs from the internal voltage only by the effect of armature reaction, i.e., we neglect leakage flux and armature winding resistance. More discussion on this topic is provided in the book by Machowski, Bialek, and Bumby, "Power System Dynamics and Stability," (1997) pp 61-63.

The implication of (15) is that the reference *speed* is constant, no matter what happens to the rotor.

Let's define the rotor mechanical *torque angle*, δ_m. This is the angle by which the rotor leads the synchronously rotating reference. Since the rotor is in phase with the mmf it produces, this angle is also the angle of the rotor mmf.

Conveniently, recall that each (time varying) mmf is assumed to induce a voltage in the a-phase windings. These voltages are denoted by $e(t)$ and $v_a(t)$ (or by their phasor representations $\overline{E}$ and $\overline{V}$). Since

- The voltages each lag their respective mmfs by 90°,
- The rotor mmf leads the a-phase terminal voltage by δ_m,

then $\overline{E}$ leads $\overline{V}$ by δ_m.

Since the rotor position is designated by θ, and the reference position is designated by θ_{ref}, we have that

$$\delta_m = \theta - \theta_{ref} \qquad (16)$$

The relation between the three defined angles is illustrated in Fig. 6.

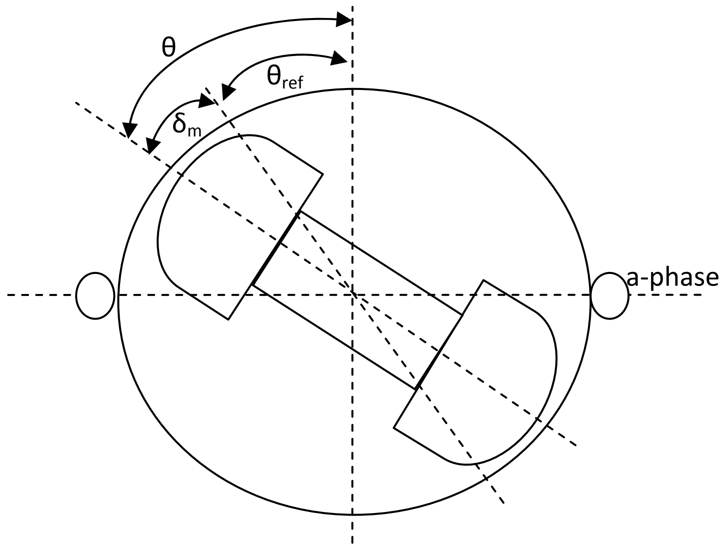


Fig. 6

From (16) we can write that

$$\theta = \theta_{ref} + \delta_m \tag{17}$$

Substituting (14) into (17) yields

$$\theta = \omega_R t + \alpha + \delta_m \tag{18}$$

Observe that under steady-state conditions, θ increases linearly with time in exactly the same way that θ_{ref} increases with time, and therefore δ_m is constant, and so we might rewrite (18) as

$$\theta(t) = \omega_R t + \alpha + \delta_m \quad (18a)$$

However, under transient conditions, because of rotor acceleration, $\delta_m = \delta_m(t)$, and we can rewrite (18) as

$$\theta(t) = \omega_R t + \alpha + \delta_m(t) \quad (18b)$$

Considering the transient condition, by taking the first derivative of (18b), we have:

$$\dot{\theta}(t) = \omega_R + \dot{\delta}_m(t) \quad (19)$$

Differentiating again results in

$$\ddot{\theta}(t) = \ddot{\delta}_m(t) \quad (20)$$

Substituting (20) into (12), repeated here for convenience,

$$\ddot{\theta} = \frac{T_a}{J} \quad (12)$$

results in

$$\ddot{\delta}_m(t) = \frac{T_a}{J} \qquad \Rightarrow \qquad J\ddot{\delta}_m(t) = T_a \quad (21)$$

We observe at this point that all of what we have done is in mechanical radians, and because we have focused on a 2-pole machine, the angles in electrical radians are the same. However, we want to accommodate the general case of a p-pole machine. To do so, recall (1), repeated here for convenience:

$$\omega_e = \frac{p}{2}\omega_m \tag{1}$$

Differentiating, we have

$$\dot{\omega}_e(t) = \frac{p}{2}\dot{\omega}_m(t) \tag{22}$$

which is just

$$\ddot{\delta}_e(t) = \frac{p}{2}\ddot{\delta}_m(t) \tag{23}$$

Substitution of (23) into (21) results in

$$J\frac{2}{p}\ddot{\delta}_e(t) = T_a \tag{24}$$

From now on, we will drop the subscript "'e" on δ and ω with the understanding that both are given in electrical radians. Therefore (24) becomes:

$$\frac{2J}{p}\ddot{\delta}(t) = T_a \tag{25}$$

or since $\dot{\omega} = \ddot{\delta}$,

$$\frac{2J}{p}\dot{\omega}(t) = T_a = T_m - T_e \tag{26}$$

Equation (26) is one form of the swing equation. We shall derive some additional forms in what follows.

5.0 A second form of the swing equation

Because power system analysis is more convenient in per-unit, let's normalize (26) by dividing by a base torque chosen to be

$$T_B = \frac{S_{B3}}{\omega_R} \tag{27}$$

where S_{B3} is a chosen 3-phase MVA rating. Dividing both sides of (26) by T_B results in

$$\frac{2J\omega_R}{pS_{B3}}\dot{\omega}(t) = \frac{T_a}{T_B} = T_{au} \tag{28}$$

We can express the kinetic energy W_K of the turbine-generator set, when rotating at ω_R, as

$$W_k = \frac{1}{2} J \omega_R^2 \tag{29}$$

where the units are watt-seconds or joules.

Solving (29) for J results in

$$J = \frac{2W_k}{\omega_R^2} \tag{30}$$

Substituting (30) into (28) yields

$$\frac{2 \dfrac{2W_k}{\omega_R^2} \omega_R}{pS_{B3}} \dot{\omega}(t) = T_{au} \tag{31}$$

Simplifying:

$$\frac{4W_k}{pS_{B3}\omega_R} \dot{\omega}(t) = T_{au} \tag{32}$$

Let's write one of the 2's in the numerator as a ½ in the denominator, and group it with p and ω_R, yielding

$$\frac{2W_k}{S_{B3}\left(\frac{p}{2}\omega_R\right)}\dot{\omega}(t) = T_{au} \qquad (33)$$

Recalling that ω_R is the mechanical reference speed, the reason for the last step is apparent, because we can now identify what is inside the brackets in the denominator as the electrical reference speed, which we can denote as ω_{Re}. This would be, in North America, 377 rad/sec. Thus, (33) becomes

$$\frac{2W_k}{S_{B3}\omega_{\mathrm{Re}}}\dot{\omega}(t) = T_{au} \qquad (34)$$

Now define the inertia constant:

$$H = \frac{W_k}{S_{B3}} \qquad (35)$$

Here, when S_{B3} has units of MVA, and W_k has units of MW-sec (or Mjoules), then H has units of MW-sec/MVA or seconds.

When S_{B3} is chosen as the generator MVA rating, H falls within a fairly narrow range. I have pulled some numbers from the Appendix D of your text to illustrate.

Unit	S_{mach}	W_K	H_{mach} $=W_k/S_{mach}$	H_{sys} $=W_k/S_{sys}$ $S_{sys}=100$
H1	9	23.5	2.61	0.235
H9	86	233	2.71	2.33
H18	615	3166	5.15	31.7
F1	25	125.4	5.02	1.25
F11	270	1115	4.13	11.15
F21	911	2265	2.49	22.65
CF1-HP	128	305	2.38	3.05
CF1-LP	128	787	6.15	7.87
N1	76.8	281.7	3.67	2.82
N8	1340	4698	3.51	47.0
SC1	25	30	1.2	0.3
SC5	75	89.98	1.2	0.9

Notes:

1. On machine base, H ranges 1-7, but on system base, it ranges w/ machine size.

2. Cross-compound machines (side-by-side turbines, same steam, different gens) have a high LP H because of large blades required by low pressure steam.

3. Synchronous condensers have no turbine and therefore small H.

6.0 A third form of the swing equation

Recall (34):

$$\frac{2W_k}{S_{B3}\omega_{\mathrm{Re}}}\dot{\omega}(t)=T_{au} \tag{34}$$

Substitution of $W_k=HS_{B3}$ (from (35)) results in

$$\frac{2HS_{B3}}{S_{B3}\omega_{\mathrm{Re}}}\dot{\omega}(t)=T_{au} \tag{36}$$

or

$$\frac{2H}{\omega_{\mathrm{Re}}}\dot{\omega}(t)=T_{au} \tag{37}$$

Equation (37) is equation 2.17 in our text.

Some clarifications:

A. Comments on ω_{Re}:

1. It is the rated electrical radian/frequency (377).

2. Your text is confusing on this.

➔*Use of* ω_R in eq. 2.13, 2.14, 2.15, 2.17: it should be ω_{Re}.

B. H must be given on the same base as S_{B3} used to normalize the right-hand side torque.

C. You can convert H's from one base to another as follows:

$$W_k = H_{mach} S_{mach} = H_{sys} S_{sys} \tag{38}$$

$$H_{sys} = H_{mach} \frac{S_{mach}}{S_{sys}} \tag{39}$$

7.0 Comments on Inertia

We make three additional comments about representing inertia in the swing equation.

7.1 Use of *M* for inertia

Another quantity often used in the literature for inertia is *M*, the angular momentum at rated speed, where

$$M = J\omega_R \tag{40}$$

We can see how *M* is related to the kinetic energy according to the following. The kinetic energy of the rotor at speed ω_R is

$$W_k = \frac{1}{2}J\omega_R^2 \tag{41}$$

Solving for J in (40),

$$J = \frac{M}{\omega_R} \tag{42}$$

Substituting (42) into (41) yields

$$W_k = \frac{1}{2}\frac{M}{\omega_R}\omega_R^2 = \frac{1}{2}M\omega_R \tag{43}$$

Also, from

$$H = \frac{W_k}{S_{B3}} \tag{35}$$

and substituting (43) into (35) results in

$$H = \frac{\frac{1}{2}M\omega_R}{S_{B3}} \tag{44}$$

which, when solved for *M*, results in

$$M = \frac{2HS_{B3}}{\omega_R} \tag{45}$$

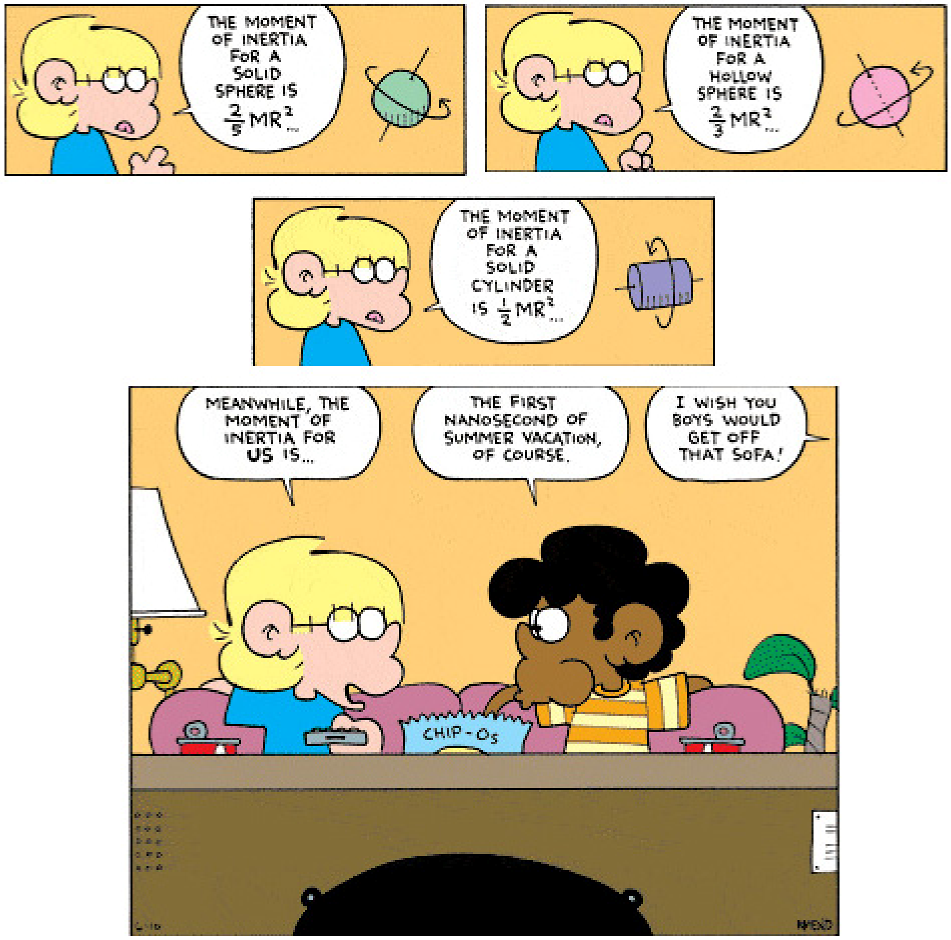
Two additional issues to note here:

1. A different "*M*" is sometimes used in the literature to denote the "mechanical starting time." This is the total time required to accelerate the unit from standstill to rated speed ω_R if rated torque (T_{au}=1.0) is applied as a step function at t=0. I will denote this as T_4, nomenclature that is consistent with the Anderson & Fouad text (see page 450). Kundur, in his book on page 132, shows that this time, in seconds, is given by

$$T_4 = 2H \tag{46}$$

where H is given on the machine base.

2. The three constants M, H, and W_k, are defined at the particular angular velocity of ω_R. However, the machine speed ω_m does deviate from ω_R during the transient conditions for which we are interested to study. Therefore, to be rigorous, we should define M, H, and W_k relative to the machine speed $\omega_m(t)$ so that M, H, and W_k vary with time. However, this considerably complicates the swing equation, and does so with negligible improvement in accuracy, since ω_m, although time varying during disturbance conditions, does not deviate much from ω_R. On the other hand, the moment of inertia J is an actual constant, i.e., it is a function of only the machine geometry and mass and does not depend on speed.



7.2 W-R Squared

Another constant that is often used by manufacturers (and it will be, usually, what you get from a manufacturer) is the "W-R-squared," denoted WR^2, which is the moment of inertia expressed in English units of lb(m)-ft^2:

$$WR^2 = [\underbrace{\text{mass of rotating parts}}_{\text{lb(m)}}][\underbrace{\text{radius of gyration}}_{\text{ft}}]^2 \qquad (47)$$

The radius of gyration is a root-mean-square average distance of all parts of the rotating object from its axis of rotation.

The conversion of units may be obtained so that the moment of inertia, *J*, in kg-m^2, is

$$J = WR^2 \text{ lb(m)ft}^2 \frac{0.4536\text{ kg}}{\text{lb(m)}}\left(\frac{0.3048\text{ m}}{\text{ft}}\right)^2 = 0.0421 \times WR^2 \quad (48)$$

We may also relate the kinetic energy at rated speed, W_k, to WR^2, by substituting (48) into the expression for W_k:

$$W_k = \frac{1}{2} J\omega_R^2 = \frac{1}{2}(0.0421 WR^2)\omega_R^2 = 0.2105(WR^2)(\omega_R^2) \quad (49)$$

where W_k is given in joules. If we wanted to write (49) as a function of RPM instead of rad/sec, where $\omega_r = 2\pi n_R/60$,

$$W_k = 0.2105(WR^2)(2\pi n_R^2 / 60) = 2.31 \times 10^{-4}(WR^2)(n_R^2) \quad (50)$$

where again W_k is in joules (this is the same as the equation at the top of pg 16 in your text). Expressing W_k in Mjoules=MW-sec, eqts. (49) and (50) become

$$W_k = 2.105 \times 10^{-8}(WR^2)(\omega_R^2) = 2.31 \times 10^{-10}(WR^2)(n_R^2) \quad (51)$$

7.3 Summing up

Remember, inertia should account for all masses on the shaft. This will always be the turbine and generator, but it may or may not include an exciter (depends on whether the machine utilizes a rotating exciter or not and whether that rotating exciter is mounted on the same shaft or not).

Also remember that we have five forms in which inertia can be expressed:

$$J, W_k, H, M, \text{ and } WR^2$$

You should be able to convert from any one form to any other form.